Institut de Physique de Nice

Interaction and disorder :

Light scattering by cold atoms:

Localization and Cooperative Scattering

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Lecture 1 : **Multiple Scattering of Light in Cold Atoms**

Steady State Results : Ohm's Law for Photons
Time dependent scattering : radiation trapping
+ Numerical Random Walk Simulations

Lecture 2 : **Interference Effects in Light Scattering by Cold Atoms**

Coherent Backscattering of Light by Cold Atoms
+Numerical Simulations with Weak Localization Corrections
Dicke Super- and Subradiance
+Numerical Simulations with Coupled Dipoles

Lecture 3 : **Anderson Localisation of Light**

Anderson Lattice Model
Effective Hamiltonian Approach
Scalar vs vectorial light : red light for Anderson localization
Outlook : towards localization of light in cold atoms

Link to Mathlab codes :

http://www.kaiserlux.de/coldatoms/LesHouches2019Kaiser.html

Lecture 3 : Anderson Localisation of light

3.1 Anderson Localisation
3.2 Effective Hamiltonian Model
3.3 Scalar vs vectorial light : red light for Anderson localization
3.4 Routes towards Anderson localisation

PHYSICAL REVIEW VOLUME 109, NUMBER 5 MARCH 1, 1958

Absence of Diffusion in Certain Random Lattices

P. W. ANDERSON
Bell Telephone Laboratories, Murray Hill, New Jersey
(Received October 10, 1957)

This paper presents a simple model for such processes as spin diffusion or conduction in the "impurity band." These processes involve transport in a lattice which is in some sense random, and in them diffusion is expected to take place via quantum jumps between localized sites. In this simple model the essential randomness is introduced by requiring the energy to vary randomly from site to site. It is shown that at low enough densities no diffusion at all can take place, and the criteria for transport to occur are given.

I. INTRODUCTION

A NUMBER of physical phenomena seem to involve quantum-mechanical motion, without any particular thermal activation, among sites at which the mobile entities (spins or electrons, for example) may be localized. The clearest case is that of spin diffusion[1,2]; another might be the so-called impurity band conduction at low concentrations of impurities. In such situations we suspect that transport occurs not by motion of free carriers (or spin waves), scattered as they move through a medium, but in some sense by quantum-mechanical jumps of the mobile entities from site to site. A second common feature of these phenomena is randomness: random spacings of impurities, random interactions with the "atmosphere" of other impurities, random arrangements of electronic or nuclear spins, etc.

Our eventual purpose in this work will be to lay the foundation for a quantum-mechanical theory of transport problems of this type. Therefore, we must start with simple theoretical models rather than with the complicated experimental situations on spin diffusion or impurity conduction. In this paper, in fact, we attempt only to construct, for such a system, the simplest model we can think of which still has some expectation of representing a real physical situation reasonably well, and to prove a theorem about the model. The theorem is that at sufficiently low densities, transport does not take place; the exact wave functions are localized in a small region of space. We also obtain a fairly good estimate of the critical density at which the theorem fails. An additional criterion is that the forces be of sufficiently short range—actually, falling off as $r \rightarrow \infty$ faster than $1/r^3$—and we derive a rough estimate of the rate of transport in the $V \propto 1/r^3$ case.

Such a theorem is of interest for a number of reasons: first, because it may apply directly to spin diffusion among donor electrons in Si, a situation in which Feher[3] has shown experimentally that spin diffusion is negligible; second, and probably more important, as an example of a real physical system with an infinite number of degrees of freedom, having no obvious oversimplification, in which the approach to equilibrium is simply impossible; and third, as the irreducible minimum from which a theory of this kind of transport, if it exists, must start. In particular, it re-emphasizes the caution with which we must treat ideas such as "the thermodynamic system of spin interactions" when there is no obvious contact with a real external heat bath.

The simplified theoretical model we use is meant to represent reasonably well one kind of experimental situation: namely, spin diffusion under conditions of

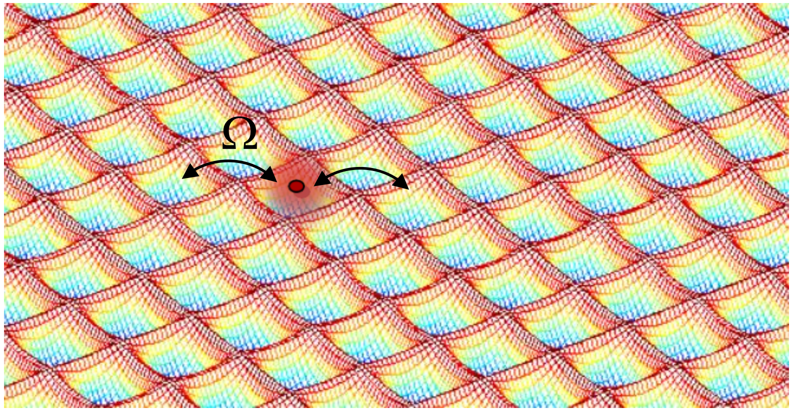
[1] N. Bloembergen, Physica 15, 386 (1949).
[2] A. M. Portis, Phys. Rev. 104, 584 (1956).
[3] G. Feher (private communication).

'The' Anderson lattice Model

3D Anderson model on N x N x N lattice
on site disorder E_j (width W) + Hopping/tunneling (Ω)

'diagonal' 'off-diagonal'

$$H_0 = \sum_{j=1}^{N} E_j |j\rangle\langle j| + \Omega \sum_{\langle i,j\rangle} (|j\rangle\langle i| + |i\rangle\langle j|)$$



$E_j \in [-W/2, W/2]$

W=0 : Bloch bands : extended states

Anderson transition in 3 D : all states are localized for W/Ω > 16.5

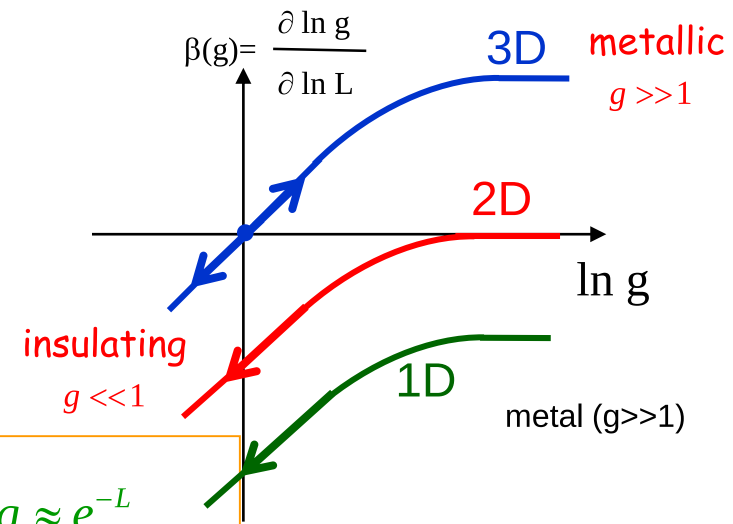
W → ∞ : 'trivial' limit: states localized on single sites

Anderson Localization in 1,2 and 3D

- **Scaling theory of localization :**

Abrahams et al., PRL **42**, 673 (1979)

$g = G / (e^2/\hbar)$: dimensionless conductance



1D : } all states are localized
2D : }

3D : threshold for disorder

$$g \approx e^{-L}$$

$$\frac{\partial \ln g}{\partial \ln L} \approx \ln g$$

$g >> 1$: metal

$g << 1$: insulator

Scaling theory of localization: $$\frac{d \ln g}{d \ln L} = \beta(g)$$

Insulator: $g \propto e^{-L} \Rightarrow \beta(g) \propto \ln g$

Metal: Ohm's law $g \propto L^{d-2} \Rightarrow \beta(g) = d-2$

3D : Quantum Phase Transition

Anderson Localization of non interacting waves in 1,2 and 3D

- **Scaling theory of localization :** Abrahams et al., PRL **42**, 673 (1979)

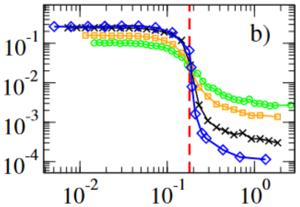
  In 3D : threshold for disorder
  In 1D&2D : all states are localized (in infinite system, ε disorder)

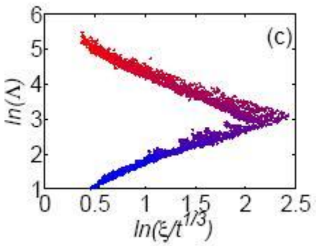
  Return probability needed for interference

- **No microscopic theory**
  - self consistent theory of localization
  - numerical simulations of toy systems

P. Wölfle, D. Vollhardt,
Int. J. Mod. Phys. B 24, 1526 (2010)

- **Many, many tools for data anaylsis**
  - eigenvalue statistics
  - partition ratio
  - finite size scaling
  - critical exponents : universality classes
  - multifractal behavior ,,,,

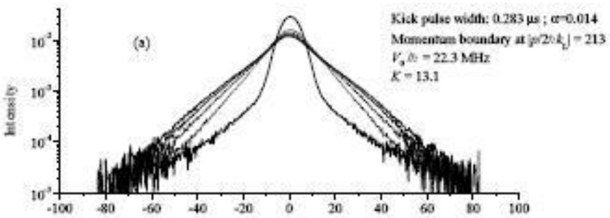




Anderson Localization in 1D

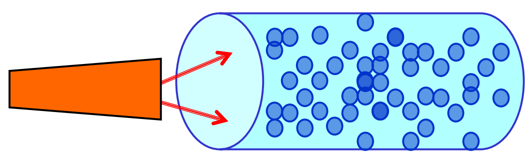
- Cold Atoms : Kicked Rotator

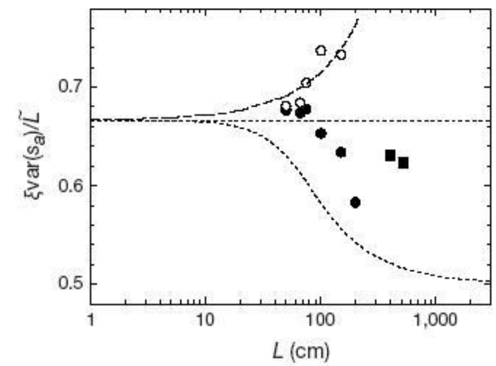
F. Moore et al., PRL **73**, 2974 (1994)



- Quasi 1D : microwave experiments :
  (A. Genack, enhanced fluctuations)

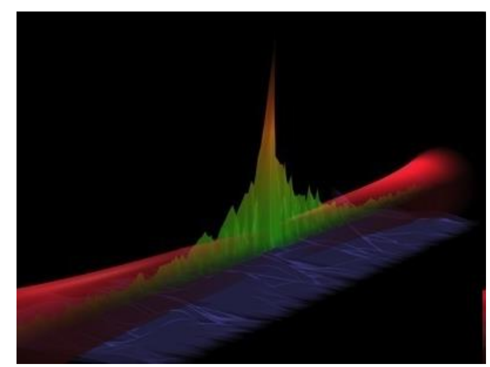
M. Stoytchev et al., PRL **79**, 309 (1997)





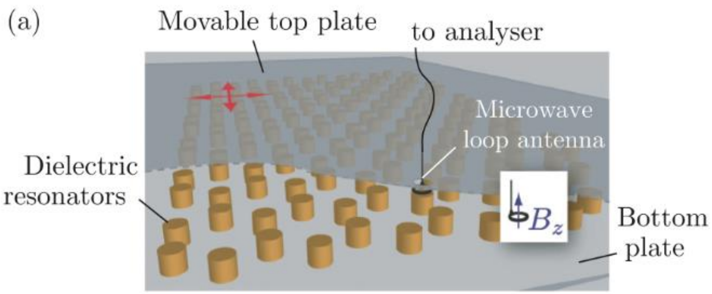
- 1D : Bose Einstein Condensate :
  (looking at localized states)

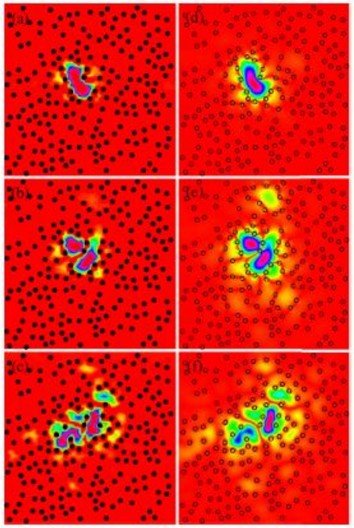
J. Billy et al. , Nature **453**, 891(2008)
G. Roati et al., Nature **453**, 895 (2008)



Anderson Localization in 2D

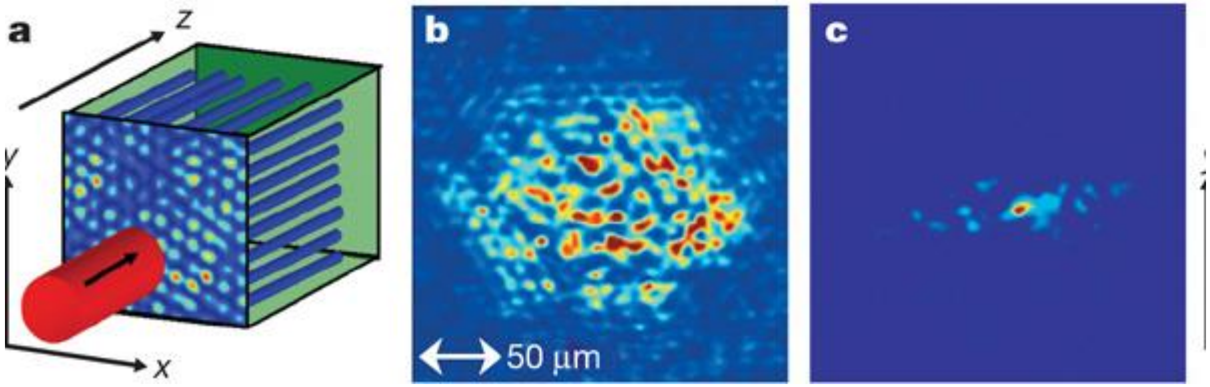
- 2D : microwave cavity





D. Laurent et al., Phys. Rev. Lett. **99,** 253902 (2007)

- Transport through 2D Photonic Crystals



T. Schwartz et al., Nature **446**, 52 (2007)

Anderson Localization in 3D :

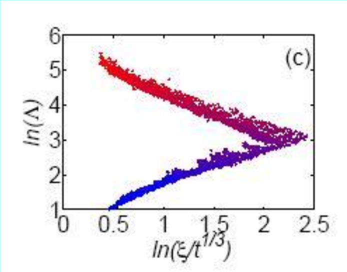
phase transition ⇒ strong scattering required

Acoustics

H.Hu et al., Nature 2008

Connected network
of beads

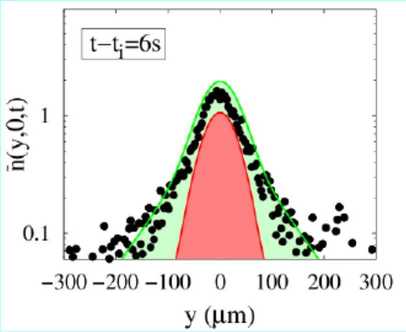
Cold Atoms



J.Chabé et al., PRL 2008

Modulated
1D kicked rotor
= 3D analogue

Matter Waves



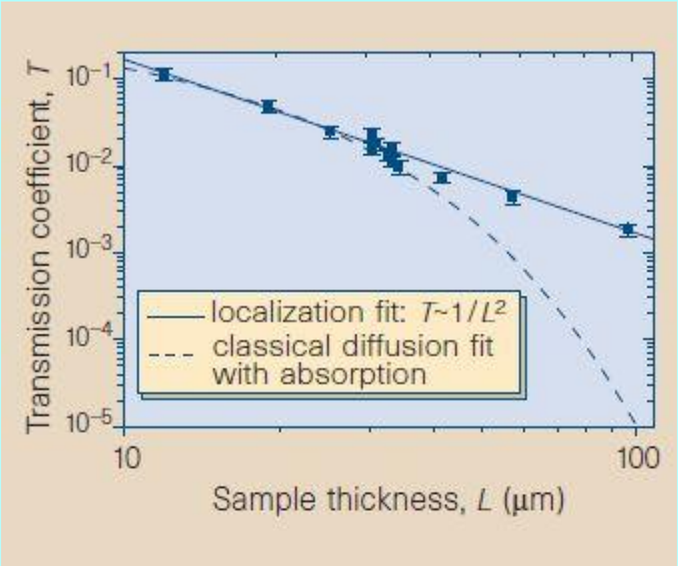
F. Jendrzejewski et al.,
Nat. Phys. 2012

Matter Wave
in 3D speckle field

Anderson Localization of Light in 3D :

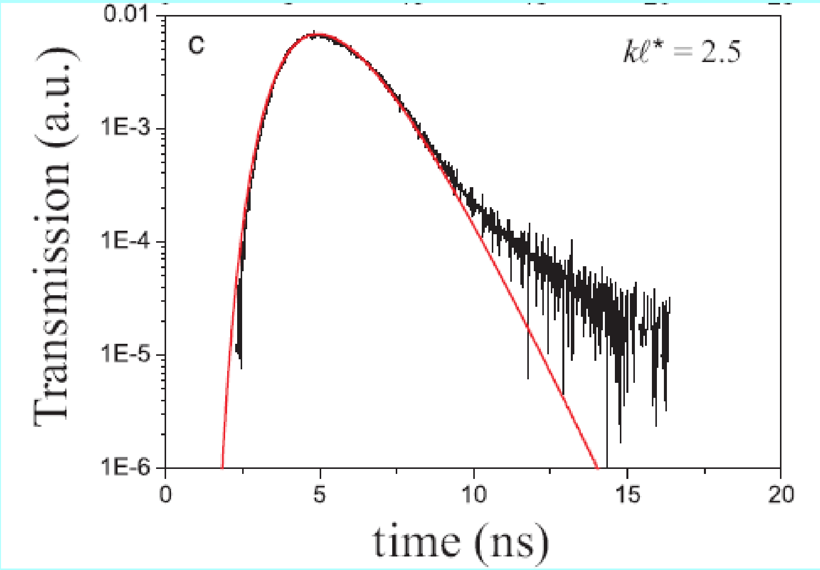
phase transition ⇒ strong scattering required

Semi-conductor powder



D.Wiersma et al., Nature 1997

White Paint



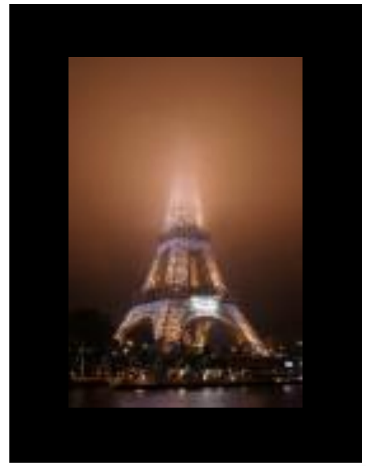
C.Aegerter et al., EPL 2006

F. Scheffold et al., Nature 398, 206(1999)
T. v. der Beek et al., PRB 85 115401 (2012)

F. Scheffold et al., Nat. Photon. 7, 934 (2013)
T Sperling et al., New J. Phys. 18, 013039 (2016)

=> Not observed so far

Multiple Scattering of Light in Atomic samples : Disorder vs cooperative effects



Anderson Localization

Multiple Scattering

Dicke States

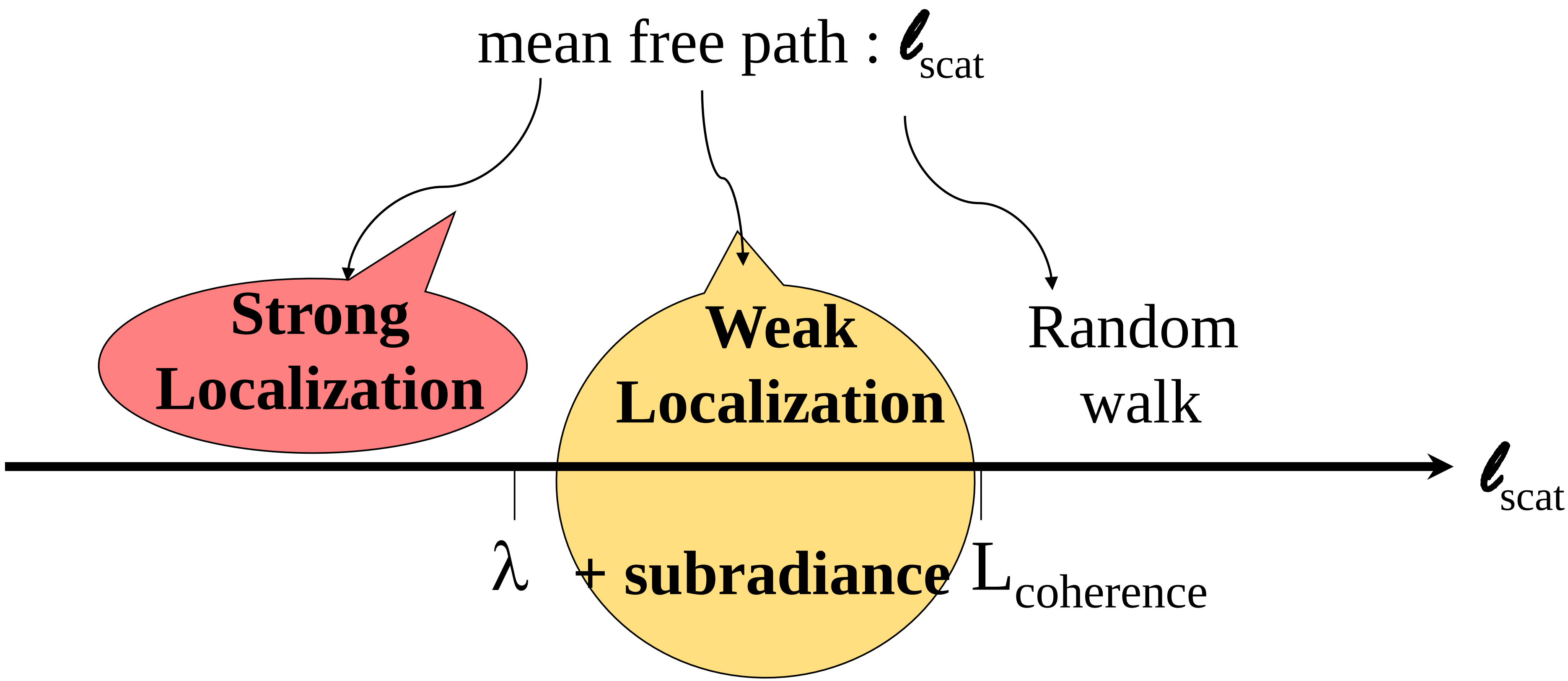
"Local"

"Global"

Interferences

Mesoscopic regime :
Interferences alter diffusion process :

GOAL : Observation of disorder induced quantum phase transition



Theory : Effective Hamiltonian in the single excitation manyfold

$$H_{eff} = (\hbar\omega_0 - \mathrm{i}\frac{\hbar\Gamma_0}{2}) \sum_i S_i^z + \frac{\hbar\Gamma_0}{2} \sum_{i\neq j} V_{ij} S_i^+ S_j^-$$

Diagonal :
On site energy

Off diagonal :
transport

$$V_{ij} = \beta_{ij} - i\gamma_{ij}$$

$$\beta_{ij} = \frac{3}{2}\left[-p\frac{\cos k_0 r_{ij}}{k_0 r_{ij}} + q\left(\frac{\cos k_0 r_{ij}}{(k_0 r_{ij})^3} + \frac{\sin k_0 r_{ij}}{(k_0 r_{ij})^2}\right)\right]$$

$$\gamma_{ij} = \frac{3}{2}\left[p\frac{\sin k_0 r_{ij}}{k_0 r_{ij}} - q\left(\frac{\sin k_0 r_{ij}}{(k_0 r_{ij})^3} - \frac{\cos k_0 r_{ij}}{(k_0 r_{ij})^2}\right)\right]$$

- Open System
- Reminiscent of Anderson Hamiltonian
- Heisenberg model with global coupling
- Long range hopping
- No decoherence (coupling to phonons, …)

Photon Escape Rate Distribution

Assume 1 initial excitation,
but unknown on which atom (or superposition of atoms)

$$H_{eff} = \left(\hbar\omega_0 - i\frac{\hbar\Gamma_0}{2}\right)\sum_{i=1}^{N}\sum_{\alpha}|e^i_\alpha\rangle\langle e^i_\alpha| - \frac{\hbar\Gamma_0}{2}\sum_{i\neq j}\sum_{\alpha\beta} g_{\alpha\beta}(\mathbf{r}_{ij}) S^{(+)}_{i,\alpha} S^{(-)}_{j,\beta},$$

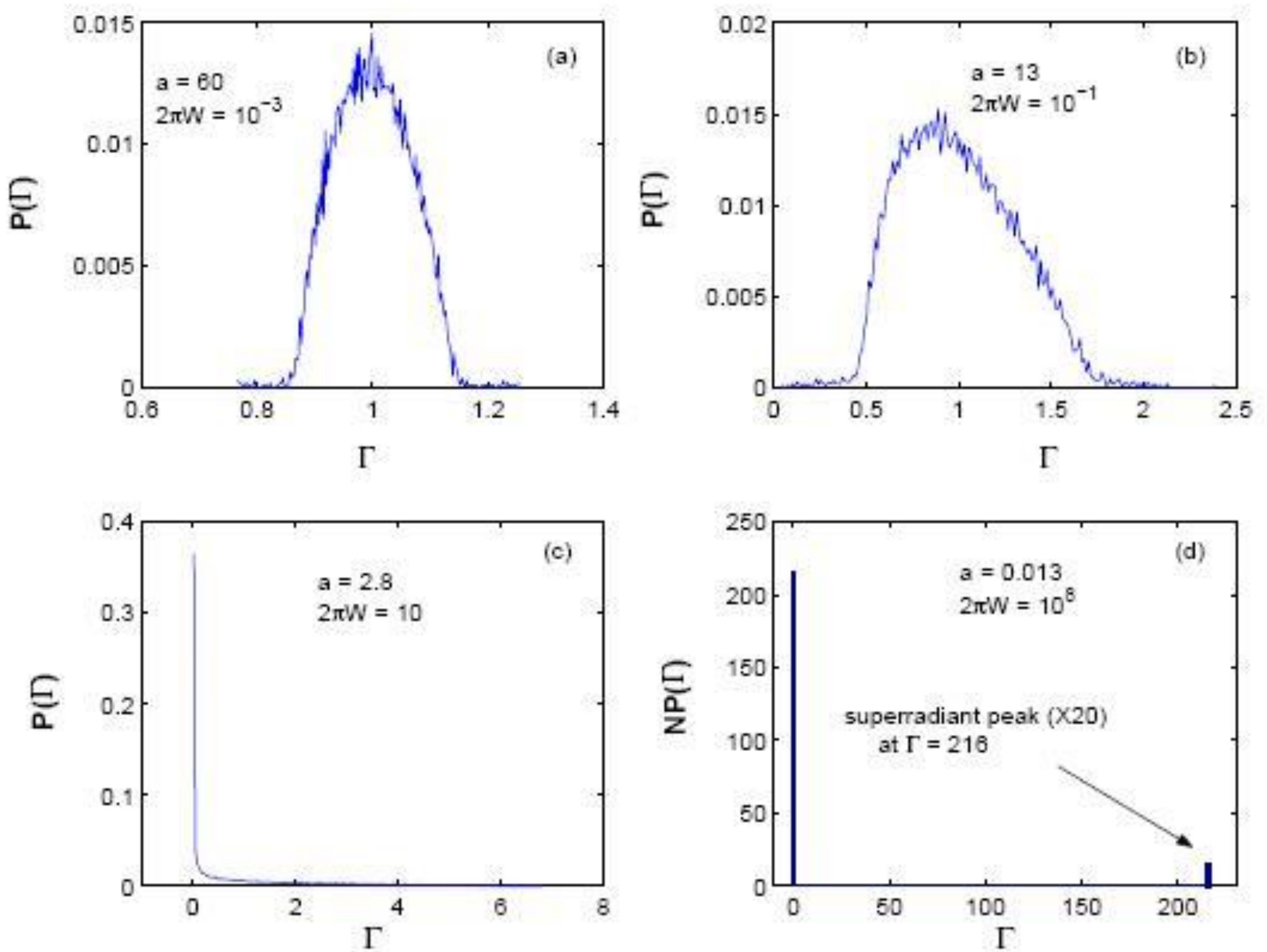
$$S^{(+)}_{i,\alpha} = |e^i_\alpha\rangle\langle g^i|$$

$$\frac{\mathrm{d}\rho_{GG}}{\mathrm{d}t} = \Gamma_0 \sum_{i,j=1}^{N}\sum_{\alpha\alpha'} \Im(g_{\alpha\alpha'}(r_{ij}))\langle G|S^{(-)}_{i,\alpha}\hat{\rho}_A S^{(+)}_{j,\alpha'}|G\rangle$$

$$\sum_{i=1}^{N} -\Im(g_{\alpha\alpha'}(r_{ij}))\mathbf{u}_j = \Gamma^j_{h\nu}\mathbf{u}_j$$

$$\Pi(t) \propto \left\langle \sum_{ij} \frac{\sin k_0 r_{ij}}{k_0 r_{ij}} b_i^\dagger(t) b_j(t) \right\rangle \propto \mathrm{Im}(\mathrm{H}_{\mathrm{eff}})$$

Photon Escape Rate Distribution



$a=L/\lambda$

$W=1/kl$

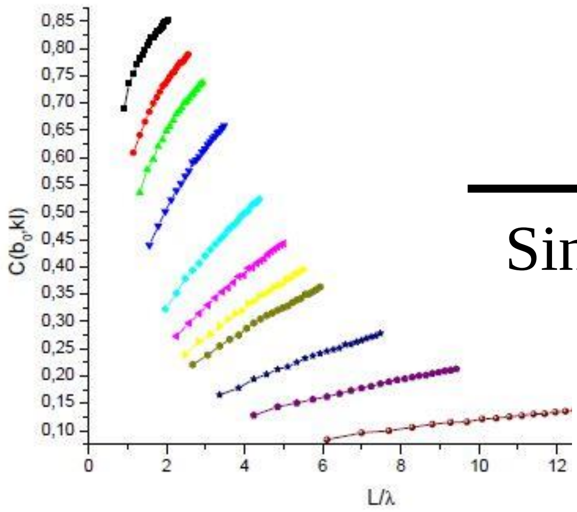
Pinheiro et al., Phys. Rev. E **69**, 026605 (2004)

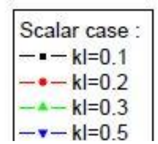
E. Akkermans et al., PRL, **101**, 103602 (2008)

Photon Escape Rates : single parameter scaling

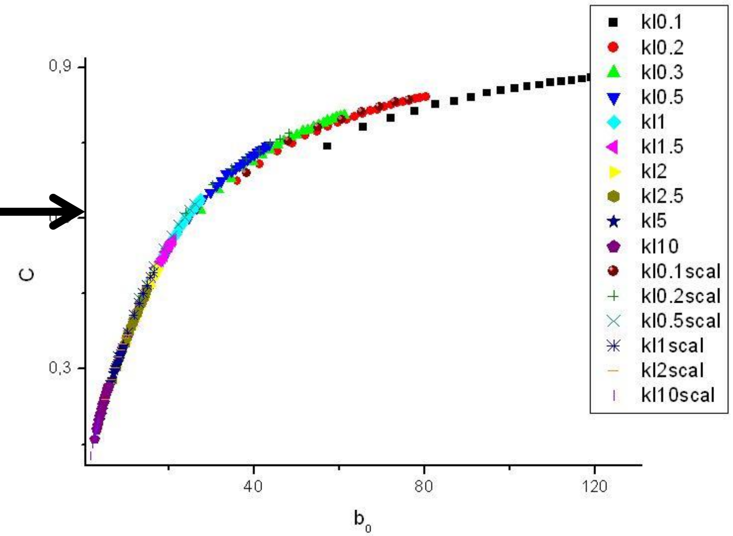
$$C(a,W) = 1 - 2\int_1^{\infty} d\Gamma \;\; P(\Gamma)$$

A measure of long lived photons





Single parameter scaling
$b_0 = N/N_{\perp}$



cooperative effects dominate over disorder !
no phase transition observed with P(Γ)

Dicke > Anderson

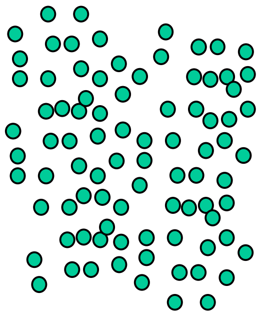
Mayor issue : how to measure this quantity?

Scalar model : E. Akkermans et al., PRL, **101**, 103602 (2008)

Vectorial model : L. Bellando et al., arxiv 1906.06966

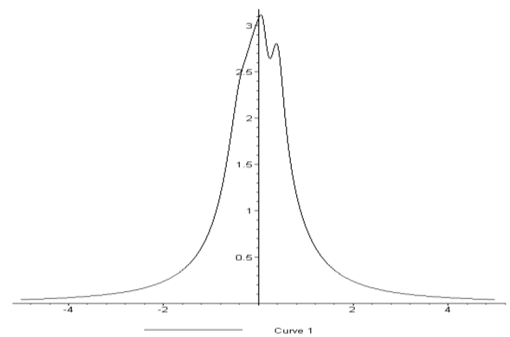
Eigenvalues of H_{eff}

Cloud of Atoms = Large ‘Molecule’ (with 10^{10} atoms)

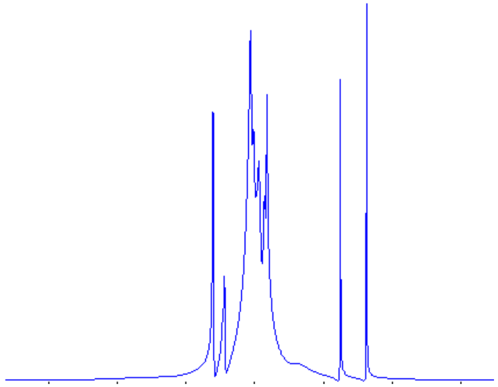


molecular spectrum ?

‘dilute’ molecule



‘dense’ molecule



SHENG LI AND ERIC J. HELLER PHYSICAL REVIEW A **67**, 032712 (2003)

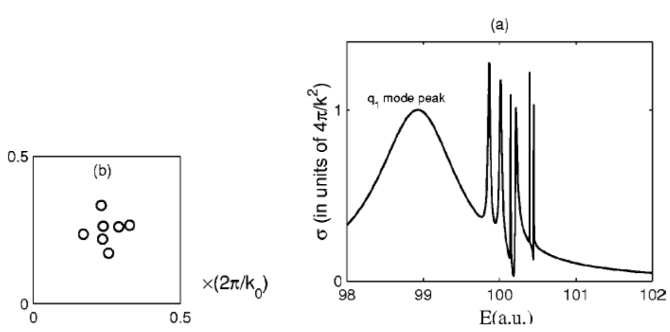
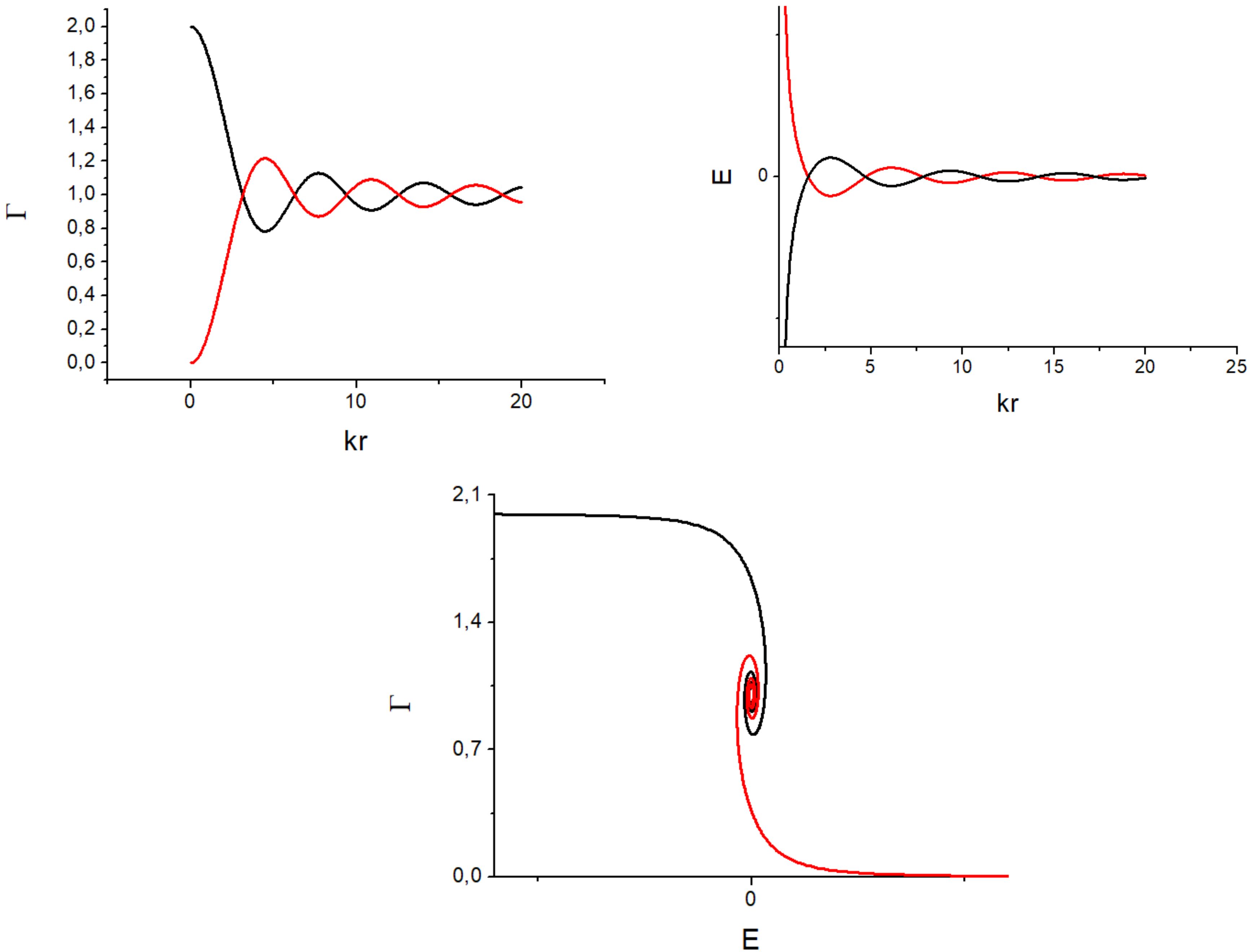


FIG. 4. (a) Total cross section as a function of energy for a system of seven identical scatterers randomly placed on a plane. Each scatterer is the same as used in Fig. 1. The positions of the scatterers are shown in (b).

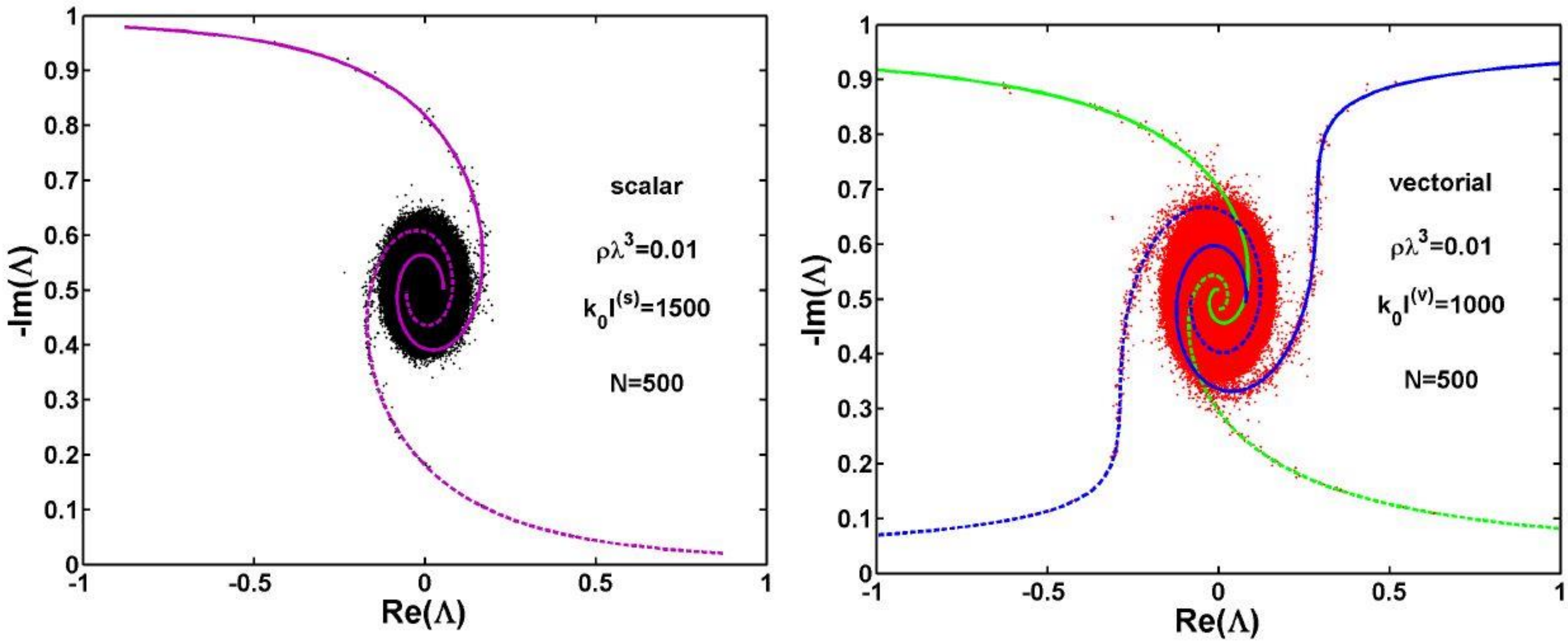
proximity resonances
doorway states
giant oscillator strength

Eigenvalues for N=2 coupled dipoles

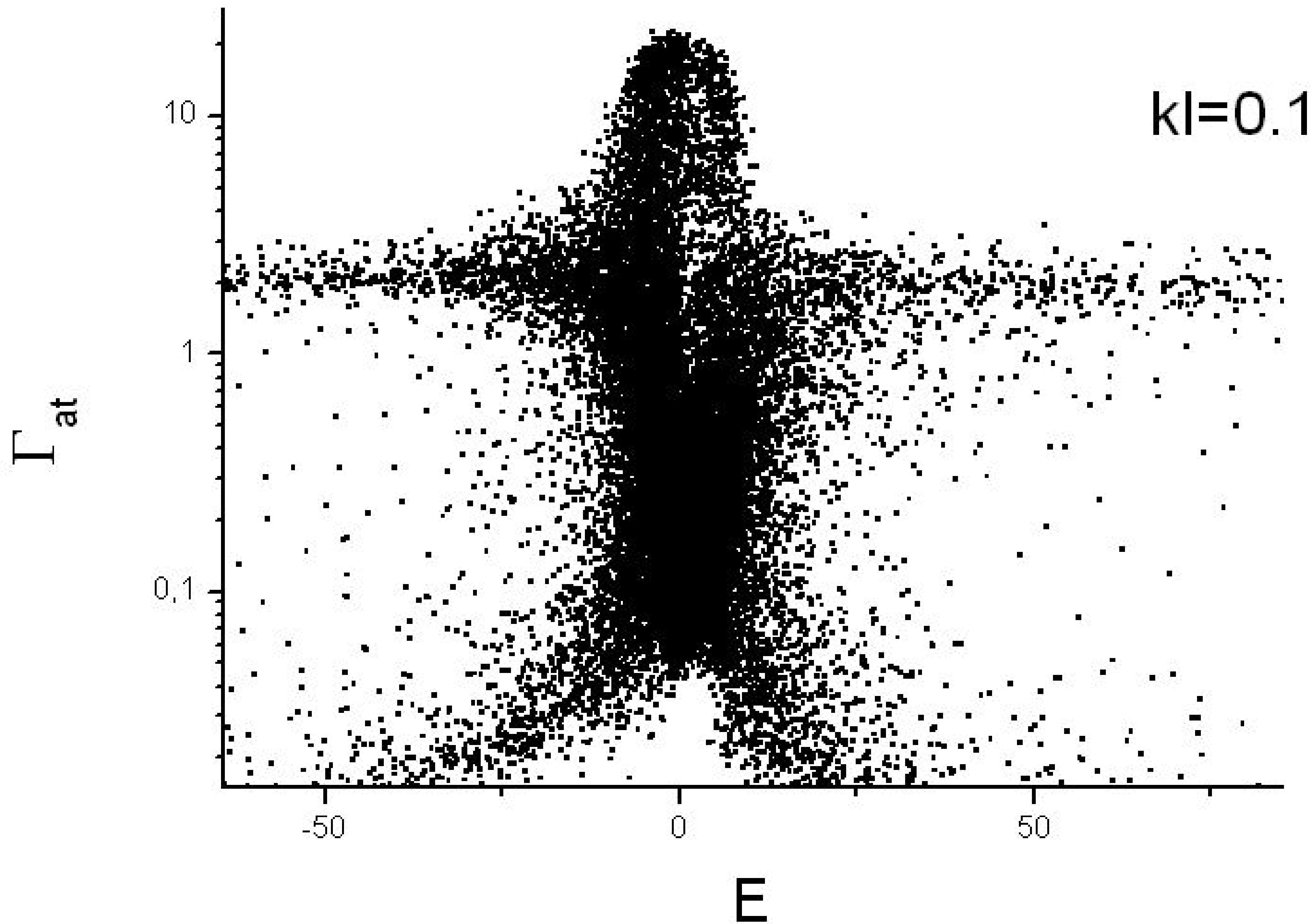


Eigenvalues for N coupled dipoles

Important near field terms for high densities

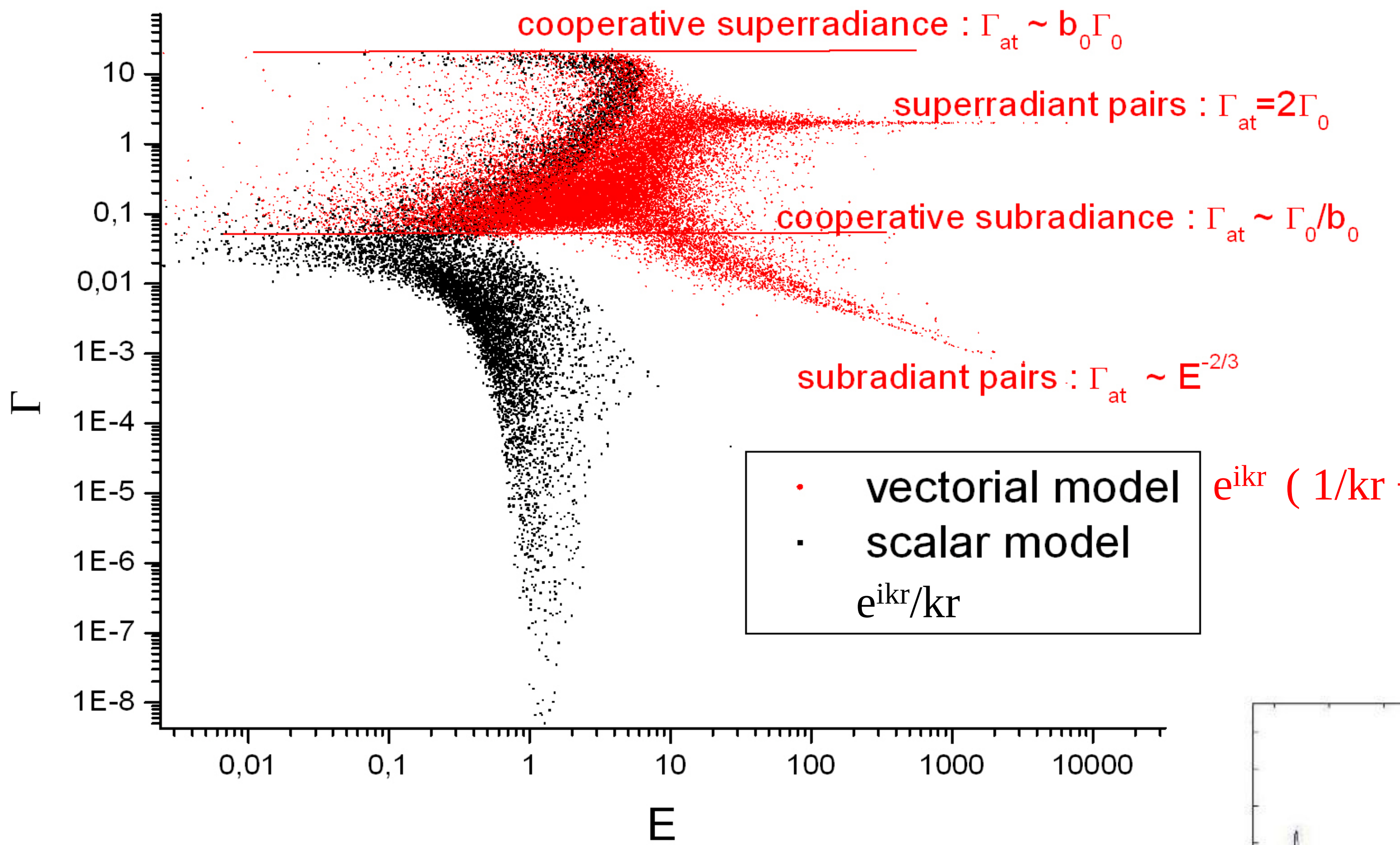


Eigenvalues of H_{eff}

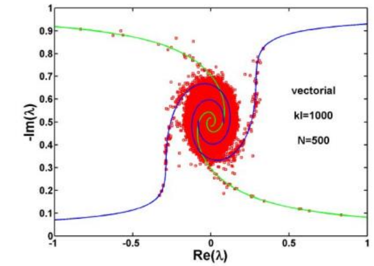


Eigenvalues for N coupled dipoles

kl=0.1

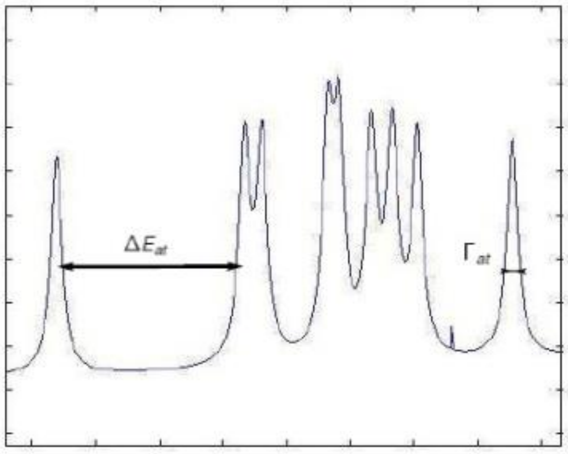


$e^{ikr}\ (1/kr + 1/kr^2 + 1/kr^3)$



Resonance Overlap (« Thouless »)

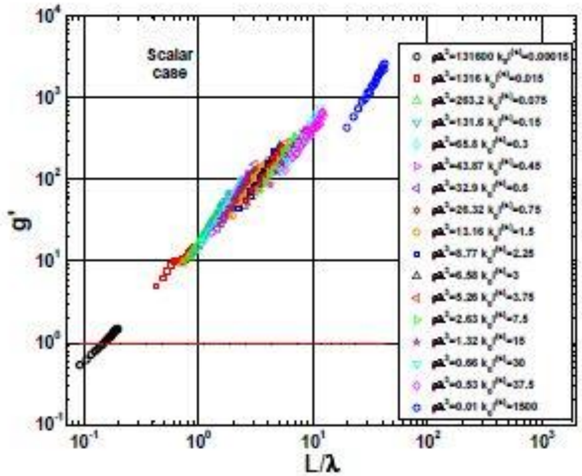
$$g = \left\langle \frac{1}{\langle 1/\Gamma \rangle \langle \Delta E \rangle} \right\rangle$$

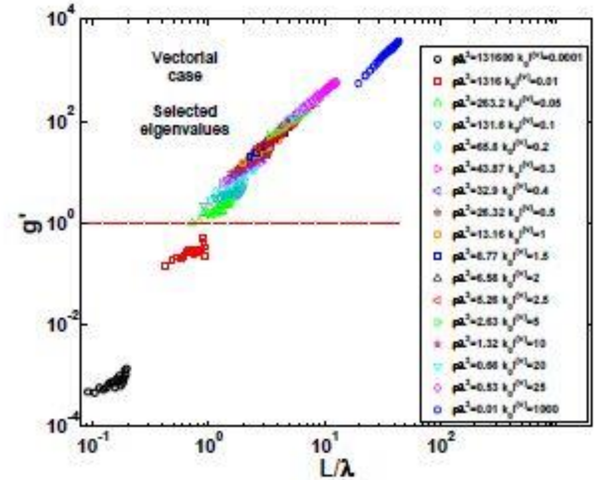


Level repulsion vs Level Width

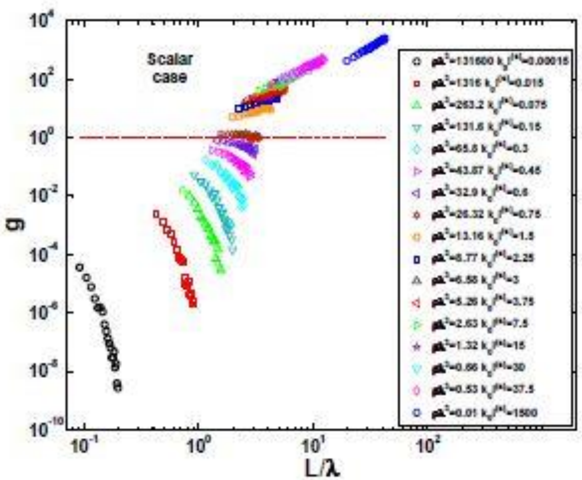
Resonance overlap (Γ/ΔE)in **superradiant transition** and **Anderson transition** (Thouless criterion)

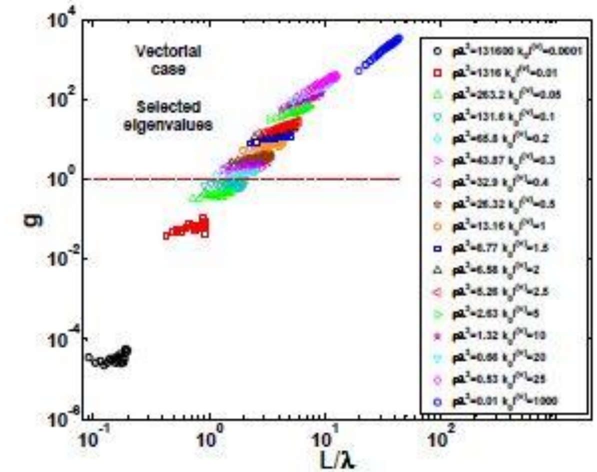
g = <Γ> / <ΔE>



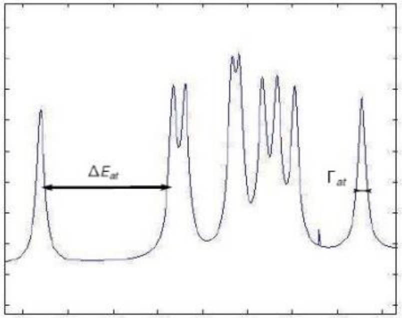


g = 1 / <1/Γ><ΔE>

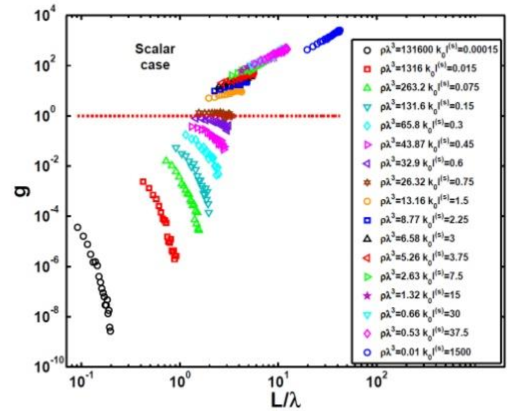


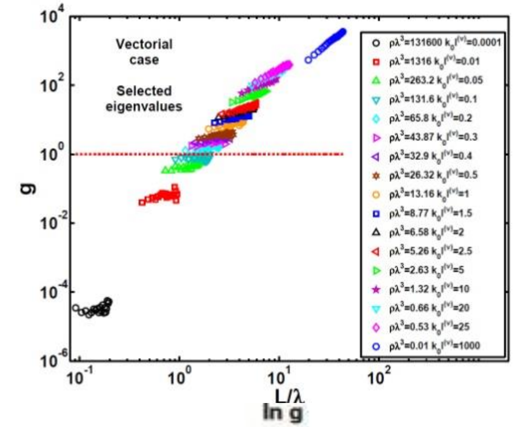


Resonance Overlap (« Thouless »)

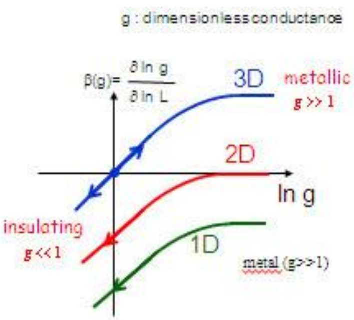


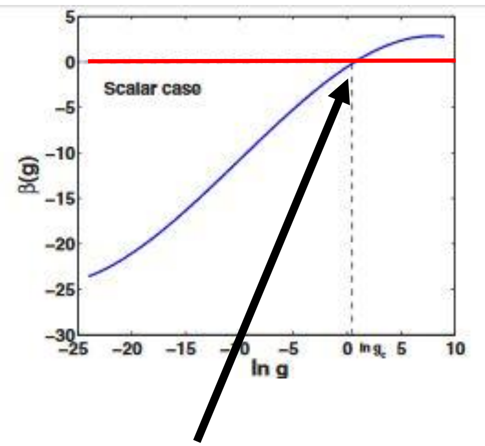
$$g = \left\langle \frac{1}{\langle 1/\Gamma \rangle \langle \Delta E \rangle} \right\rangle$$

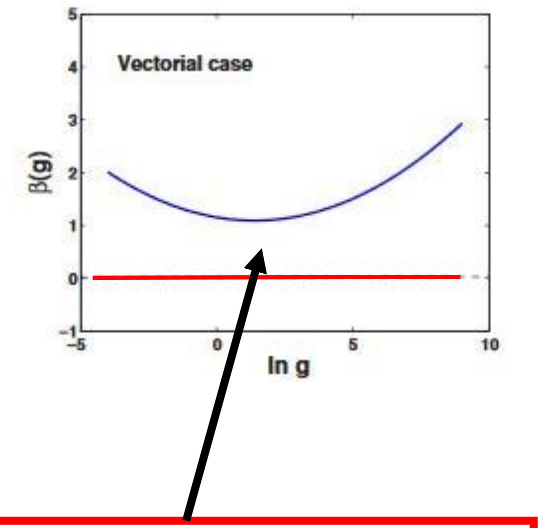




Scaling function β(g)







Phase transition

No phase transition

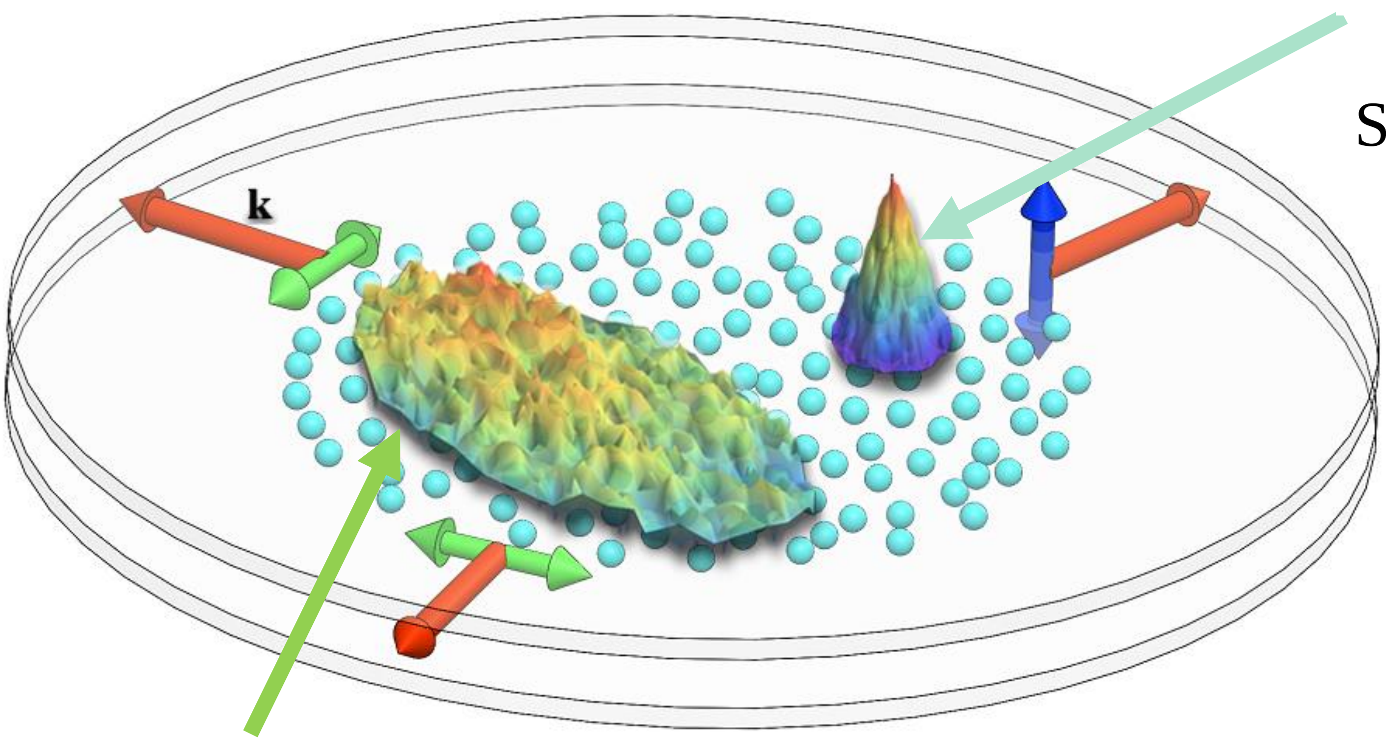
S. Skipetrov, I. Sokolov, PRL 112, 023905 (2014)

Bellando et al,. Phys. Rev. A 90, 063822 (2014)

Similar surprise in 2D : No localisation for vectorial light !

$$\frac{d\hat{\sigma}_j^{(0)}}{dt} = -\frac{\Gamma_0}{2}\hat{\sigma}_l^{(0)} - \frac{\Gamma_0}{2}\sum_{l=1}^{N} H_0(kr_{jl})\hat{\sigma}_l^{(0)},$$

Spatially localized mode
(scalar case)



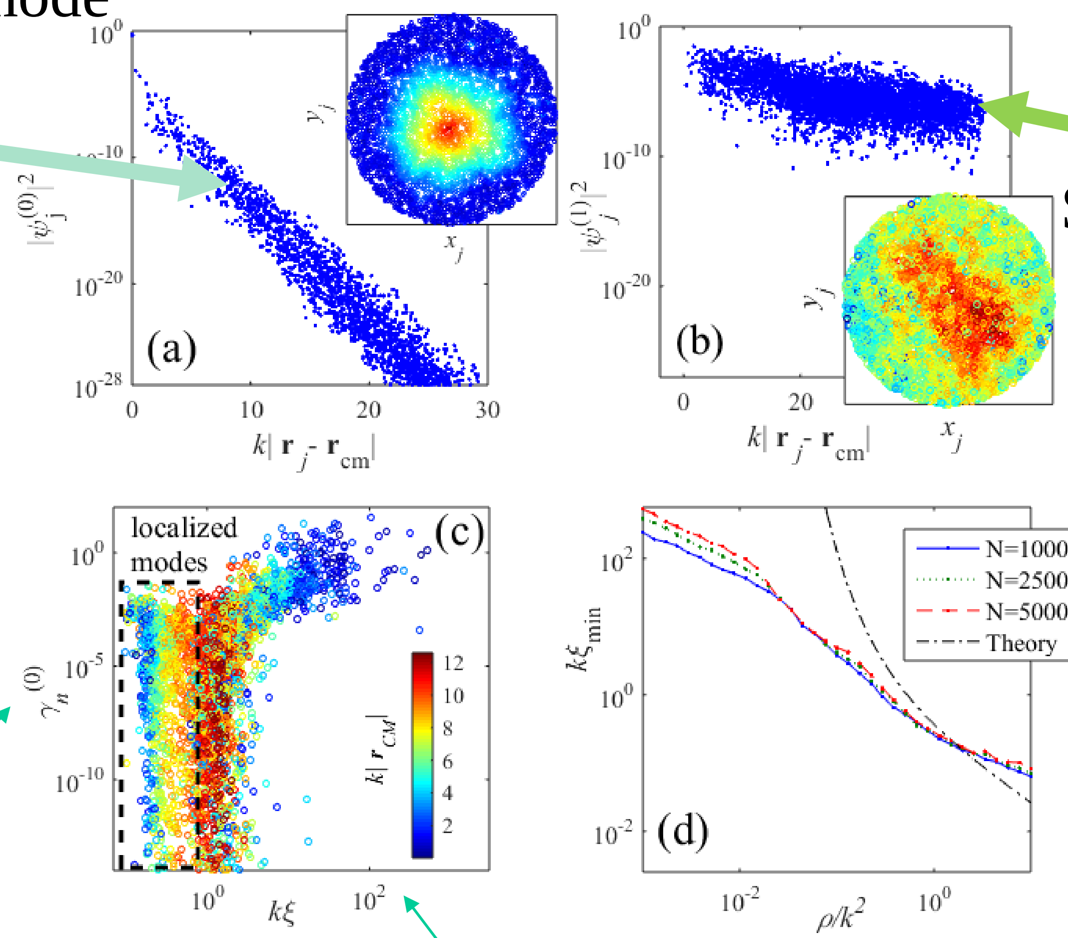
Spatially extended mode
(vectorial case)

$$\frac{d\hat{\sigma}_j^{(\pm1)}}{dt} = -\frac{\Gamma_1}{2}\hat{\sigma}_j^{(\pm1)} - \frac{\Gamma_1}{2}\sum_{l\neq j}\left[H_0(kr_{jl})\hat{\sigma}_l^{(\pm1)} + e^{2i\varphi_{jl}}H_2(kr_{jl})\hat{\sigma}_l^{(\mp1)}\right]$$

C. Maximo et al., Phys. Rev. A **92**, 062702 (2015).

Mode profiles

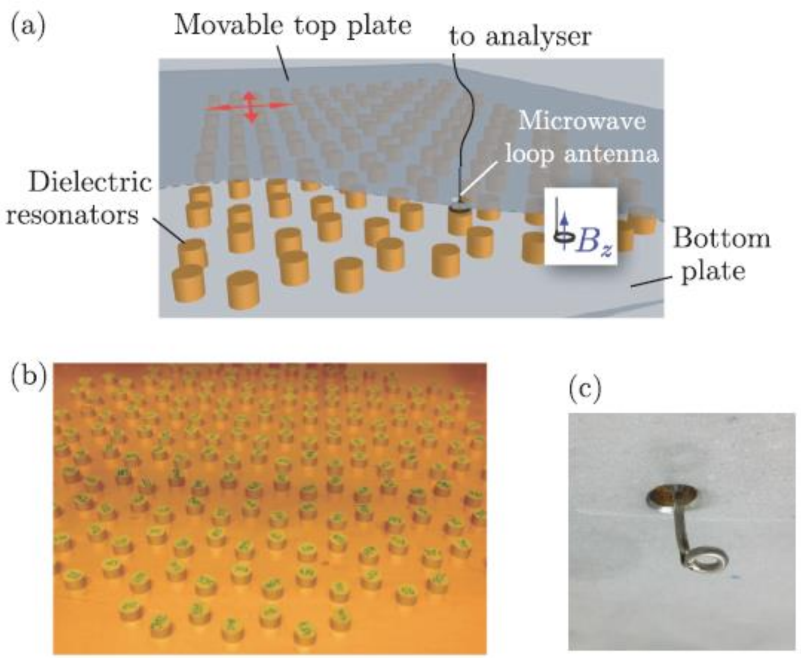
Spatially localized mode
(scalar case)

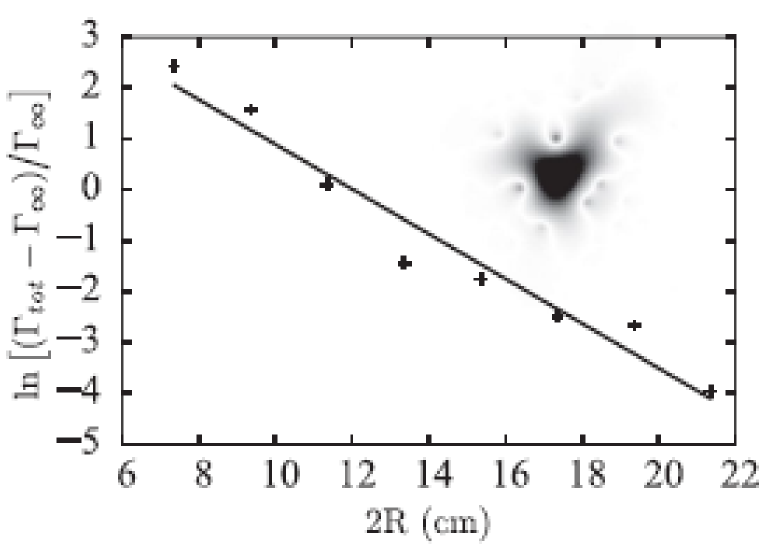


Spatially extended mode
(vectorial case)

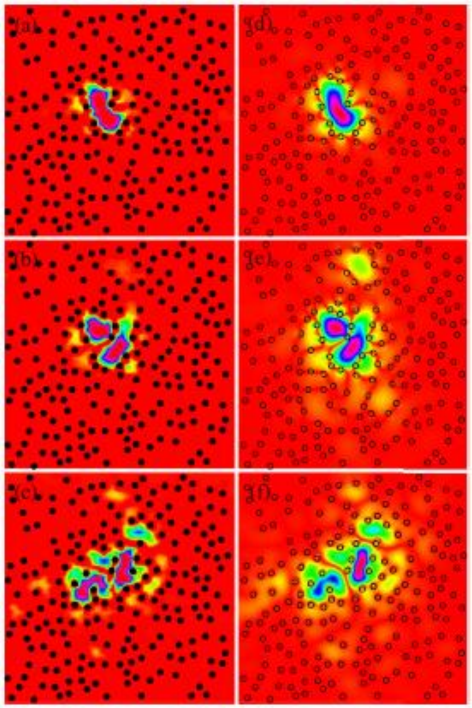
Mode width NOT correlated to localisation length :
temporal vs spatial localisation

Single Polarisation channel in 2D : microwave experiments allow to measure eigenvectors





Scanning perturbation :
Measure of local field



$$\Gamma_{\text{leak}} \propto \exp(-2R/\xi_{\text{loc}})$$

D. Laurent et al., Phys. Rev. Lett. **99,** 253902 (2007)

No Anderson Localization
for Vectorial Light in 3D !

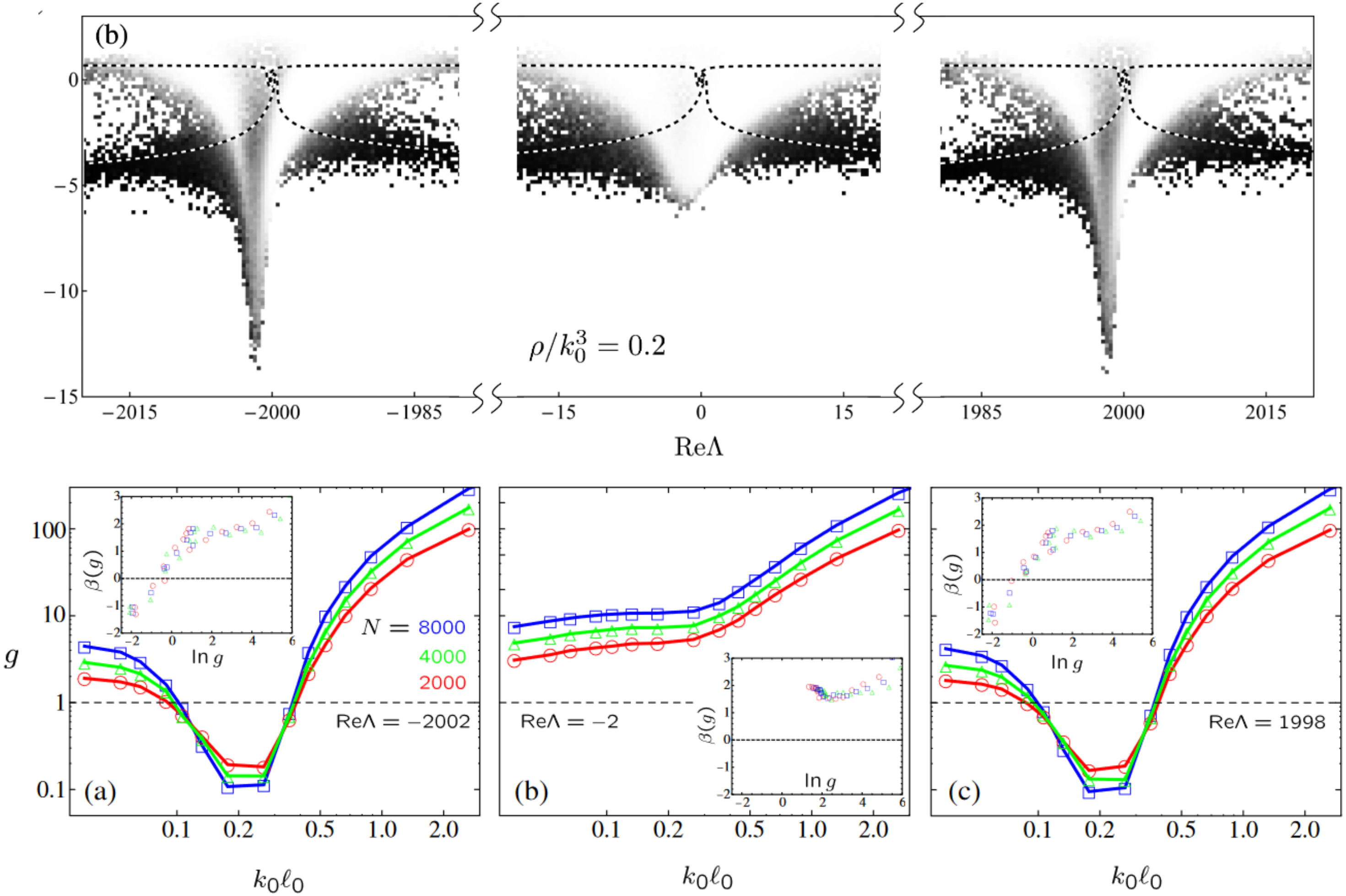
NOT captured by general scaling Ansatz by the gang of four
nor by self-consistent theory of localization !

Universal theory does not care about details
But details can matter !

2 solutions proposed since 2014 :

1) Magnetic field assisted Anderson localization

Dense sample + magnetic field :
partial suppression of near field dipole dipole interactions



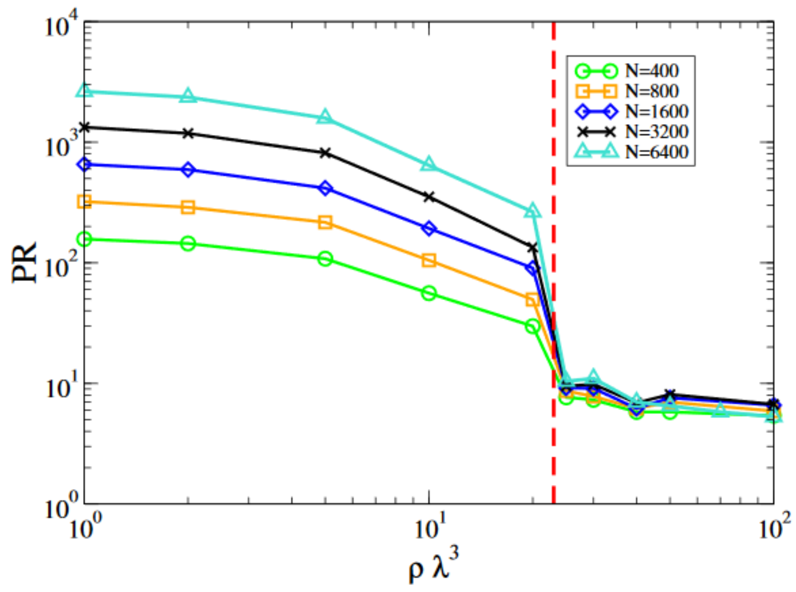
S. Skipetrov, I. Sokolov, PRL **114**, 053902 (2015)

2 solutions proposed since 2014 :

2) Diagonal disorder in dilute samples

A warning for what to expect :

- **Against main stream** credo that high spatial densities are required !
- Change the analysing criteria :
  (inverse) partition ratio to see how many atoms are involved in one state

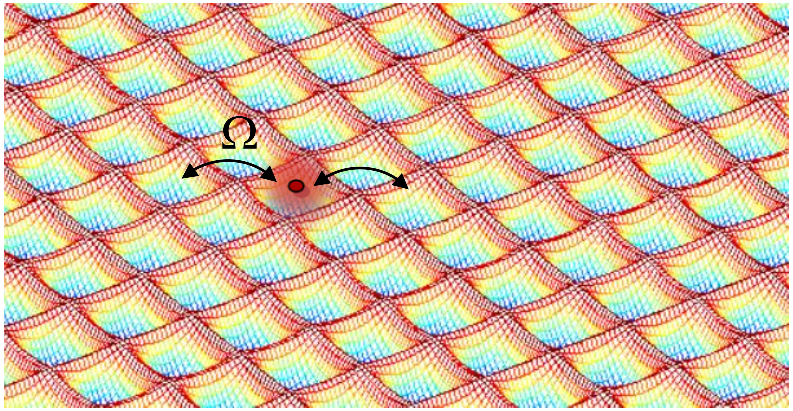


PR for $0.01 < \Gamma < 1$

Combining Anderson and Dicke
Toy Model : Open Disordered System:

3D Anderson model on 10 x10 x10 lattice
hopping (Ω) + on site disorder (W)

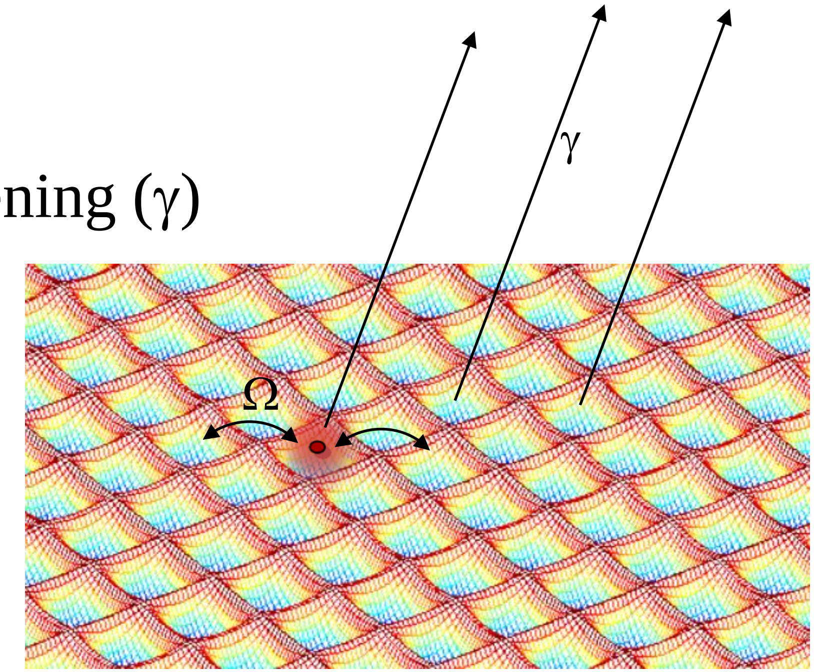
$$H_0 = \sum_{j=1}^{N} E_j |j\rangle\langle j| + \Omega \sum_{\langle i,j\rangle} (|j\rangle\langle i| + |i\rangle\langle j|)$$



Anderson transition in 3 D : all states are localized for W/Ω > 16.5

Combining Anderson and Dicke
Toy Model : Open Disordered System:

3D Anderson model on 10 x10 x10 lattice
hopping (Ω) + on site disorder (W) + opening (γ)

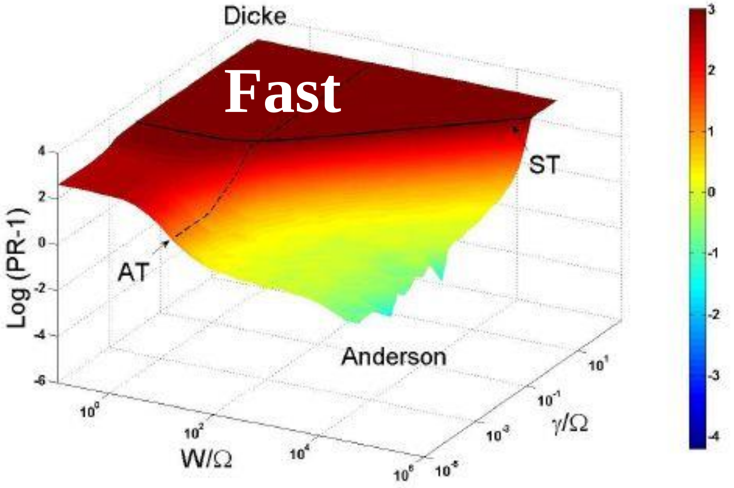


$$H_0 = \sum_{j=1}^{N} E_j |j\rangle\langle j| + \Omega \sum_{\langle i,j\rangle} (|j\rangle\langle i| + |i\rangle\langle j|)$$

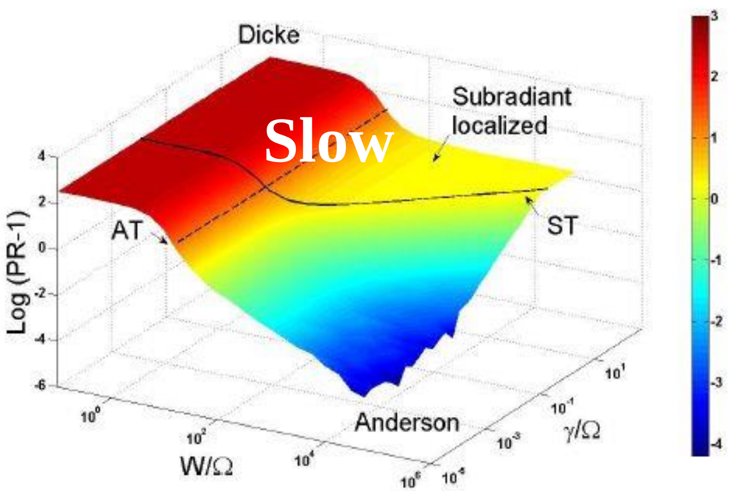
$$(H_{\text{eff}})_{ij} = (H_0)_{ij} - \frac{i}{2}\sum_c A_i^c (A_j^c)^* = (H_0)_{ij} - i\frac{\gamma}{2} Q_{i,j}$$

All sites coupled to one single decay channel : $Q_{ij}=1$

Toy Model : Anderson lattice model + coupling to one open mode

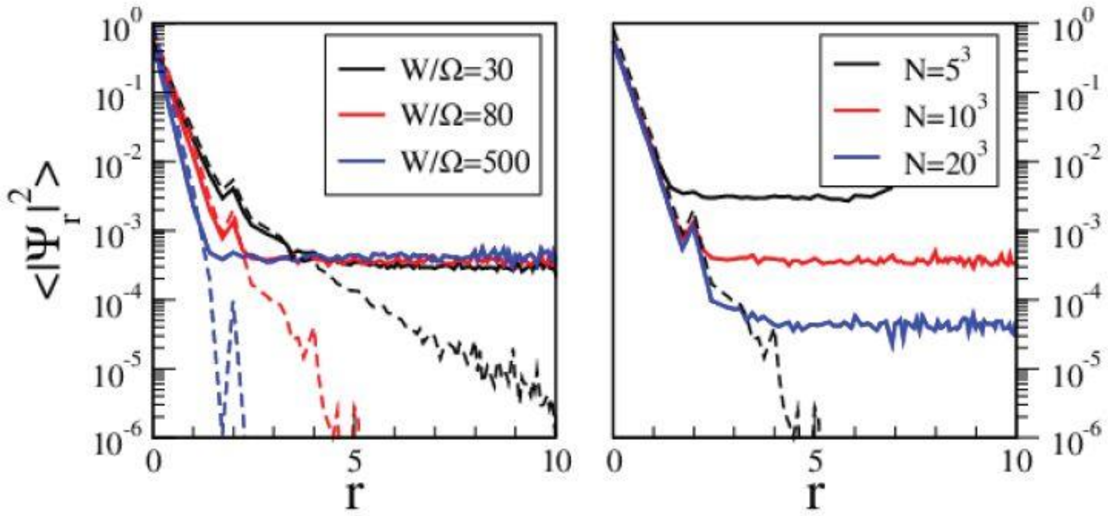


(a)



(b)

Hybrid Subradiant States
« decoupled » from outside world



A.Biella, et al., EPL, 103, 57009 (2013)

Apply this idea to the coupled dipole model

$$\mathcal{H} = \sum_{i=1}^{N} (E_i - i\frac{\Gamma_0}{2}) |i\rangle \langle i| + \frac{\Gamma_0}{2} \sum_{i \neq j}^{N} V_{i,j} |i\rangle \langle j|$$

$$V_{i,j} = \frac{exp(ik_0 \cdot r_{ij})}{k_0 \cdot r_{ij}}$$

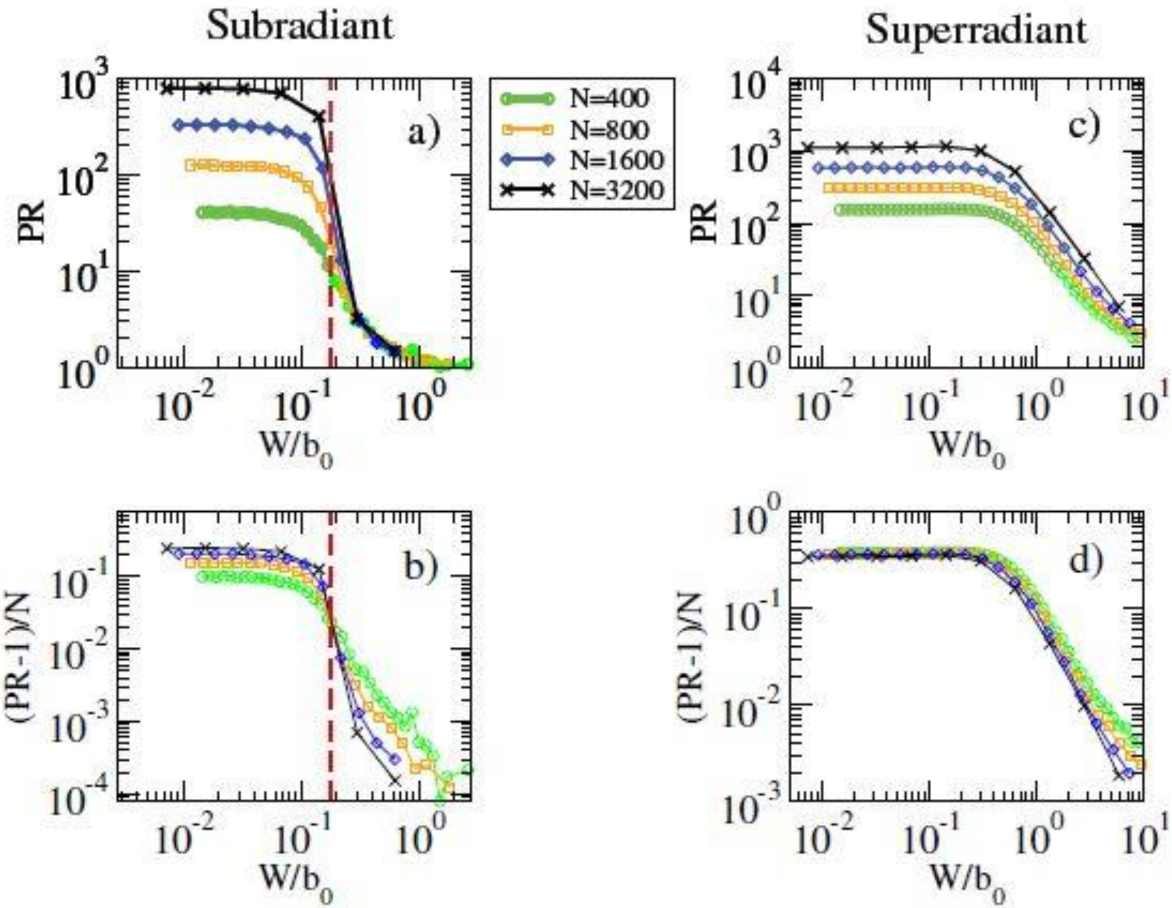
Diagonal disorder : random light shifts : $E_j \in [-W/2, W/2]$

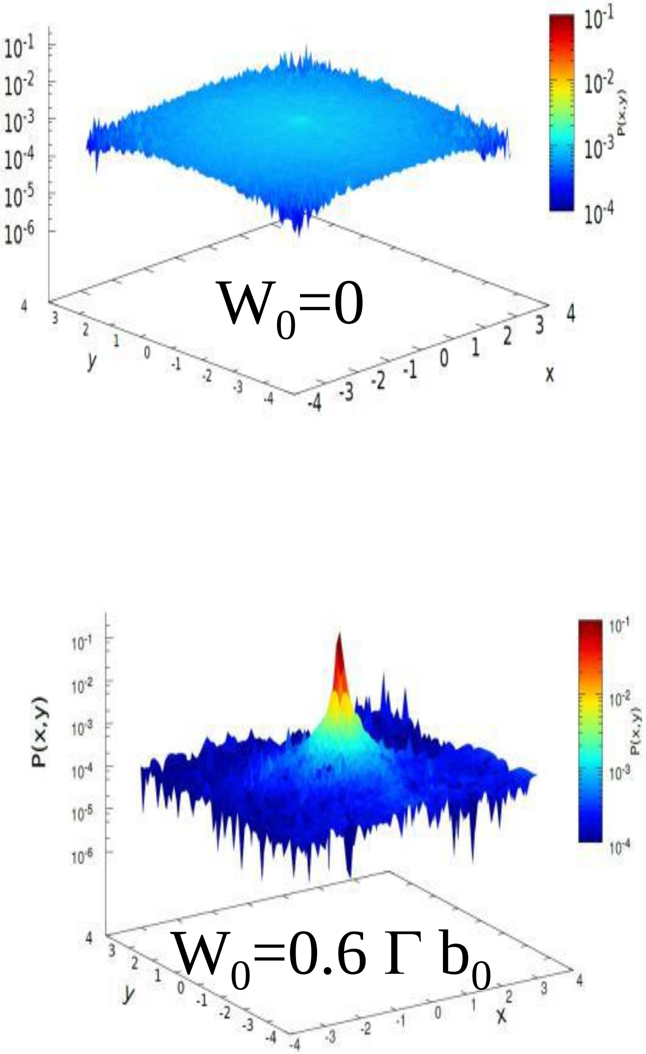
+ remain in the dilute limit : $\rho\lambda^3 < 5 < \rho_{cr}\lambda^3 = 24$

$$PR = \left\langle 1/\sum_i |\langle i|\psi\rangle|^4 \right\rangle$$

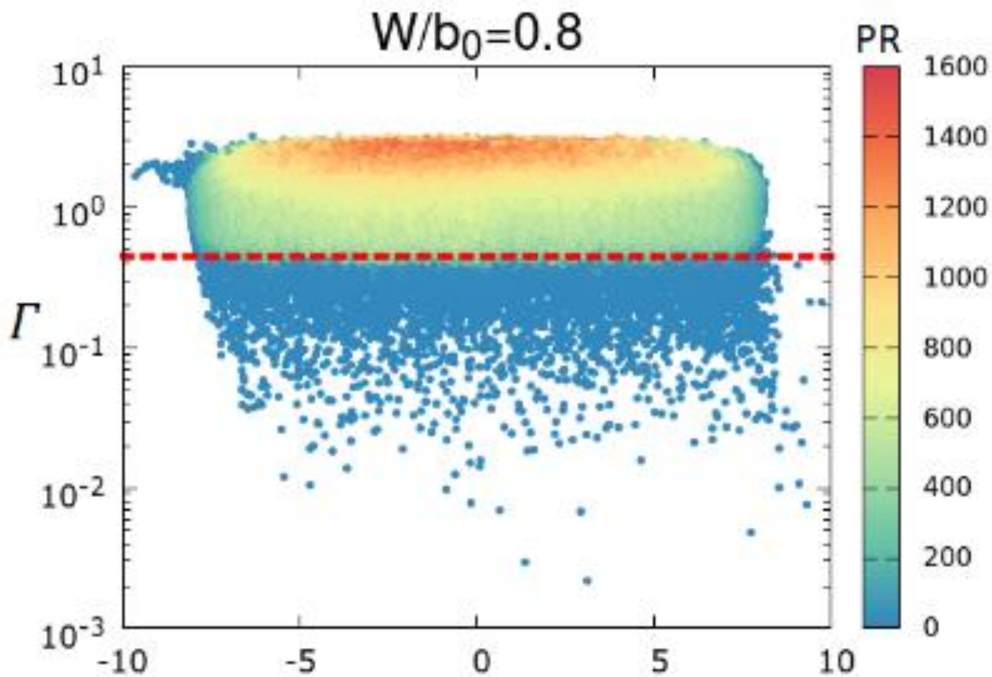
Scaling law for partition ratio

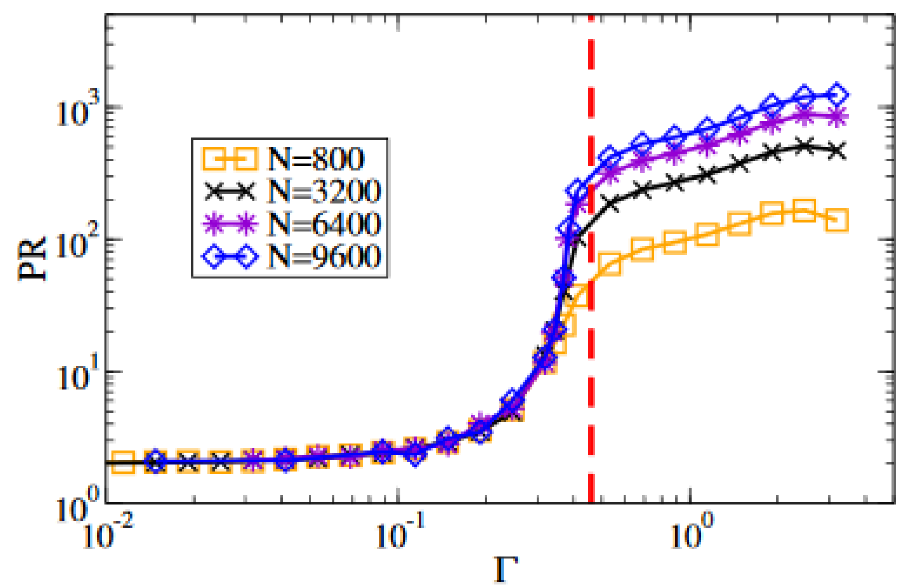
$$PR = \left\langle 1/\sum_i |\langle i|\psi\rangle|^4 \right\rangle$$





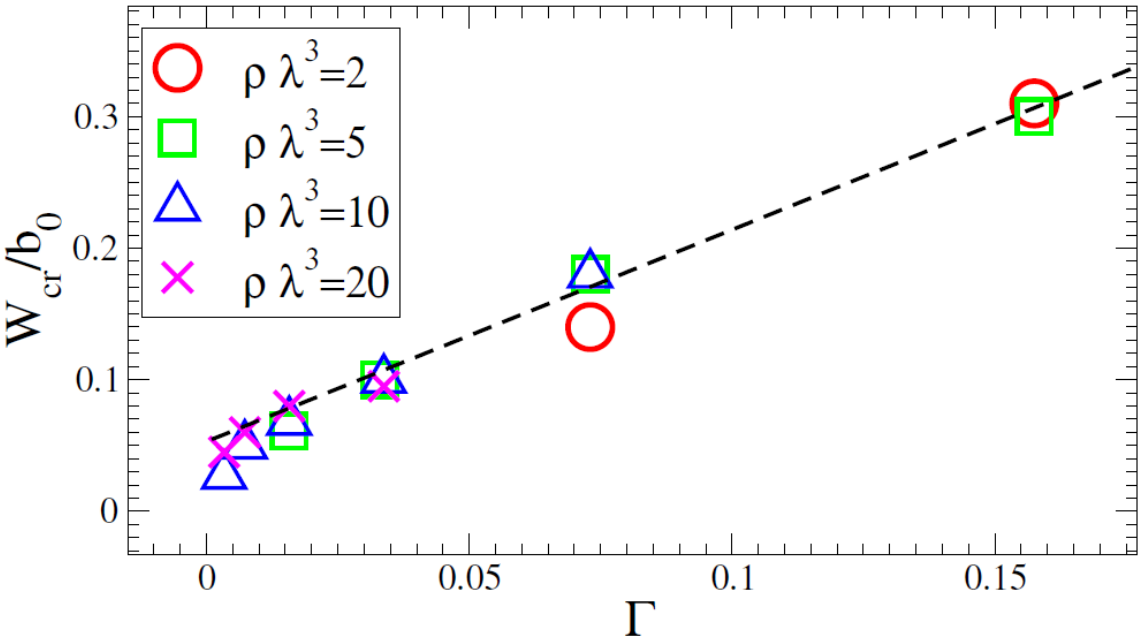
Mobility edge along the imaginary axis





Longer lifetimes need less disorder to get localized

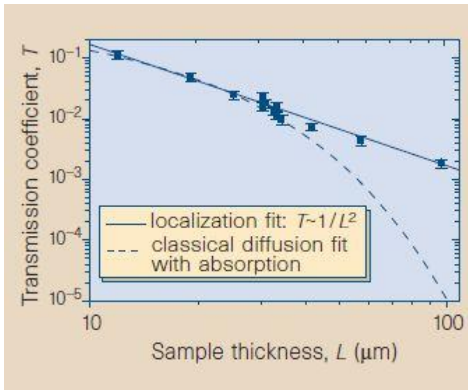
Critical disorder

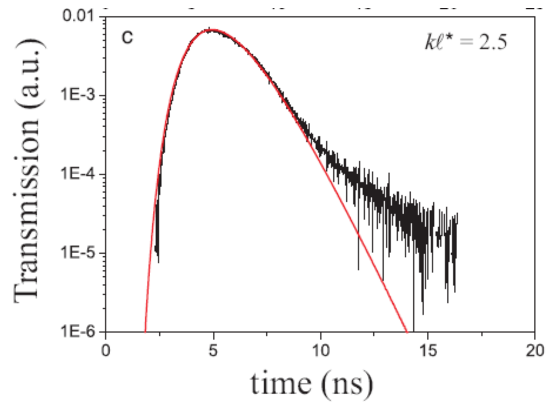


- Scaling law : $W_{cr} / b_0 \propto \Gamma_{mode}$
- All states get localized in the infinite W limit
  (same trivial limit as in the Anderson lattice model)
- How to connect to kl=1?
  (same question for the Anderson lattice model)

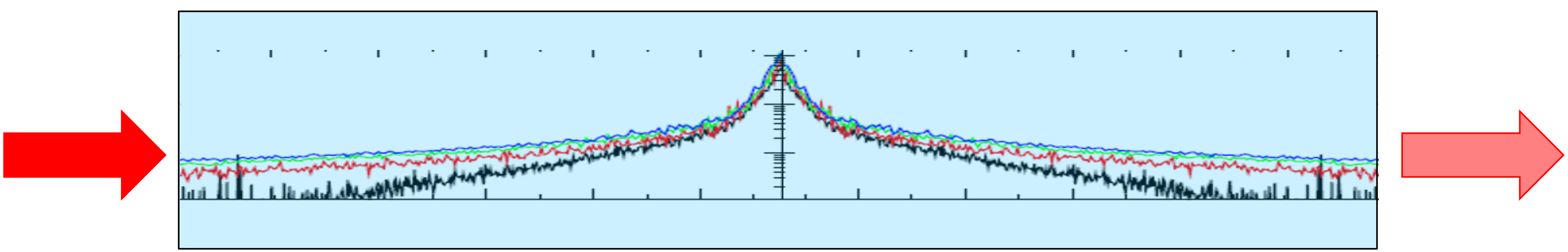
How to detect Anderson localisation of light ?

No insulating behaviour from (time resolved) average transport ? ☹





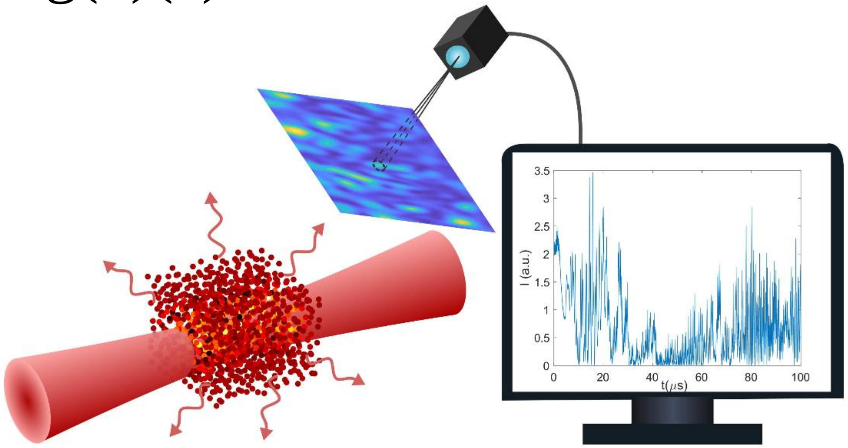
Localized states have extended (cooperative) tails



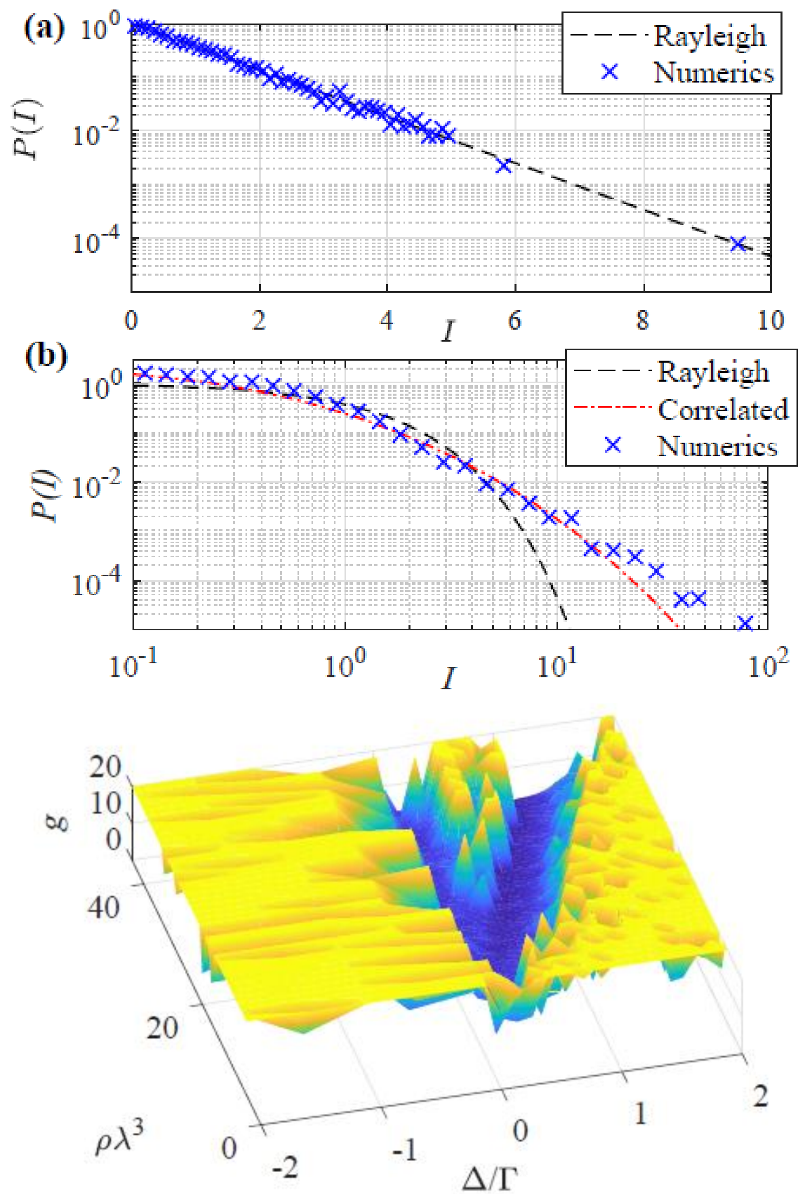
Macroscopic observables of Anderson localisation

Critical behavior at the phase transition :
Enhanced fluctuations at the phase transition

Noise is the signal ☺ :
intensity correlations : g(2)(τ)



Microscopic (ab initio) model of observables of Anderson localisation



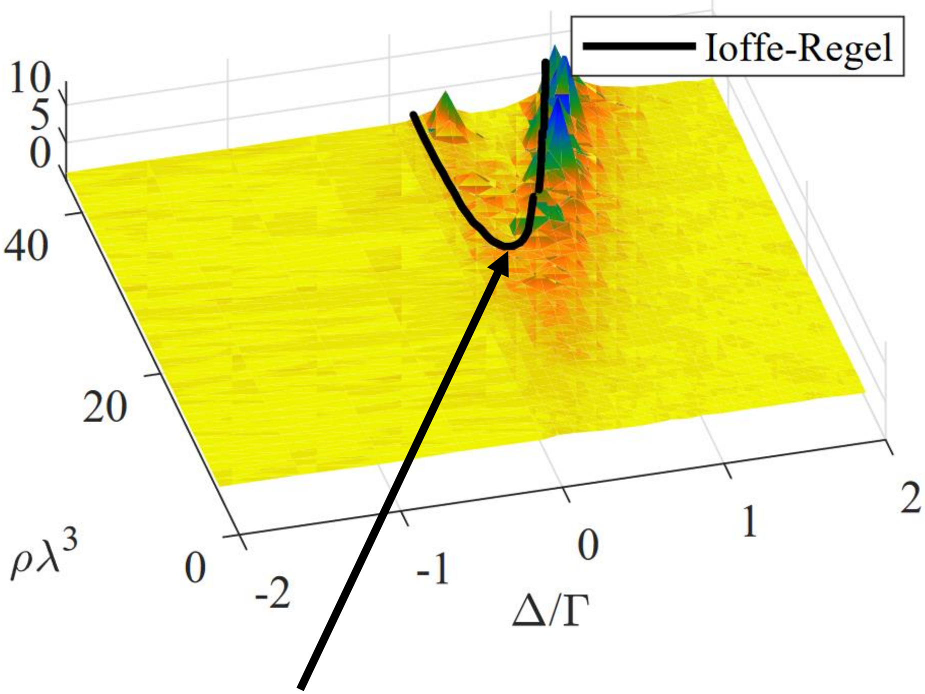
$$P(I) = \int_{-i\infty}^{i\infty} \frac{dx}{\pi i} K_0(2\sqrt{-xI}) \exp\left(-\Phi_c(x)\right),$$

$$\Phi_c(x) = g \int_0^1 \frac{dy}{y} \log\left(\sqrt{1+\frac{xy}{g}} + \sqrt{\frac{xy}{g}}\right),$$

M. v. Rossum, T. Nieuwenhuizen, Rev. Mod. Phys. 71, 313 (1999)

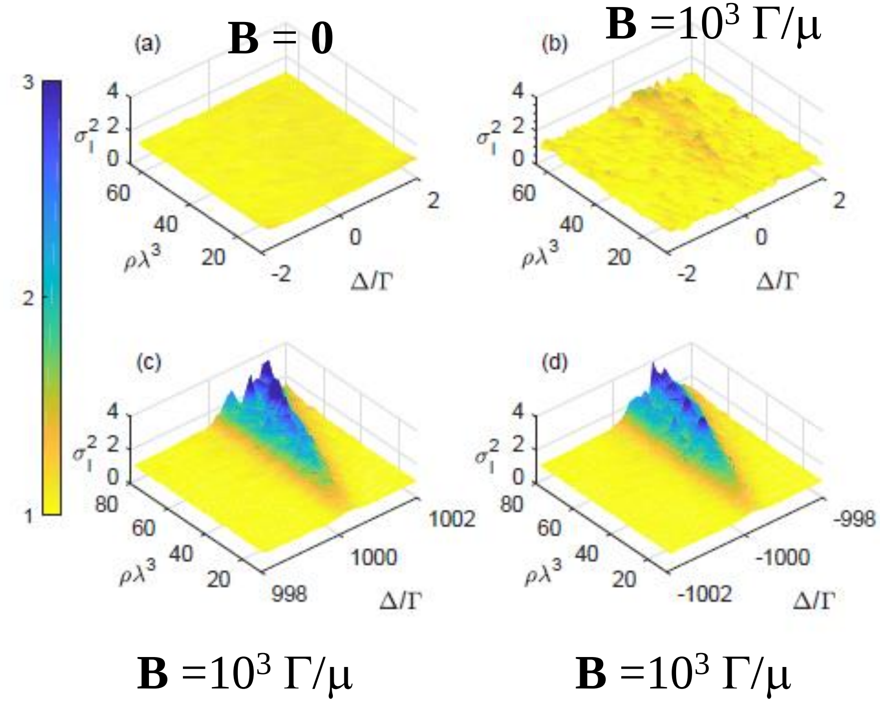
g : conductance
= number of open transport channels

Variance of scattered intensity



$$\delta_c = \frac{\rho\lambda^3}{8\pi^2} \pm \frac{1}{2}\sqrt{3\alpha\frac{\rho\lambda^3}{4\pi^2} - 1}$$

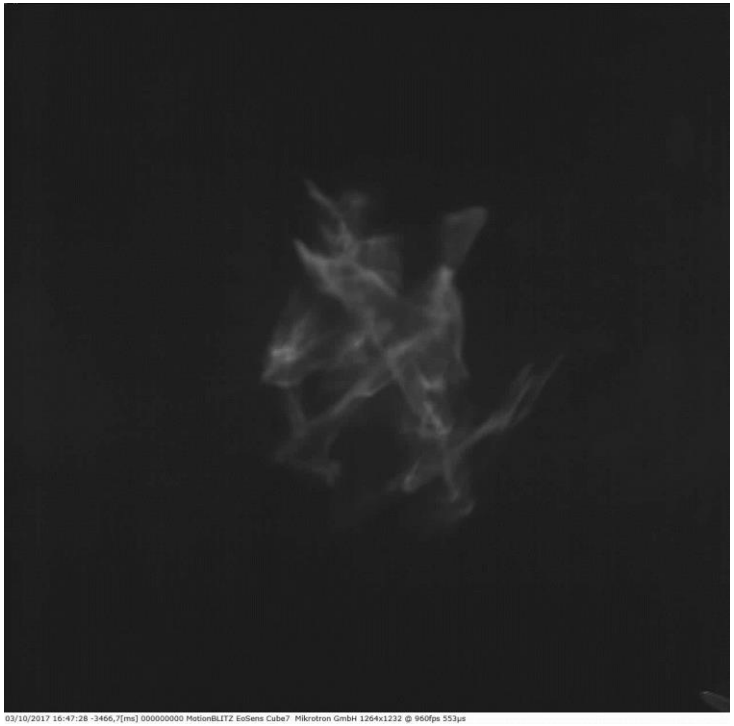
Scalar Model



Vectorial Model

Lectures not given ☺

Correlation functions : g^1, g^2, $g^{3/2}$



Collective shifts (atomic clocks)

Collective Forces

Quantum optics : beyond single excitation

Non linear propagation = quantum fluids of light

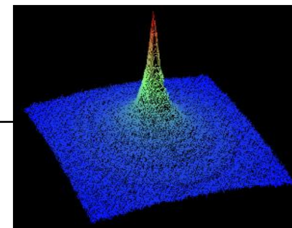
From cold atom to astrophysics

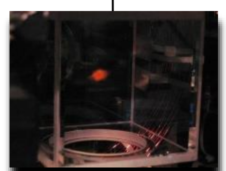
Princeton

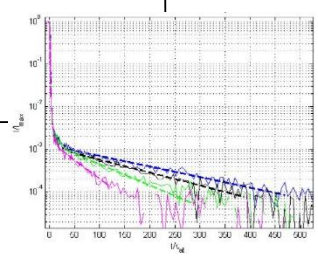
Anderson

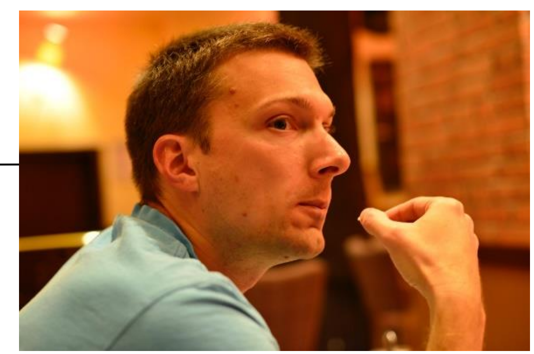
Nice

Labeyrie









Dicke

Guerin

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Thank you
and have a safe trip back
home