



Interaction and disorder :

Light scattering by cold atoms:

Localization and Cooperative Scattering



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Lecture 1 : Multiple Scattering of Light in Cold Atoms

1.1 Steady State Results : Ohm's Law for Photons

+ Numerical Random Walk Simulations

1.2 Time dependent scattering : radiation trapping

+ Numerical Random Walk Simulations

1.3 Random laser

Lecture 2 : Interference Effects in Light Scattering by Cold Atoms

2.1 Coherent Backscattering of Light by Cold Atoms

+Numerical Simulations with Weak Localization Corrections

2.2 Dicke Super- and Subradiance

+Numerical Simulations with Coupled Dipoles

Lecture 3 : Anderson Localisation of Light

3.1 Anderson Lattice Model

3.2 Effective Hamiltonian Approach

3.3 Scalar vs vectorial light : red light for Anderson localization

3.4 Outlook : towards localization of light in cold atoms

Link to Mathlab codes :

<http://www.kaiserlux.de/coldatoms/LesHouches2019Kaiser.html>

Lecture 2 : Interference Effects in Light Scattering by Cold Atoms

2.1 Coherent Backscattering of Light

[Numerical Simulations](#)

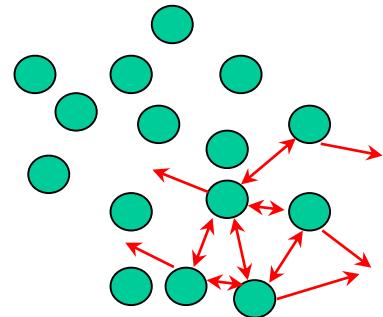
2.2 Dicke Super- and Subradiance

Coupled Dipoles

Cooperative effects in time dependant scattering

[Numerical Simulations](#)

Photons ...



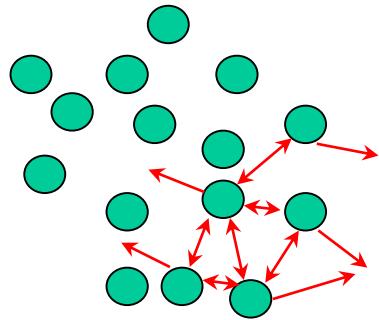
Random walk :

Diffusion

$$\text{coefficient } D_0 \approx \ell^2 / \tau$$

$$\ell = 1/n \sigma$$

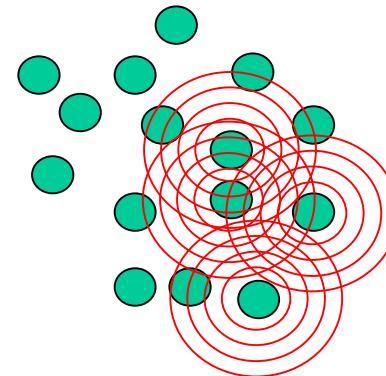
Photons ...



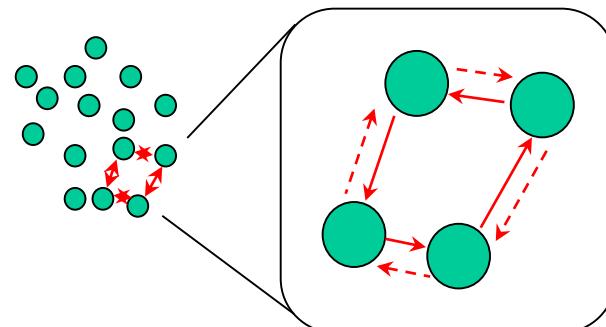
Random walk :
Diffusion
coefficient $D_0 \approx \ell^2 / \tau$

$$\ell = 1/n \sigma$$

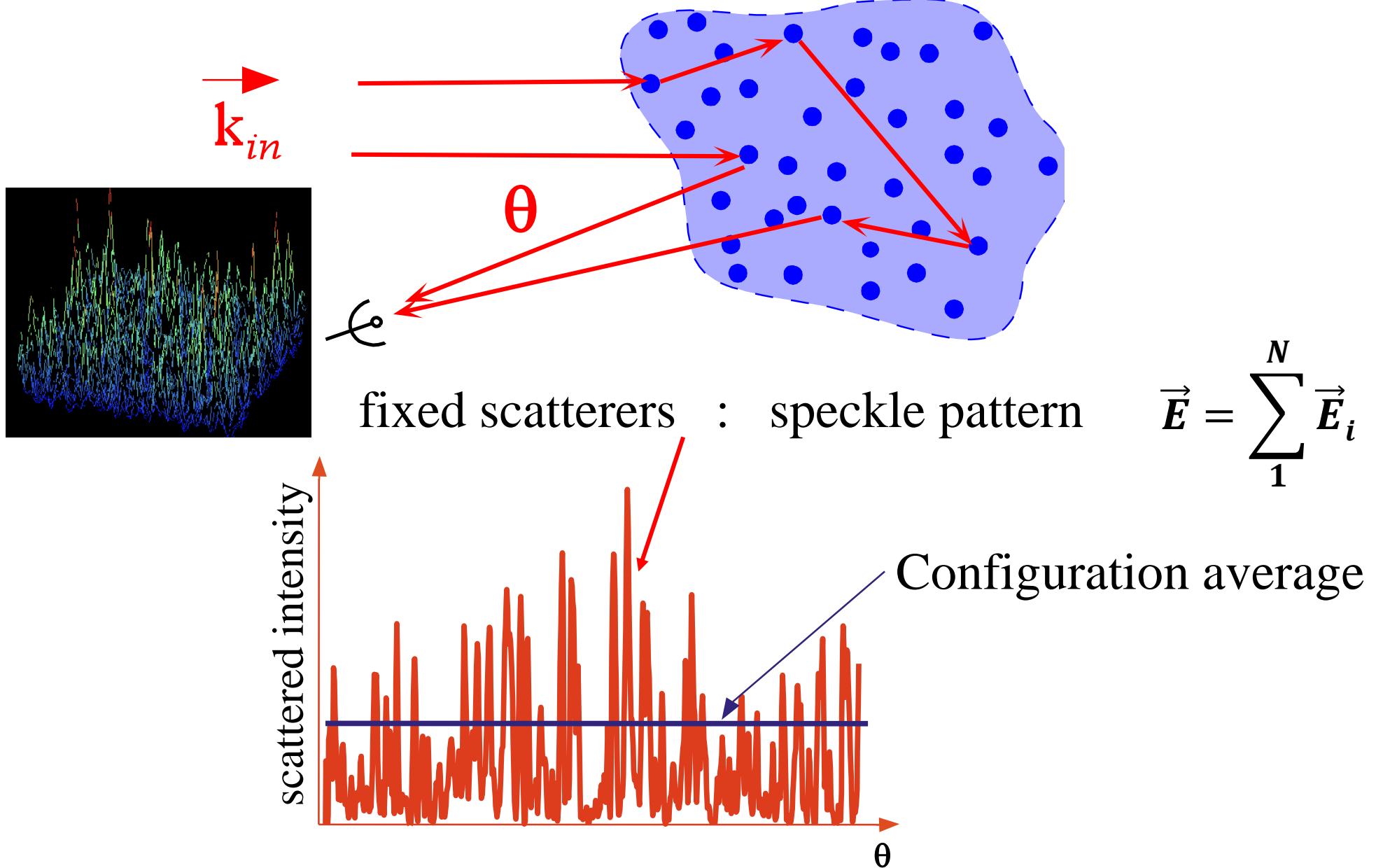
... are waves



Interference correction to Diffusion coefficient
 $D \approx D_0 [1 - 1/(k\ell)^2]$
Strong Localization ($D=0$) :
Ioffe-Regel criterium : $k\ell \approx 1$
(near field scattering $\ell \approx \lambda$)

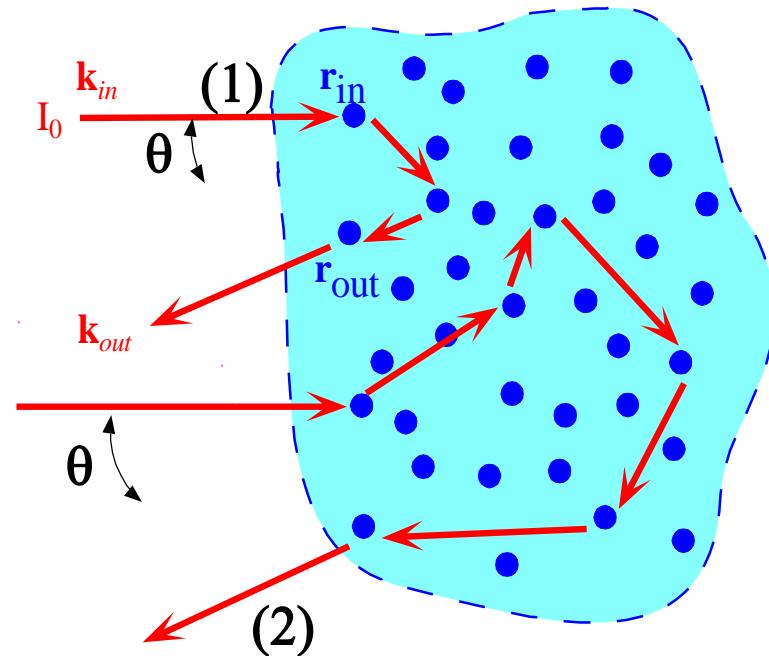


Wave effects : Interferences and speckle



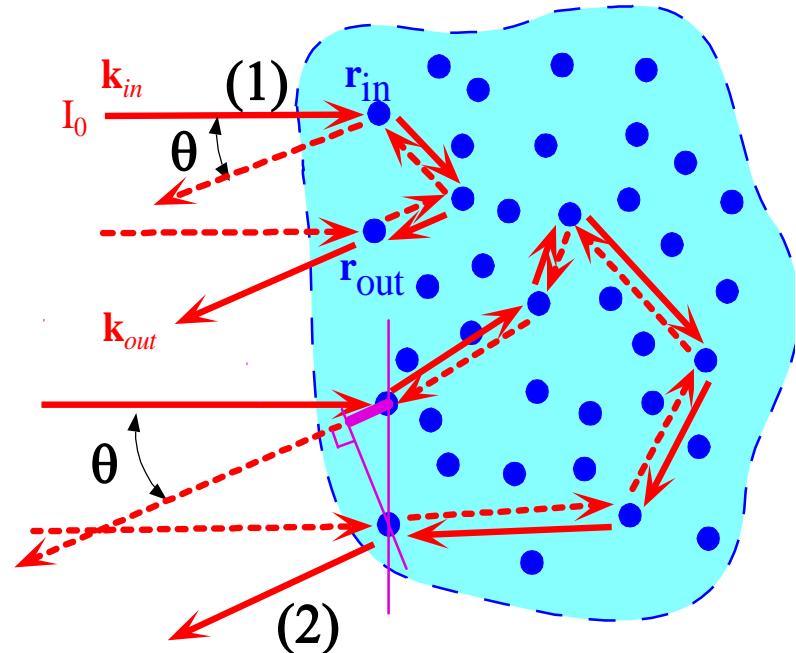
Configuration Averaged Intensity

- **uncorrelated** paths add incoherently



Configuration Averaged Intensity

- **uncorrelated** paths add incoherently
- **correlated** (i.e. reciprocal) paths add coherently

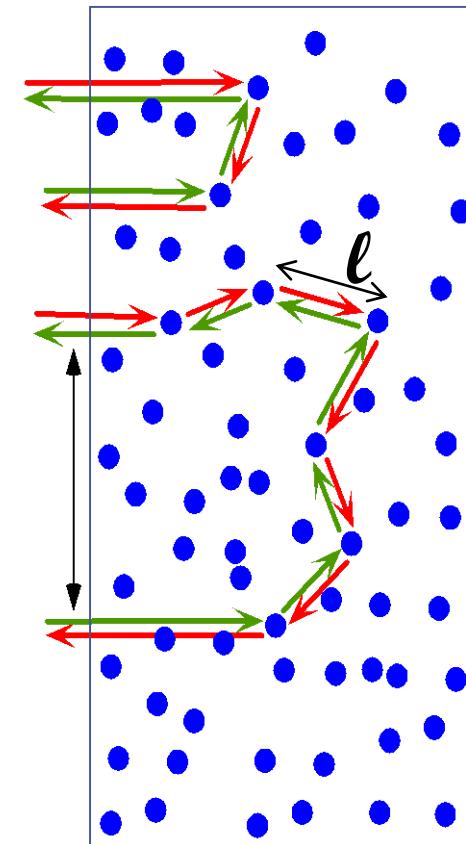
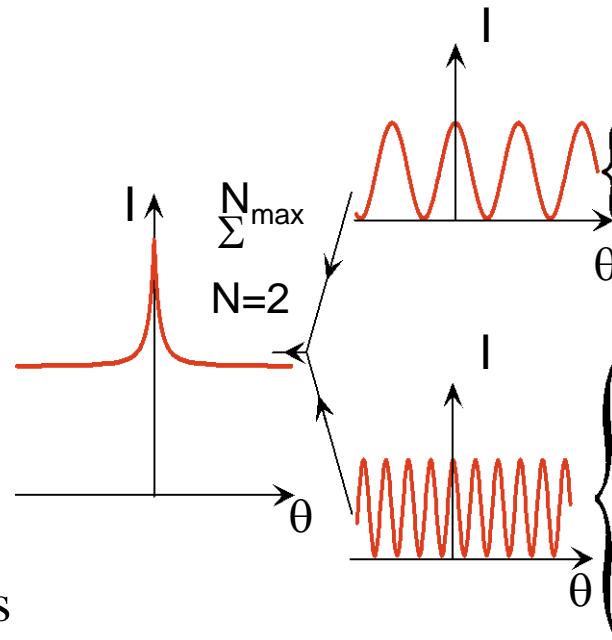


$$\Delta\varphi = (k_{in} + k_{out}) \cdot (r_{in} - r_{out}) \quad \theta=0 \Rightarrow \Delta\varphi = 0 \text{ for any path}$$

Coherent
Backscattering

$$\frac{\langle I(0) \rangle}{\langle I(\theta) \rangle} = 2$$

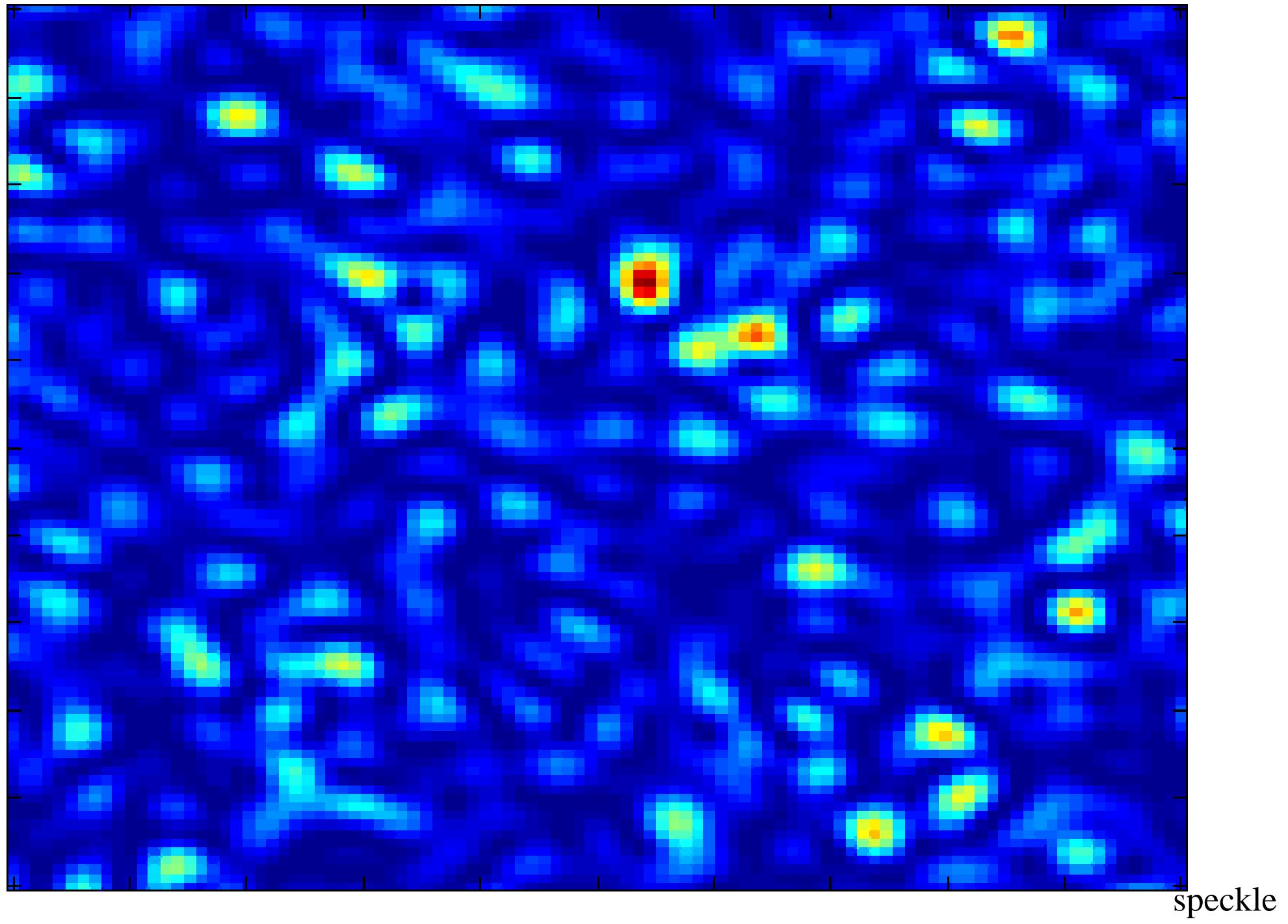
$$\text{CBS width : } \Delta\theta \approx \frac{\lambda}{l} \approx \frac{1}{kl}$$



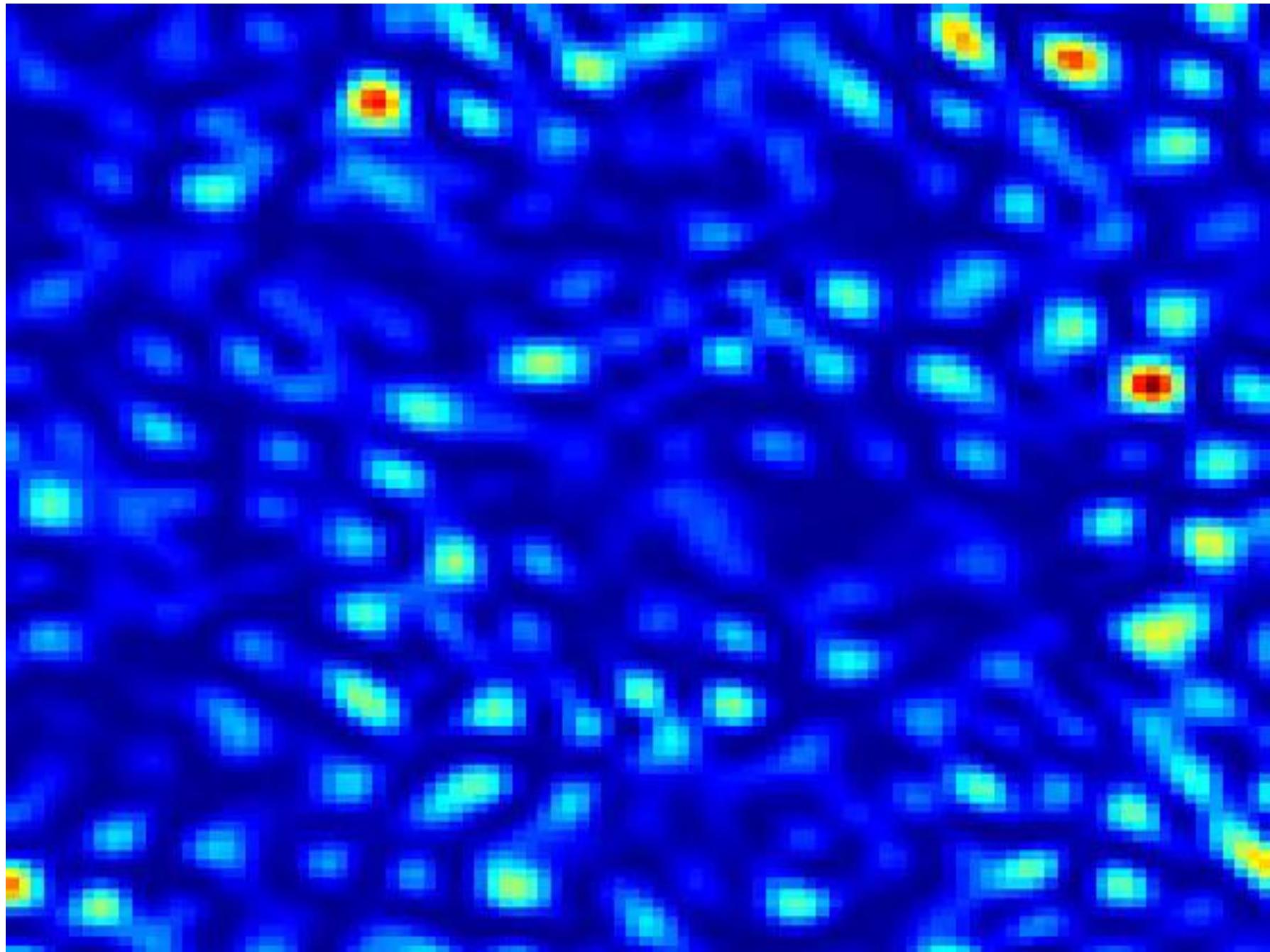
CBS height : reciprocal amplitudes
(phase, intensity)

Young double slits /
white light fringe /
self-aligned multiple
Sagnac interferometer

Fluctuating Speckle Pattern

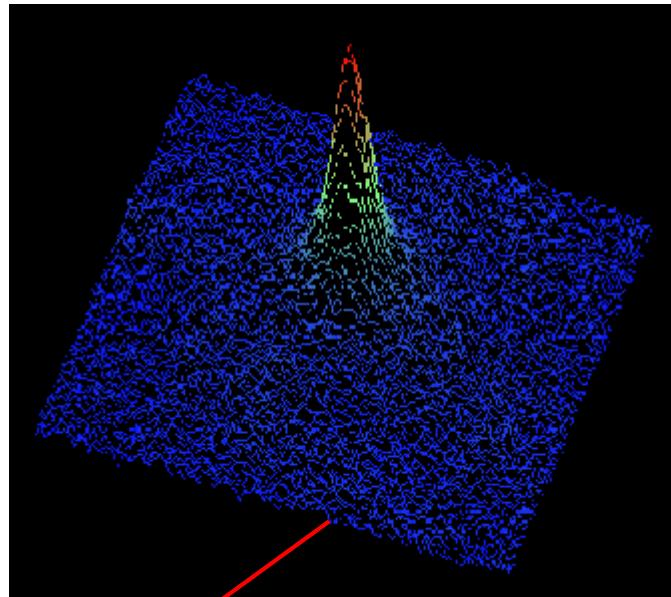
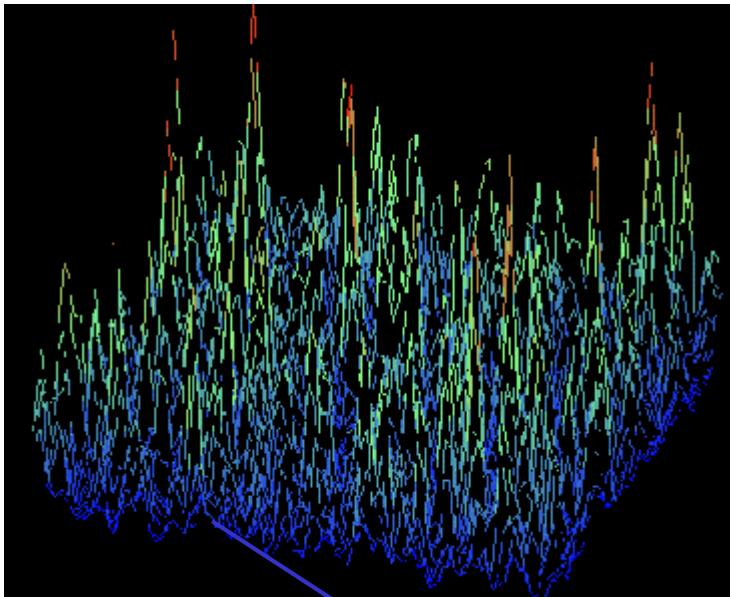


Integrated signal (configuration average)

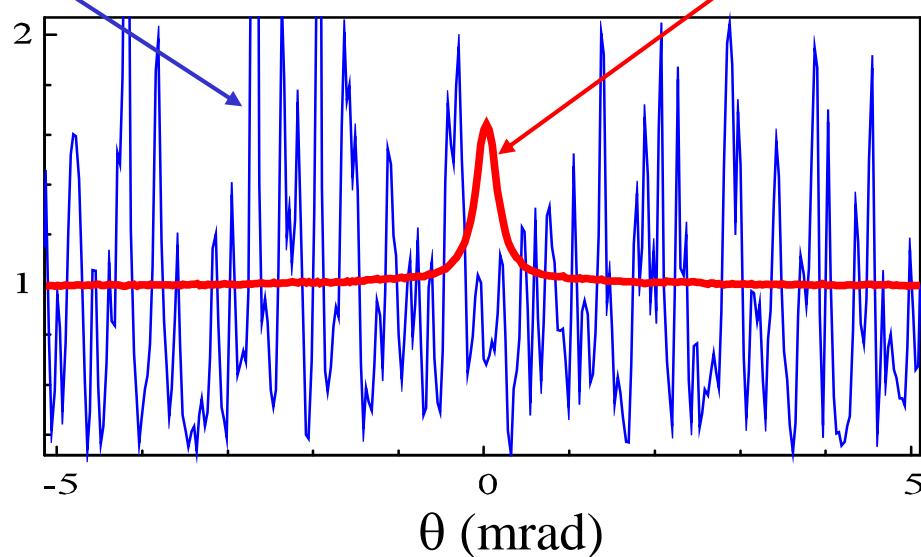


cone

Configuration Average

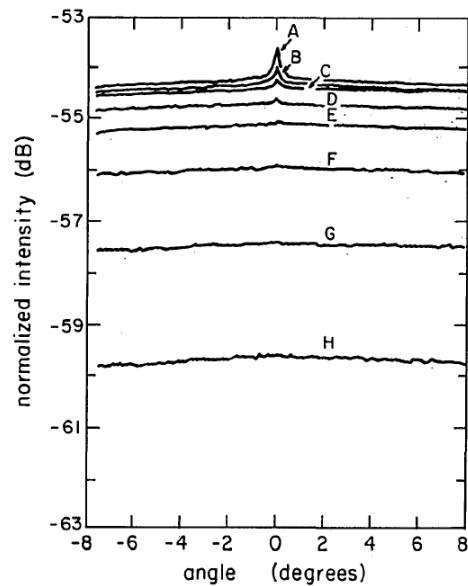


Single
realization

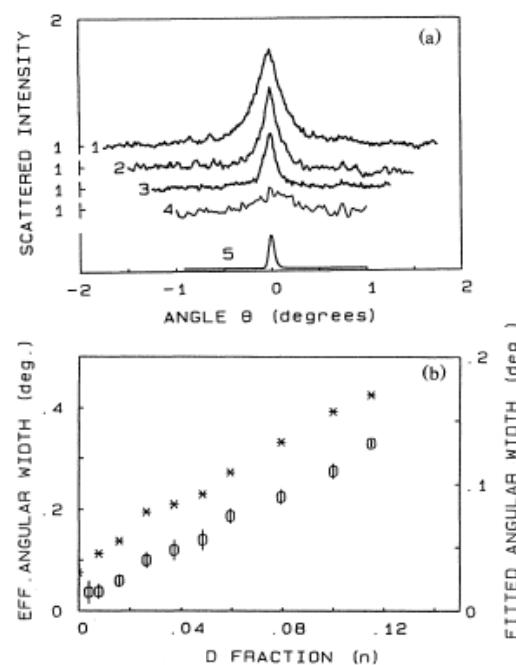


Configuration
average

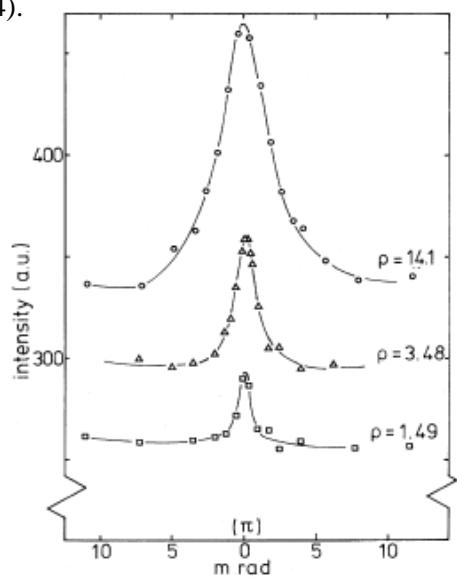
Coherent Backscattering and weak localization



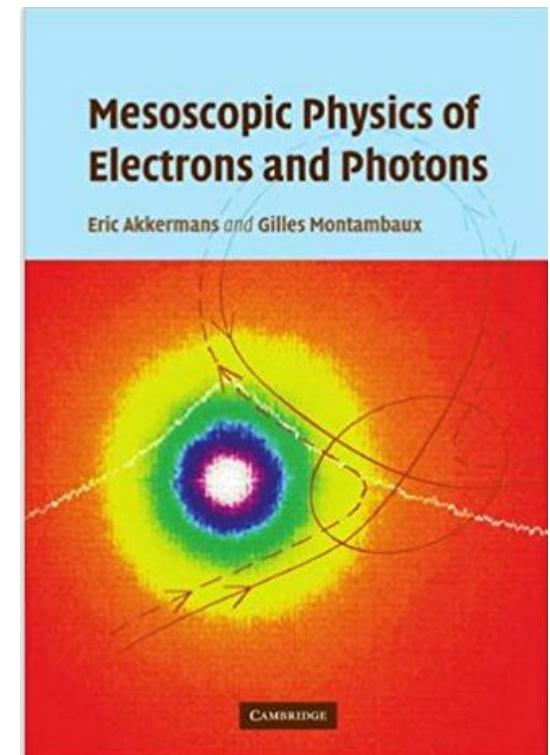
Y. Kuga and A. Ishimaru,
J. Opt. Soc. Am. A **8**, 831 (1984).



M. P. van Albada and A. Lagendijk,
Phys. Rev. Lett. 55, 2692 (1985).



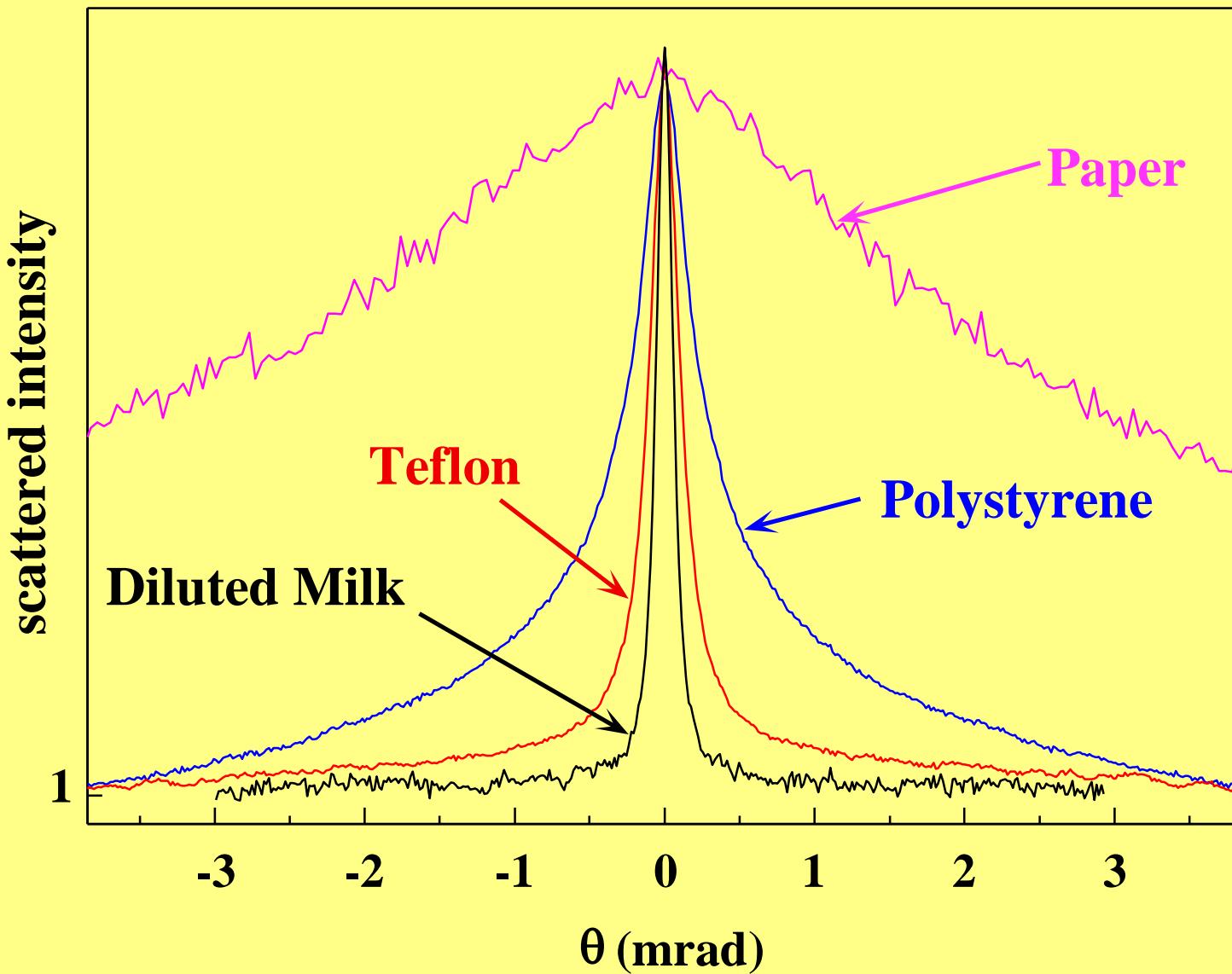
P. E. Wolf and G. Maret,
Phys. Rev. Lett. 55, 2696 (1985).



Diffusion approximation

$$I = I(0) \left[1 + \frac{3}{7} \frac{1}{(1 + \theta k l)^2} \left(1 + \frac{1 - e^{-4\theta k l / 3}}{\theta k l} \right) \right]$$

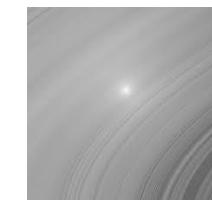
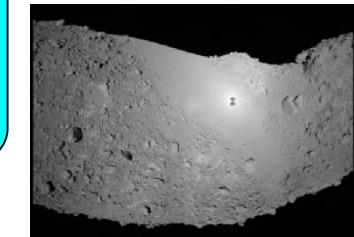
E. Akkermans et al., J. Phys. France 49, 77 (1988)



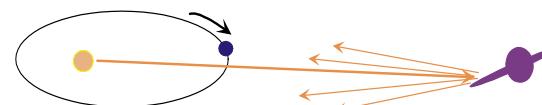
Coherent Backscattering

Light waves :

white paint (TiO_2), teflon, milk, paper, tissue

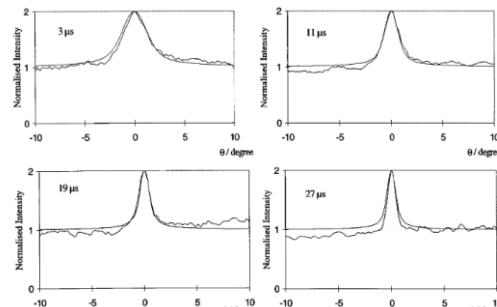


opposition surge : rings of Saturn



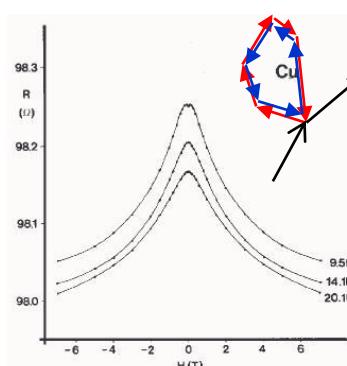
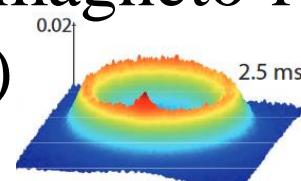
Acoustic waves :

metal rods

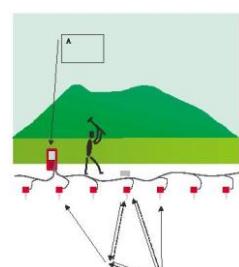


Matter waves :

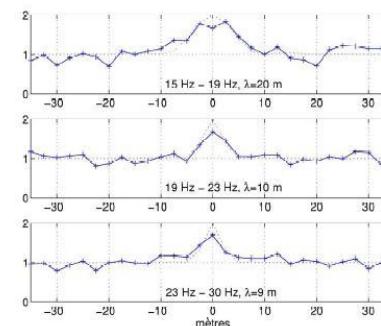
electrons : negative magneto-resistance



ultracold atoms (2D)



Seismic waves :



Coherent Backscattering vs Weak Localization:

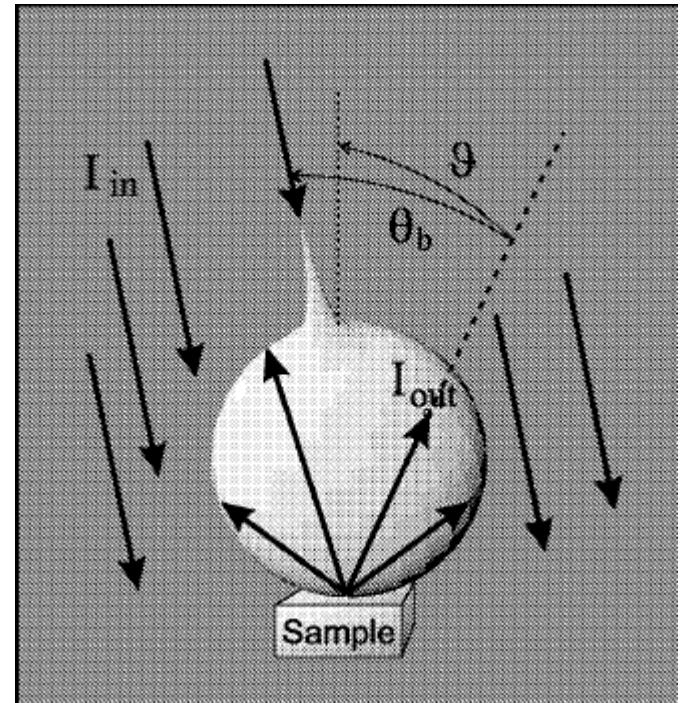
Interference correction to Diffusion coefficient

$$D \approx D_0 [1 - 1/(k\ell)^2]$$

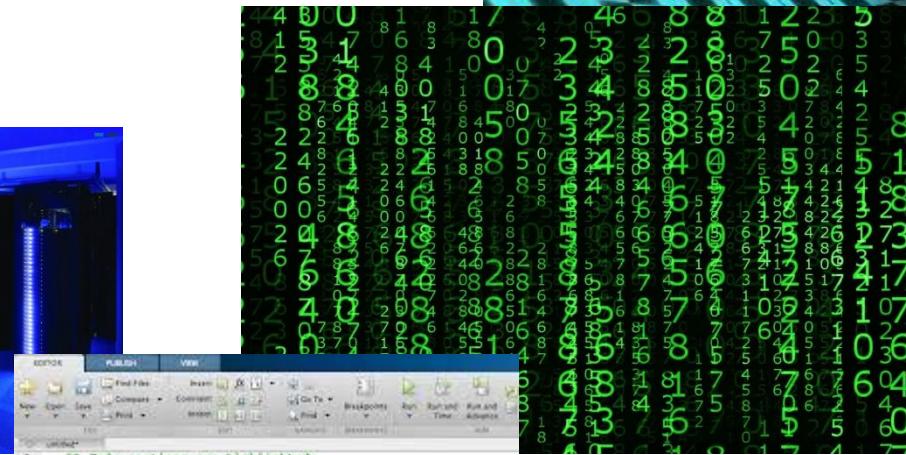
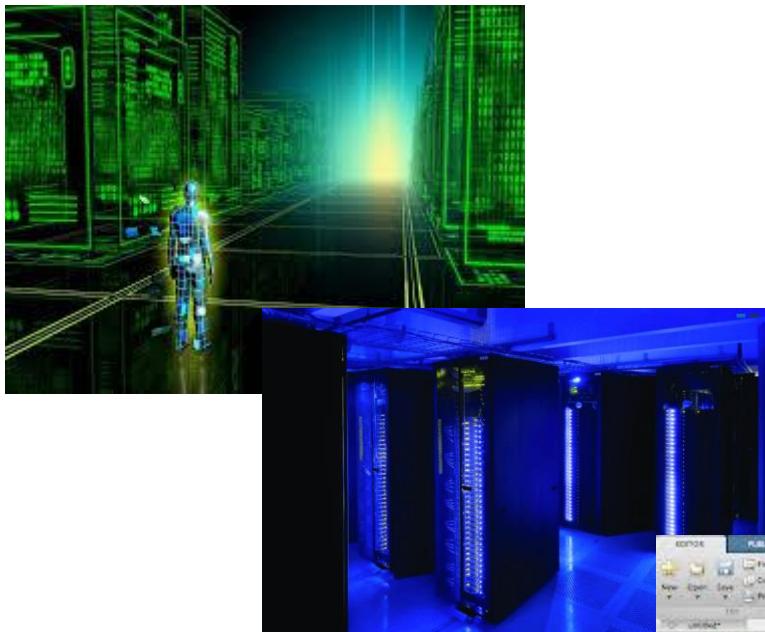
$$\langle r^2 \rangle \approx l^2 (1 - PCBS/P_{tot})$$

$$\langle r^2 \rangle \approx l^2 (1 - \Delta\theta_{CBS}^2/4\pi)$$

$$\langle r^2 \rangle \approx l^2 (1 - 1/k l^2)$$



Numerical Simulations



Link to Matlab codes of Robin Kaiser's lecture in Les Houches 2019

Random Walk based codes :

- [RandomWalk LH2019.m](#)
- [Ohm LH2019.m](#)
- [TransmissionProfile N100000.m](#)
- [RT LH2019.m](#)
- [RandomWalk CBS.m](#)

Coupled Dipole based codes :

- [CoupledDipoles LH2019.m](#)

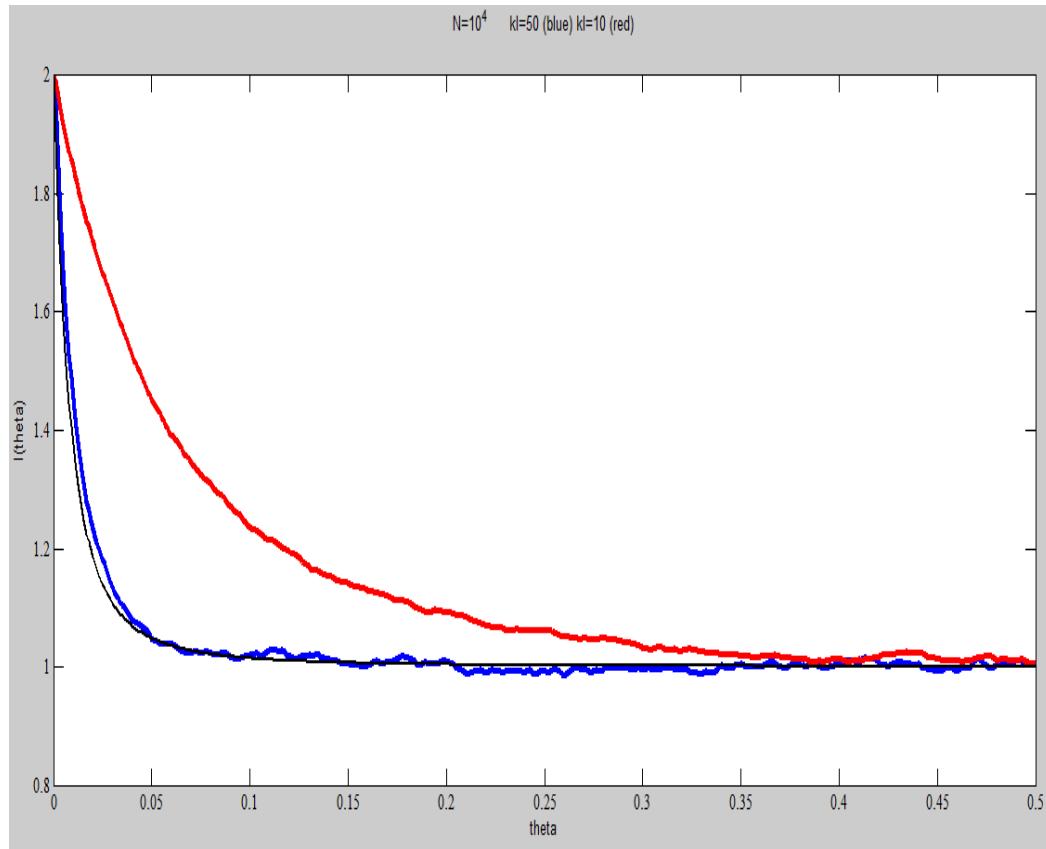
```

1 % Random walk (RW) in a finite slab
2 % All lengths (Lz, R, ell, x0, y0) have to be given in the same unit.
3 % The outputs are: Nsc the number of scattering event (0 for transmitted photons), the output angles theta, phi, the output positions x, y
4 %
5 % Parameters of the slab
6 Lz=1000; % length of the slab
7 R=2000; %transverse size
8 ell=20; %scattering mean free path
9
10 theta_out=(0:0.001:0.5);
11 phi_out=(0:0.001:0.5);
12
13 Nph=1000; % number of photons
14 for iph=1:1:Nph
15
16 % Initial Conditions
17 Nsc=0; % number of scattering events already done
18 cos_phi = 1; theta=0; % direction of propagation
19 x=0; %initial position along x
20 y=0; %initial position along y
21 z=0; % Initial impact with the vapor (NOT the 1st scattering event!)
22 r=sqrt(x^2+y^2);
23
24 % length of the 1st step using an exponential distribution (Beer-Lambert)
25 step = -ell*log(rand); % use the inverse function f^(-1) to draw the random number following a distribution f: exp -> log
26
27 % The first step is ballistic in forward direction
28 z = z+step;
29 zl=z;
30
31 while all(z<=Lz && z>0 && r<R) % Next scattering event inside the slab
32
33 Nsc = Nsc+1; % add 1 to number of scattering events
34
35 % record the trajectory (optional)
36 X(Nsc) = x;
37 Y(Nsc) = y;
38 Z(Nsc) = z;
39
40 % draw a new random direction
41 theta = 2*pi*rand;
42 cos_phi = -1+2*rand; % uniform between -1 and 1 (always check that the distribution is uniform: mistakes often happen with cos, sin etc)
43 sin_phi = sqrt(1-cos_phi^2); % always >0 since phi is between 0 and pi
44
45 % length of the next step in an exponential ditribution (Beer-Lambert)
46 step = -ell*log(rand);
47
48 % new position
49 x = x + step*sin_phi*cos(theta) ;
50 y = y + step*sin_phi*sin(theta);
51 z = z + step*cos_phi;
52 r = sqrt(x^2+y^2);
53
54 % record the trajectory (optional)
55 X(Nsc) = x;
56 Y(Nsc) = y;
57 Z(Nsc) = z;
58
59 end
60 phi = acos(cos_phi);
61
62 % plotting the trajectory (just for illustration)
63 %figure(1), plot3([0 X], [0 Y], [0 Z]), xlabel('x/l'), ylabel('y/l'), zlabel('z/l'), axis([-R R -R R -1 Lz+1]);
64 Nfinx(iph)=x;
65 Nfiny(iph)=y;
66 Nfinz(iph)=z;
67
68 % Adding the reverse path with the corresponding dephasing
69 %(in the y=0 plane)
70 dICBS(iph,:)=1+cos(sin(theta_out)*x+(1-cos(theta_out))*(z-zl));
71 end;
72 ICBS=sum(dICBS)/Nph;
73 plot(theta_out, ICBS,'DisplayName','ICBS','YDataSource','ICBS');figure(gcf)
74 xlabel ('theta');
75 ylabel('I(theta)');
76
77 % Diffusion limit formula
78 %IcbsNorm=l+(3/7).*l+(1-exp(-4*k1*theta_out/3))./(k1.*theta_out).^(2);

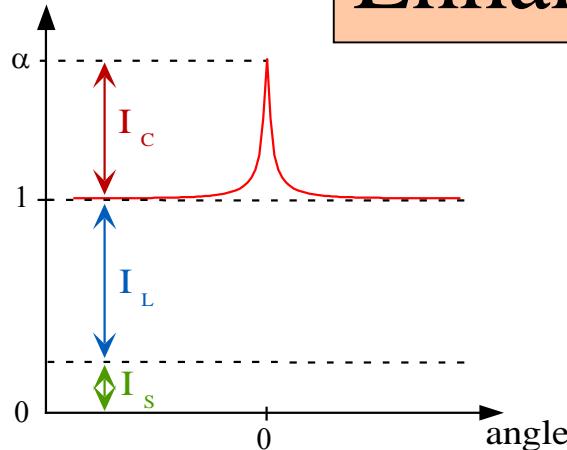
```

$$I = \frac{1}{2} \frac{3}{8\pi} \left[1 + 2 \frac{z_0}{l} + \frac{1}{(1+ql)^2} \left(1 + \frac{1 - e^{-2qz_0}}{ql} \right) \right]$$

$$I = I(0) \left[1 + \frac{3}{7} \frac{1}{(1+\theta kl)^2} \left(1 + \frac{1 - e^{-4\theta kl/3}}{\theta kl} \right) \right]$$



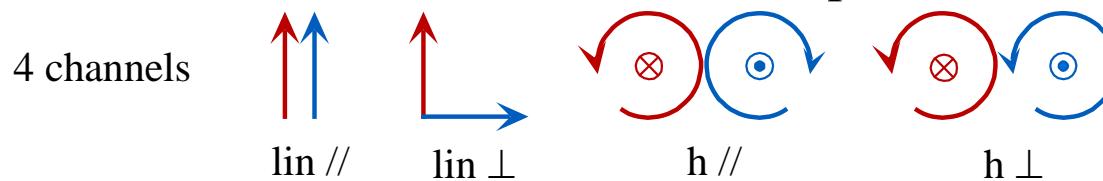
Enhancement Factor = 2 ?



CBS enhancement :

$$\alpha = 1 + I_c / (I_s + I_L) \leq 2$$

influence of polarization :



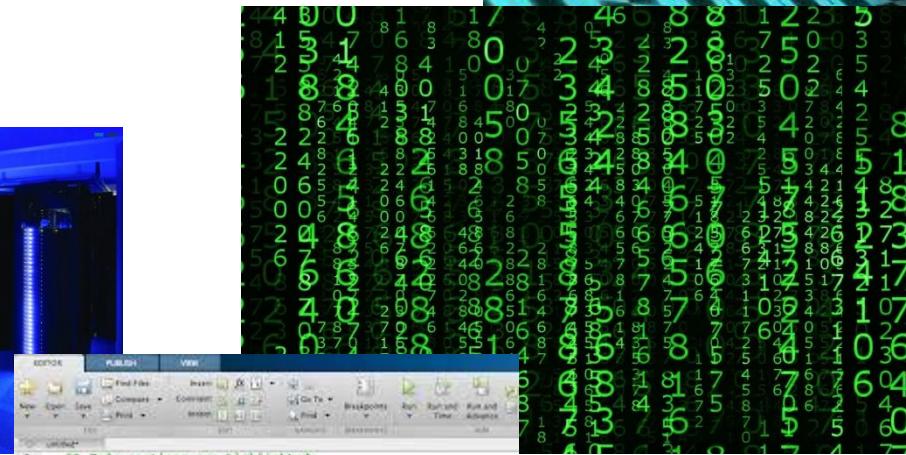
"classical"
scatterers

	lin //	lin ⊥	h //	h ⊥
I_s	> 0	$= 0$	$= 0$	> 0
I_c	$= I_L$	$< I_L$	$= I_L$	$< I_L$
α	< 2	$\ll 2$	$= 2$	$\ll 2$

Reciprocity applies

Single scattering=offset

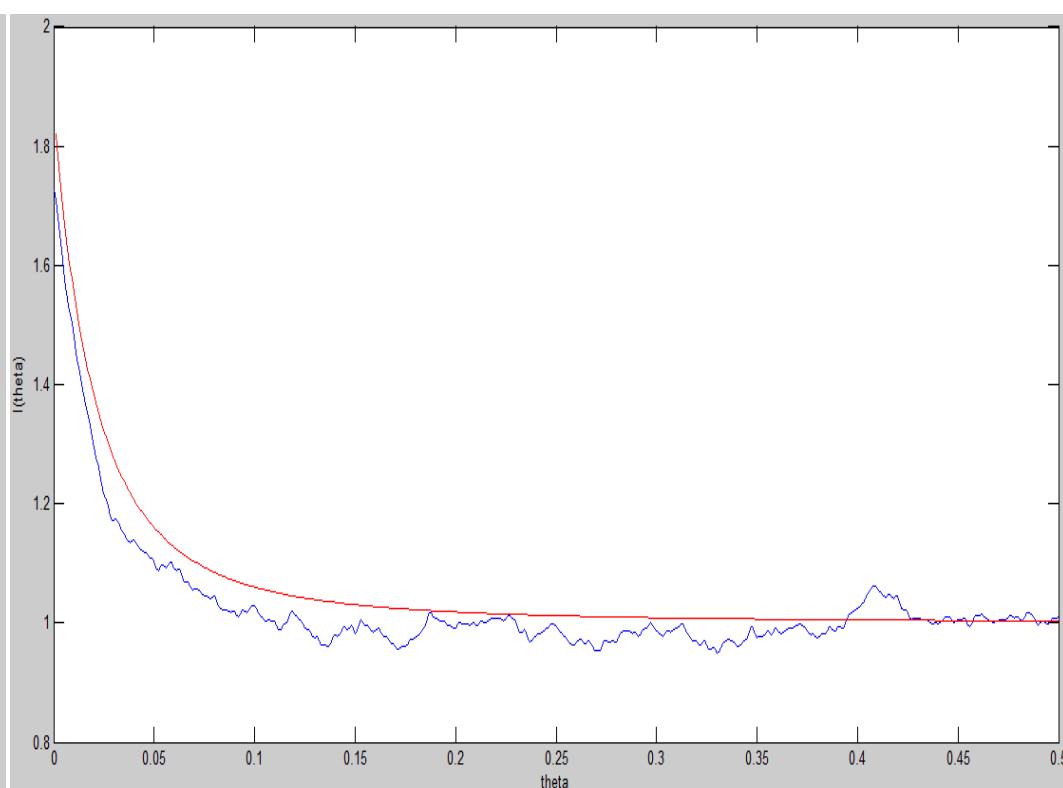
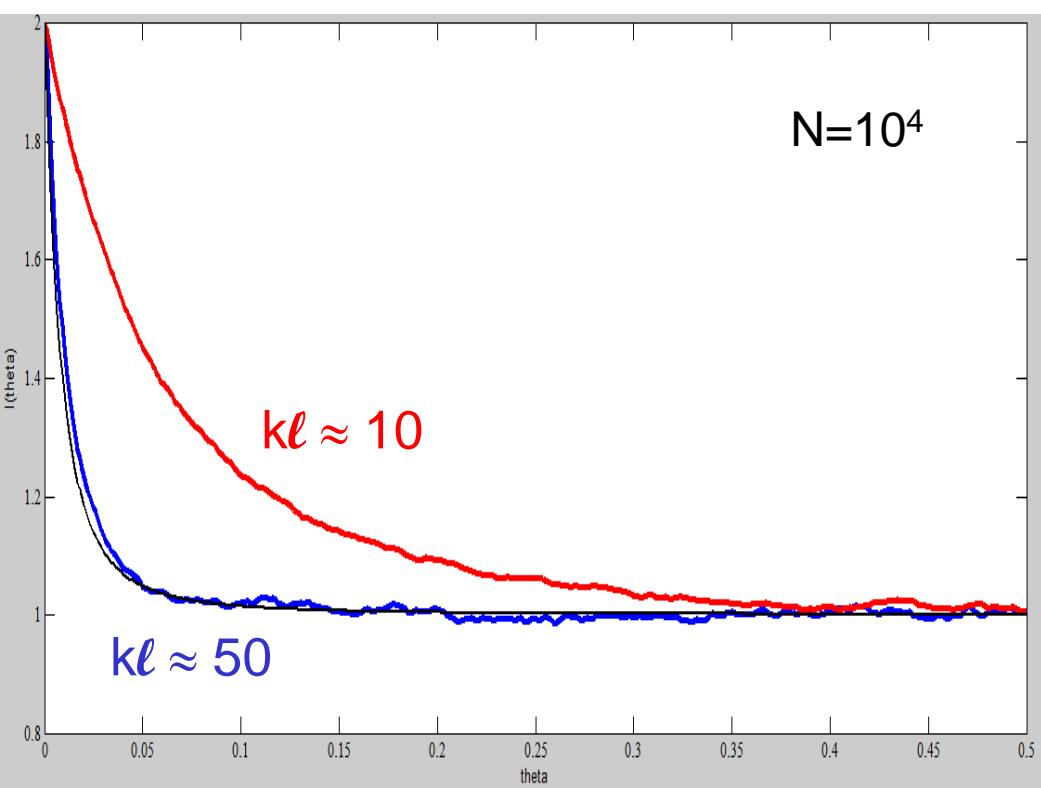
Numerical Simulations

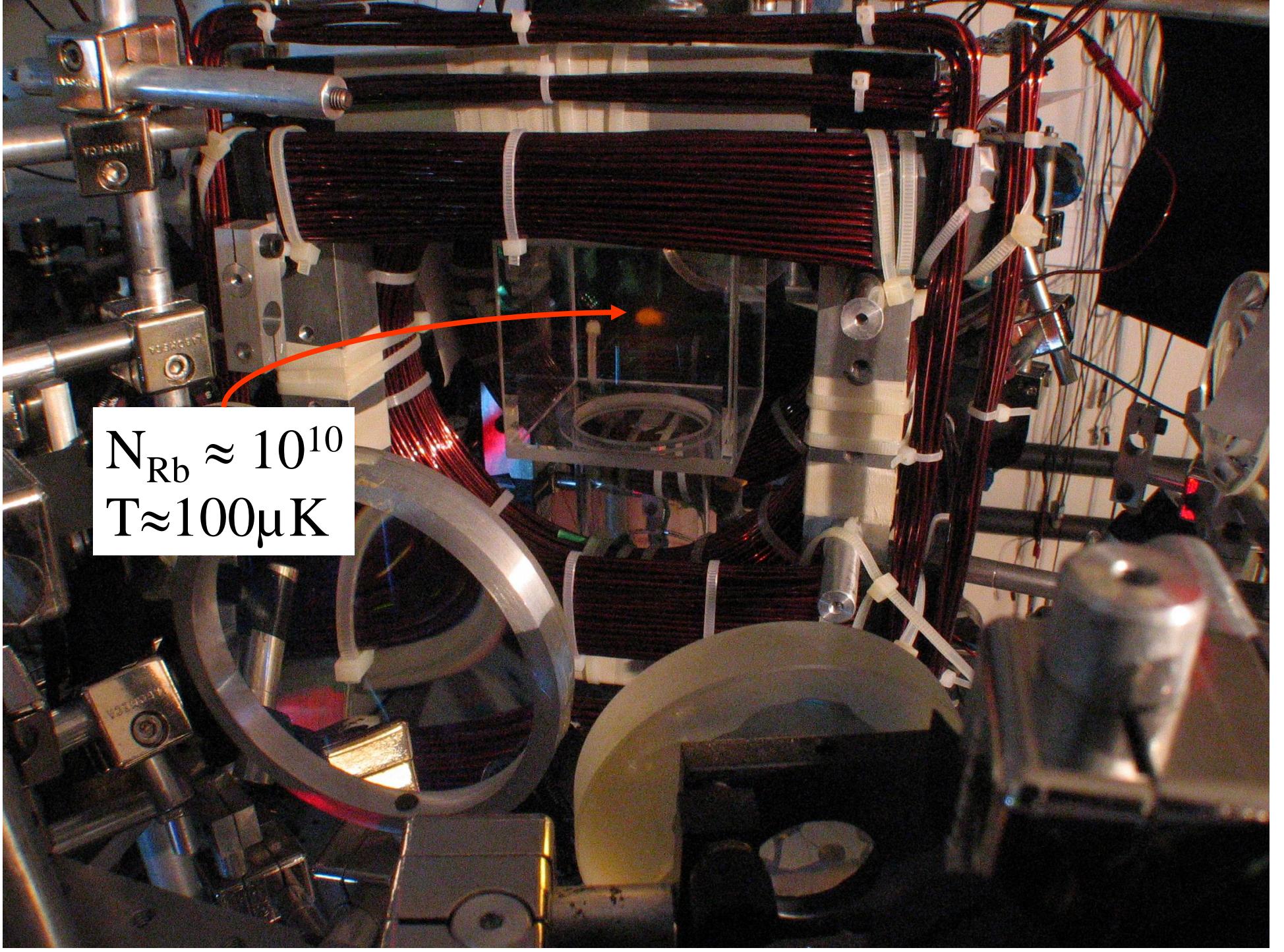


```

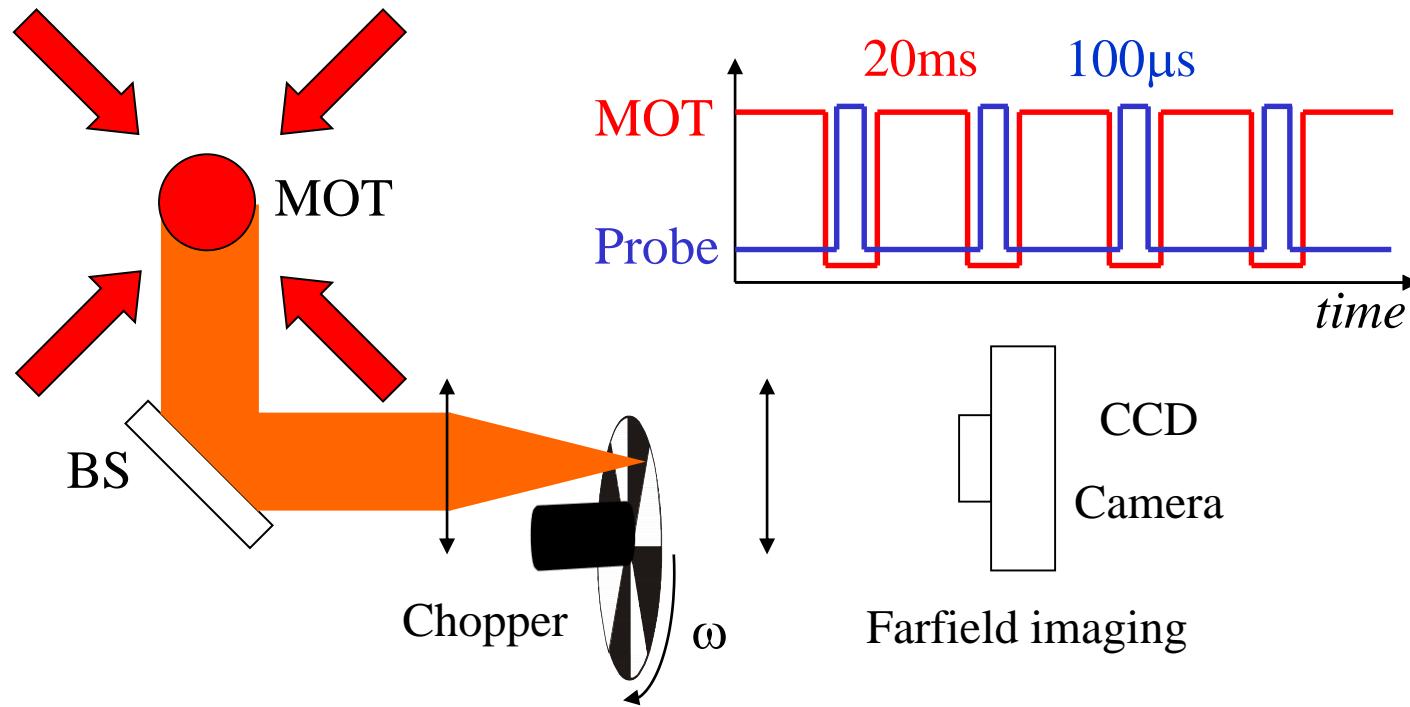
69 % Adding the reverse path with the corresponding dephasing
70 % but only for double scattering!
71 %(in the y=0 plane)
72 -
73 -     dICBS(iph,:)=1+cos(sin(theta_out)*x+(1-cos(theta_out))*(z-zl));
74 -
75 - else
76 -     dICBS(iph,:)=1;
77 end;
78 if (z<0)
79     Rdiff=Rdiff+1; %just to count diffuse transmission
80     Rdifftime(iph)=Nsc; % register number of events, which we put to time
81 end
82 end;
83 ICBS=sum(dICBS)/Nph;
84 plot(theta_out, ICBS,'DisplayName','ICBS','YDataSource','ICBS');figure(gcf)
85 xlabel('theta');
86 ylabel('I(theta)');
87
88 % Analytical Formula
89 xh=(1:1:100);
90 hh=hist(Rdifftime, xh);
91 BackScattering=sum(hh(2:1:100));
92 SingleScattering=hh(2)/BackScattering; % hh(1) is unscattered light
93 MultipleScattering=sum(hh(3:1:100))/BackScattering;
94 %Diffusion limit formula with reduction for single scattering
95 kl=ell;
96 Enhancement_MS=(1+(3/7).* (1+(1-exp(-4*kl*theta_out/3))./(kl.*theta_out))./(1+kl*theta_out).^2);
97 IcbsNorm=SingleScattering+MultipleScattering*Enhancement_MS;
98 %IcbsNorm=MultipleScattering*Enhancement/(SingleScattering+MultipleScattering);
99 hold on;
100 plot(theta_out, IcbsNorm,'Color','red','DisplayName','ICBS','YDataSource','ICBS');figure(gcf)

```



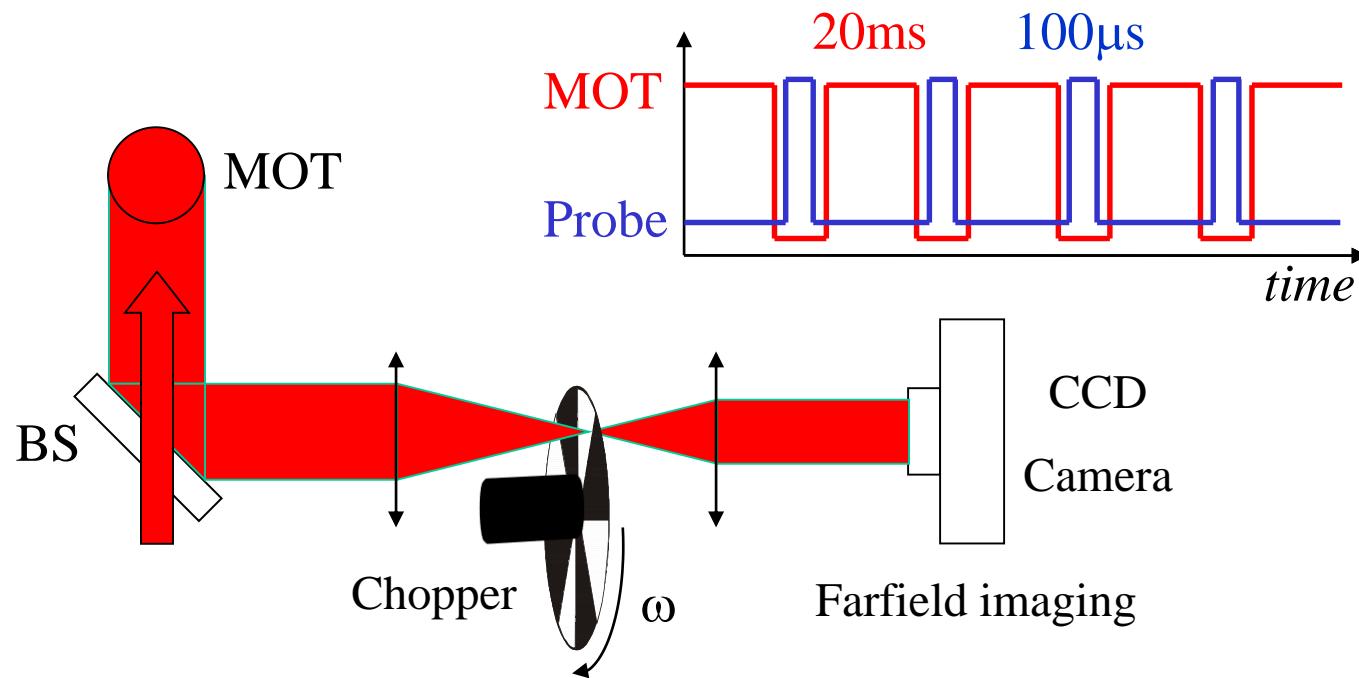

$$N_{Rb} \approx 10^{10}$$
$$T \approx 100 \mu K$$

Time sequence



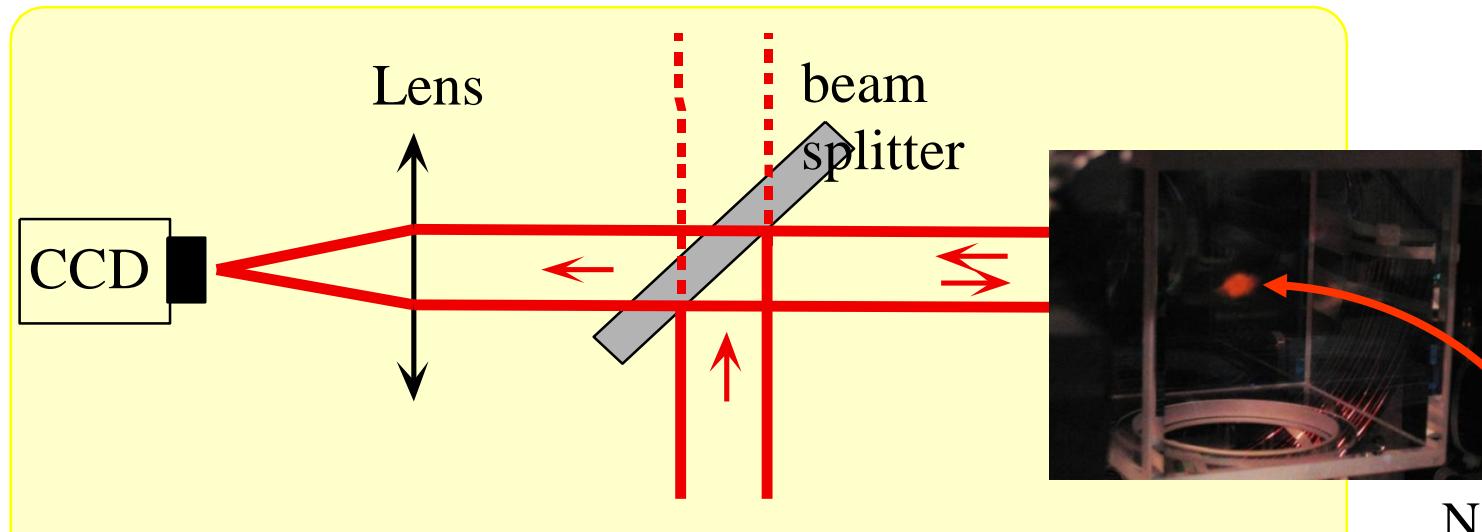
300 photons/pixel/second
1 image ~ every 10-20 minutes

Time sequence

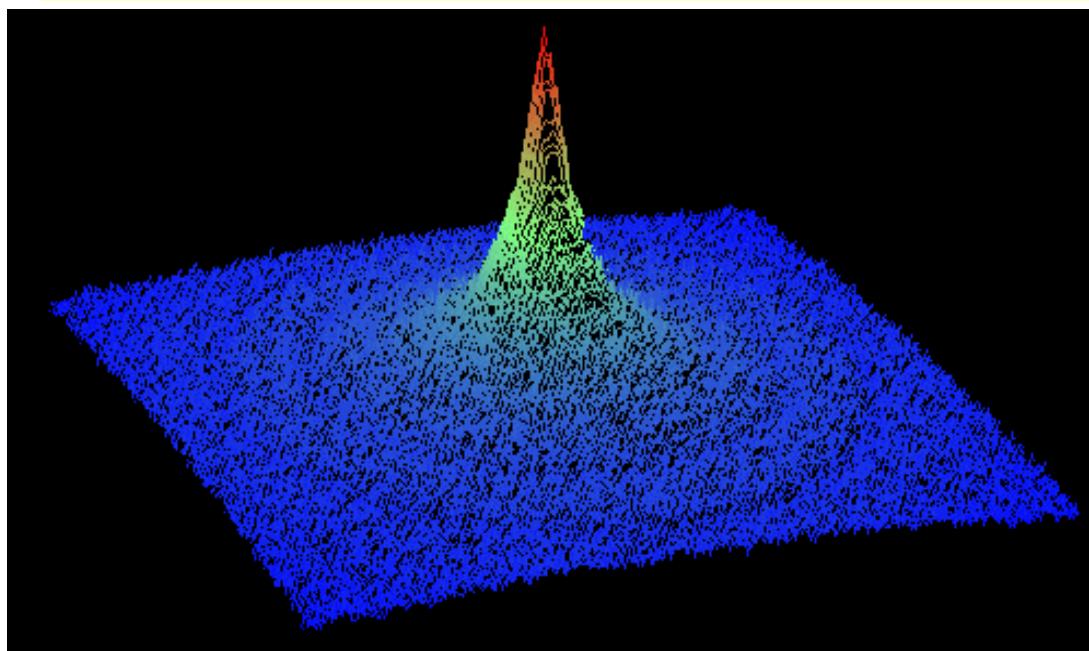


300 photons/pixel/second
1 image ~ every 10-20 minutes

Weak Localisation = precursor of strong Localisation?



$N \approx 10^{10}$
 $T \approx 100\mu\text{K}$
 $k\ell \approx 1000$

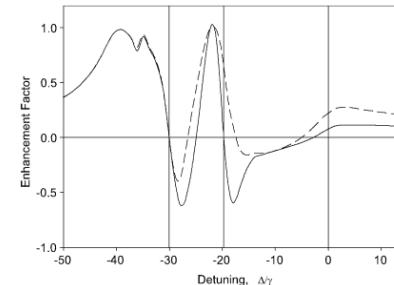


Coherence after
resonant scattering
with atoms !

also : Mark Havey group

Quantitative studies of coherent backscattering of light by cold atoms

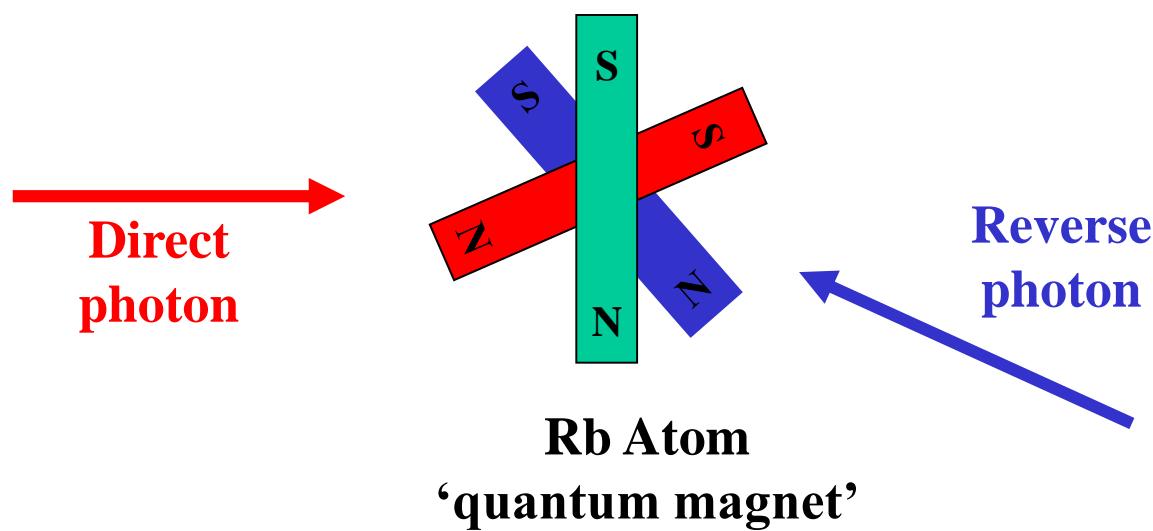
- Internal structure ✓
- Magnetic field ✓
- Atomic Motion ✓
- Inelastic scattering ✓
- Antilocalisation ✗
- High density limit ✗
- ...



D. Kupriyanov et al., Opt. Comm. 243 165 (2004)

Influence of internal Zeeman structure

degenerated ground state : “quantum magnets” \Rightarrow broken reciprocity
 \Rightarrow reduced contrast !



Polarization encoded in Zeeman substates

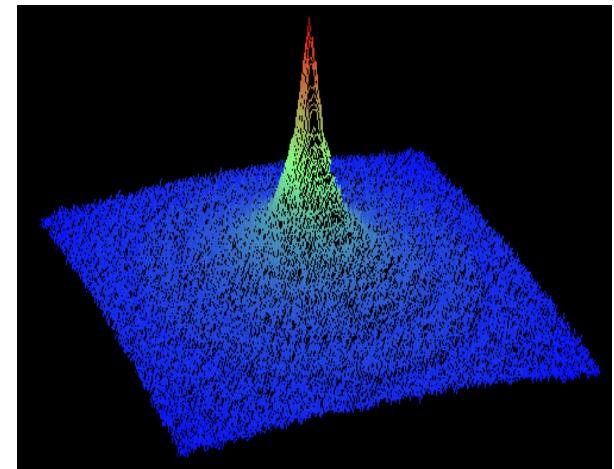
- \rightarrow reversed paths are distinguishable
- \rightarrow loss of contrast

Theory :

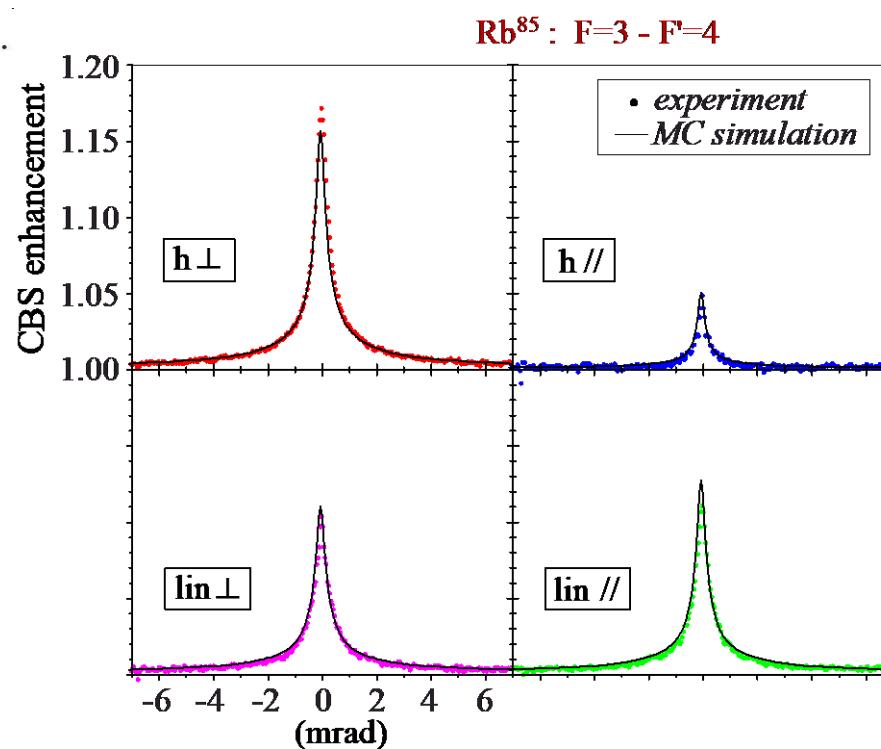
- no “exact” solution
- diagrammatic approach

$$R \approx L = \begin{array}{c} \otimes \\ \otimes \end{array} + \begin{array}{c} \otimes \quad \otimes \\ \vdots \quad \vdots \\ \otimes \quad \otimes \end{array} + \begin{array}{c} \otimes \quad \otimes \quad \otimes \quad \otimes \\ \vdots \quad \vdots \quad \vdots \quad \vdots \\ \otimes \quad \otimes \quad \otimes \quad \otimes \end{array} + \dots$$

$$C = \begin{array}{c} \otimes \quad \otimes \\ \otimes \quad \otimes \end{array} + \begin{array}{c} \otimes \quad \otimes \quad \otimes \\ \vdots \quad \vdots \quad \vdots \\ \otimes \quad \otimes \quad \otimes \end{array} + \dots$$



Excellent agreement
(no free parameter)



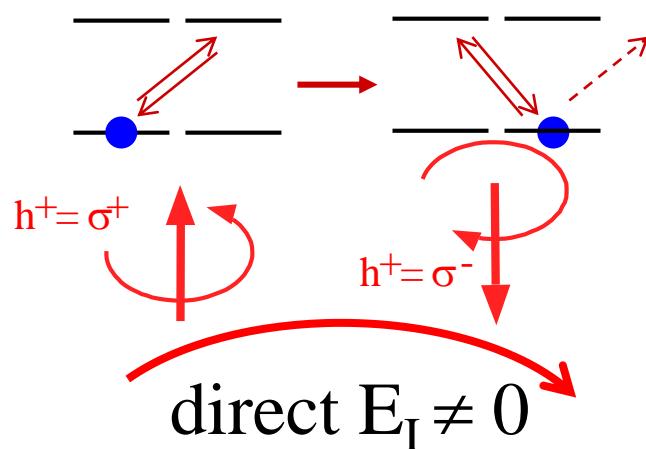
Surprise :
Not the
'correct'
polarization
channel

Influence of internal structure

degenerated ground state : “quantum magnets” \Rightarrow broken reciprocity
 \Rightarrow reduced contrast !

- Intensity effect :

e.g. Rayleigh scattering on $J=1/2 \rightarrow J'=1/2$

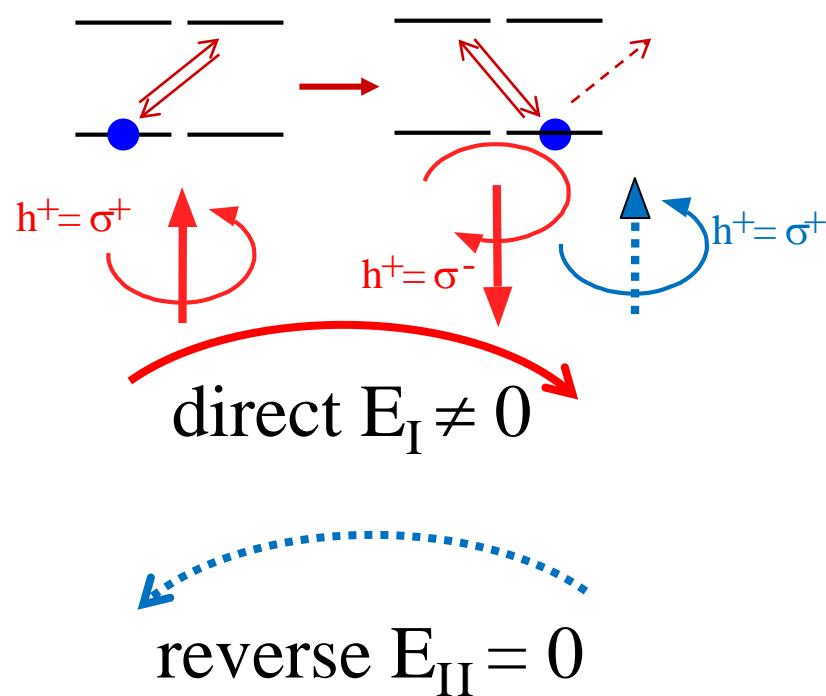


Influence of internal structure

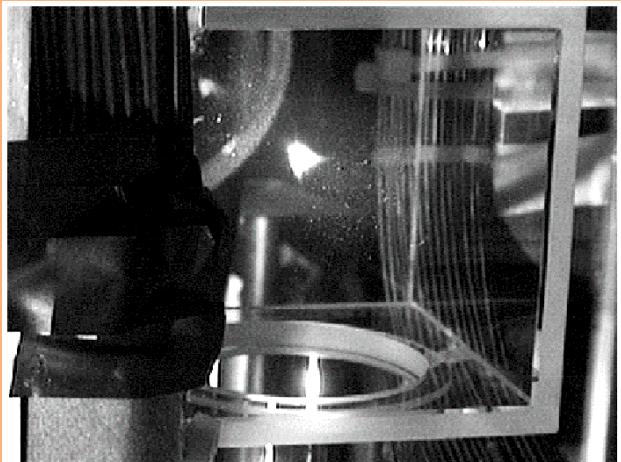
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e.g. Rayleigh scattering on $J=1/2 \rightarrow J'=1/2$

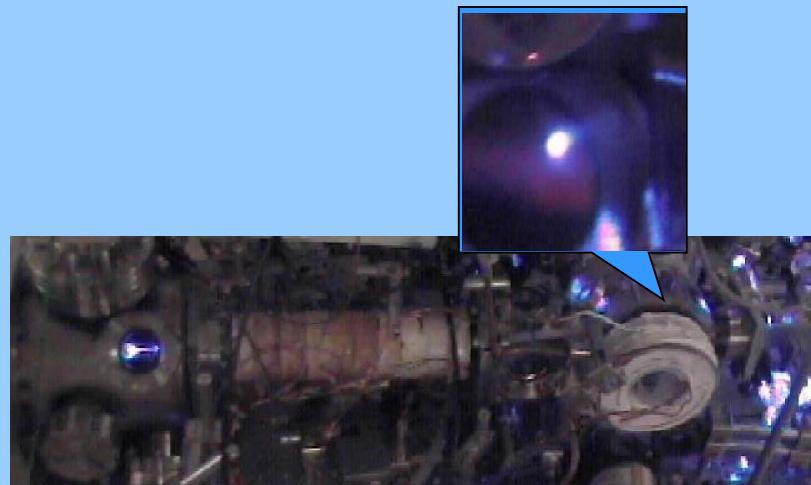


Rubidium ($F=3 \rightarrow F'=4$)



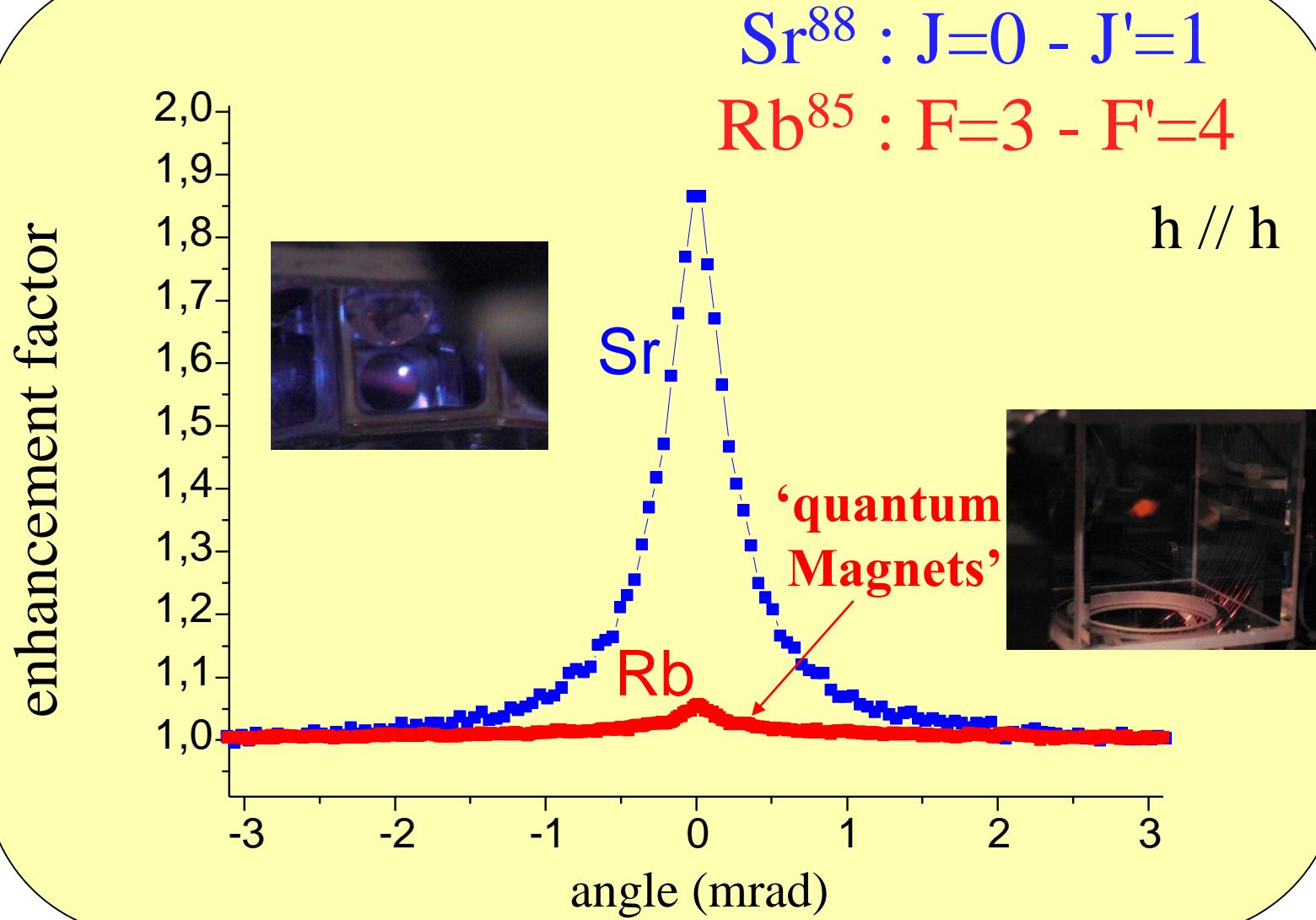
vapor trap :
optical thickness : 40

Strontium ($J=0 \rightarrow J'=1$)



Zeeman slower :
optical thickness : 3

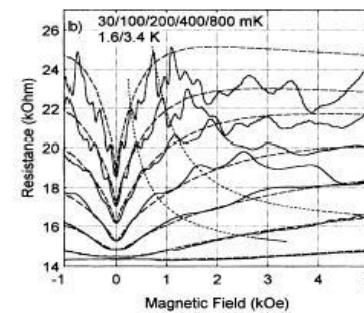
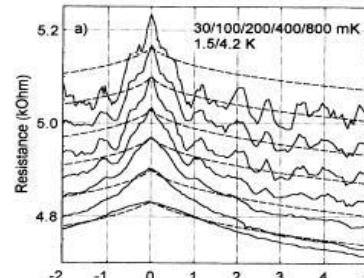
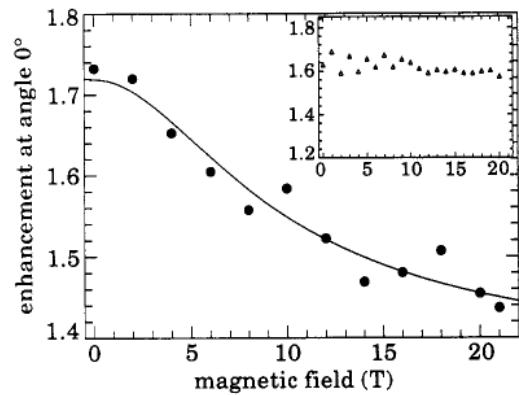
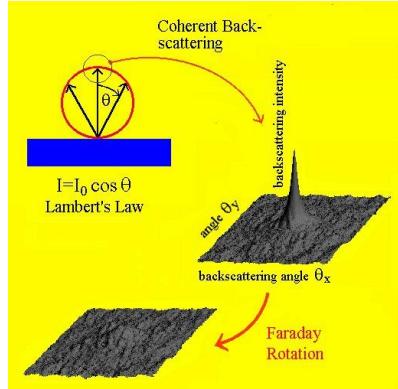
Influence of internal structure



Y. Bidel et al., Phys. Rev. Lett. **88**, 203902 (2002)

Atoms with $J=0$ to $J=1$ behaves like classical Rayleigh scatterer

Weak Localization in presence of Magnetic fields



negative
magneto-resistance

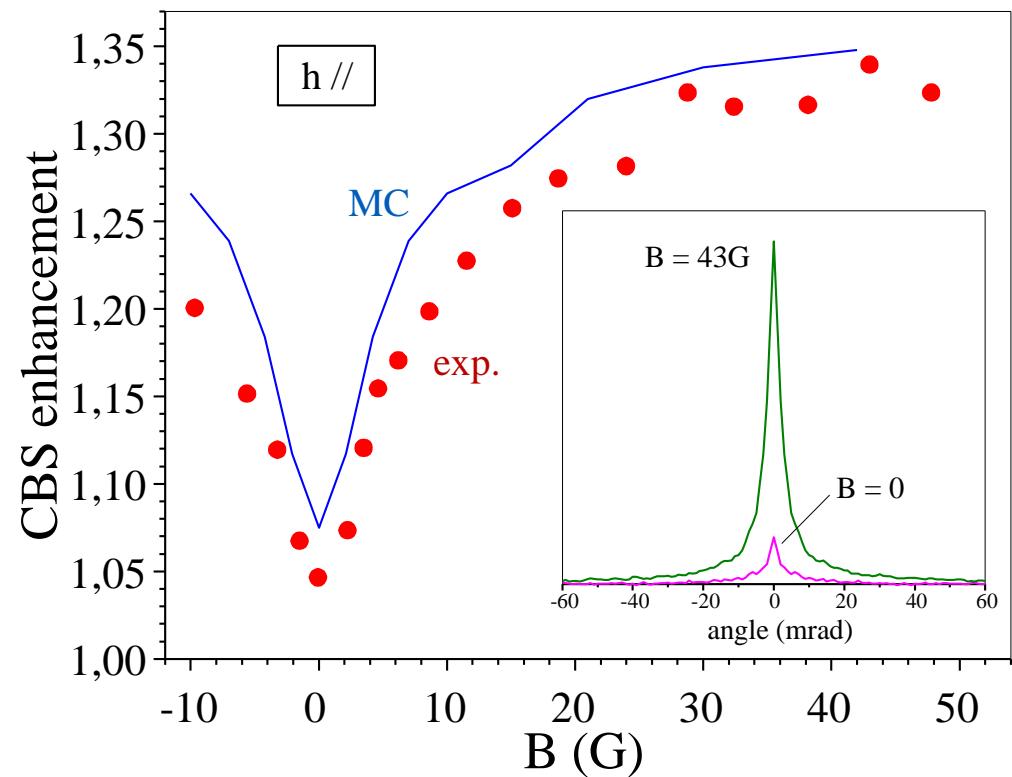
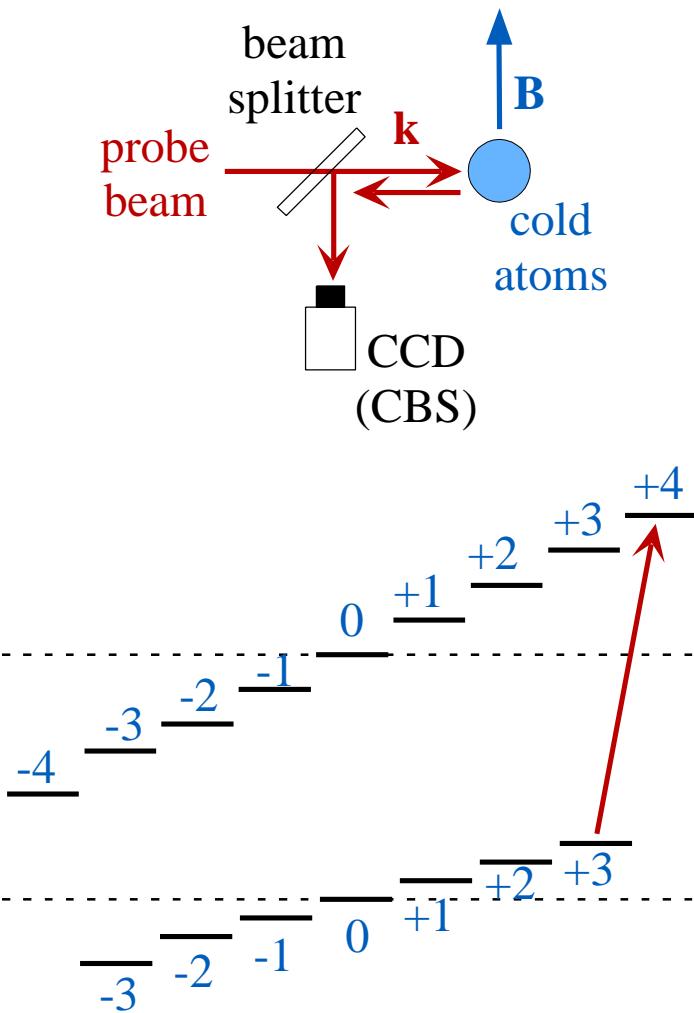
increased
weak localization :
magnetic impurities
+ magnetic field

FIG. 2. Resistance changes as a function of the magnetic field for the wires of n^+ -Cd_{1-x}Mn_xTe with $x = 0$ (a) and $x = 1\%$ (b) at various temperatures between 30 mK and 4.2 K (traces for the lowest temperatures are shifted upward). Dashed lines represent magnetoresistance calculated in the framework of 3D weak-localization theory [4,14]. Dotted lines are guides for the eye, and visualize a strong temperature dependence of the resistance features in Cd_{0.99}Mn_{0.01}Te (b).

from Phys. Rev. Lett. **75**, 3170 (1995)

F. Erbacher, R. Lenke, G. Maret, Europhys. Lett. **21**, 551 (1993)

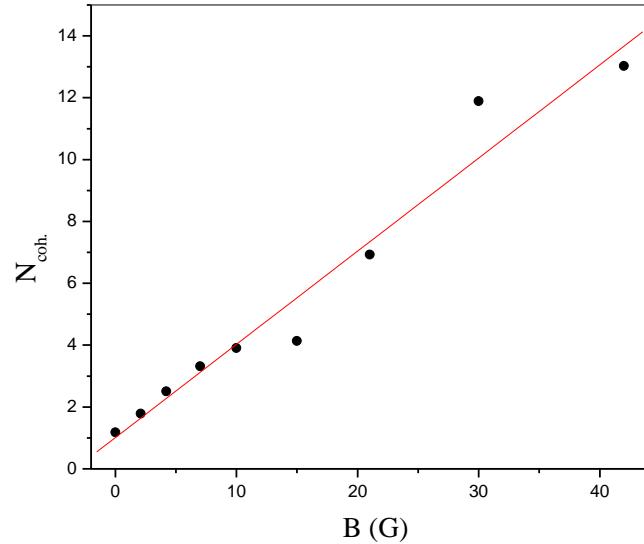
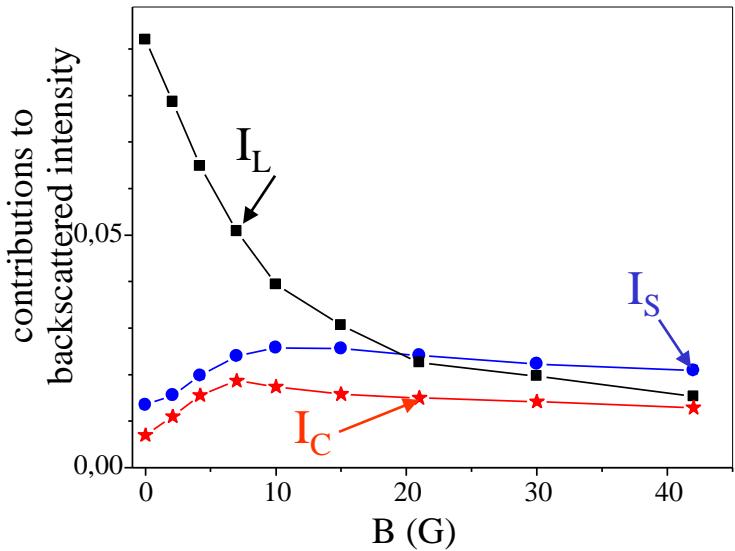
Restoring Coherent Backscattering with Magnetic Fields



Restoring Coherence Length with Magnetic Fields

$\mu B \gg \Gamma$

effective 2 level system



Coherence length :



REDUCED by internal structure ($F=3 \rightarrow F'=4'$)

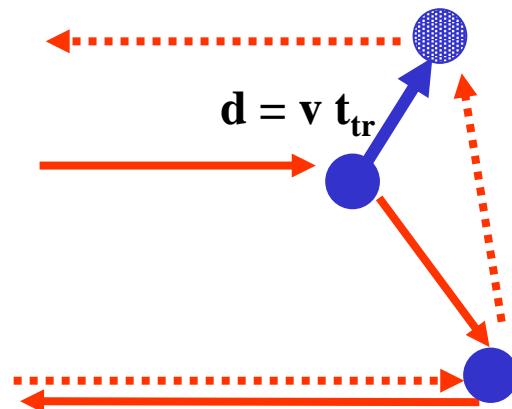


RESTORED by magnetic field

Dynamical Breakdown of CBS

scatterer should not move faster than light

A.A.Golubenstev, Sov. JETP 59, 26 (1984)



‘fast’ atomic dynamics
vs
‘slow’ light transport

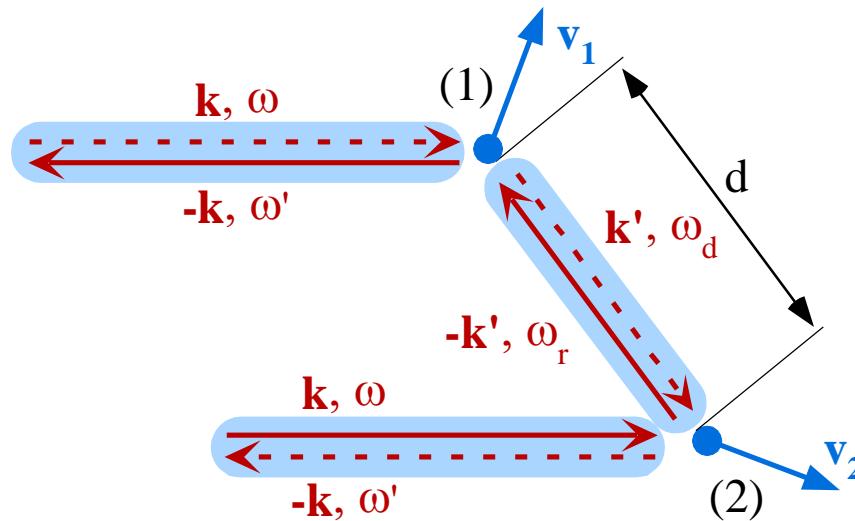
$$d = v t_{tr} \ll \lambda$$

close to resonance $t_{tr}=1/\Gamma$: $kv \ll \Gamma$

a

Dynamical breakdown of CBS

Doppler effect
⇒
 \neq frequencies



2 contributions :

scattering :

attenuation : $\sqrt{\sigma} = \frac{1}{[1+4(\delta/\Gamma)^2]^{1/2}}$

phase shift : $\phi = \arctan(\Gamma/2\delta)$

propagation in effective medium :

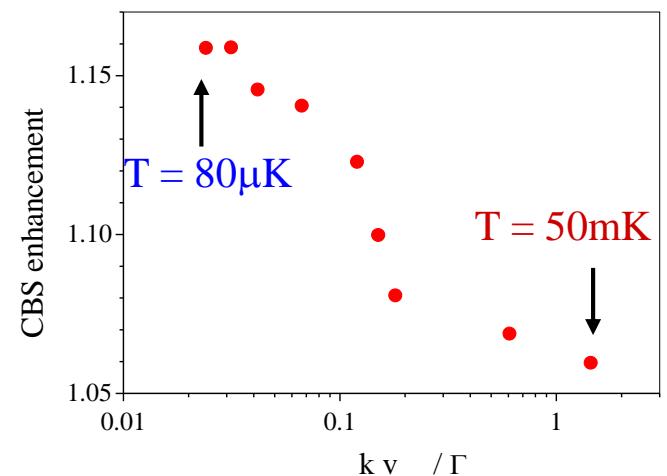
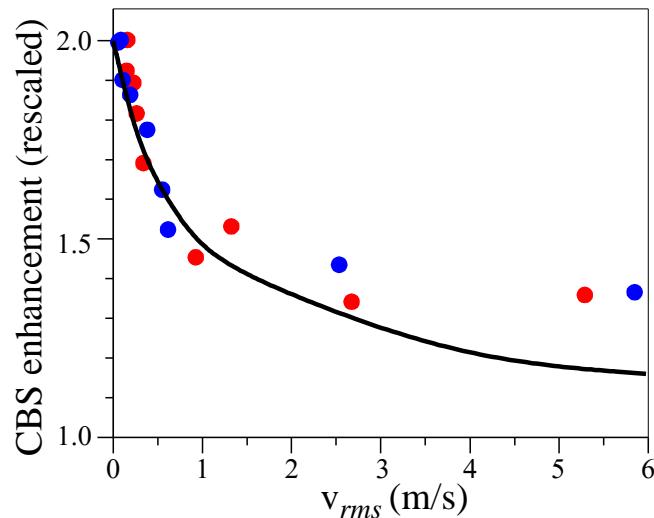
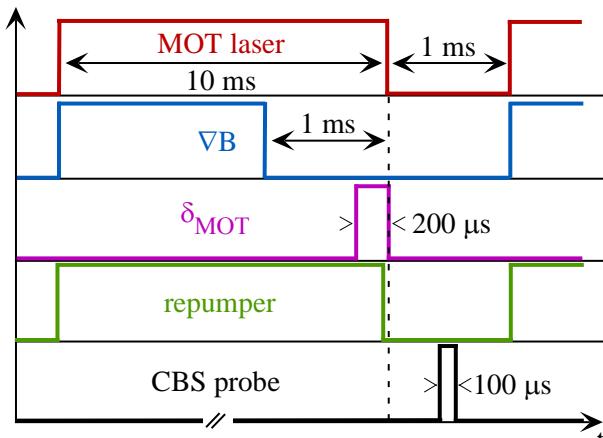
$$n = 1 + N\alpha/2$$

attenuation : $e^{-kd'n''}$

phase shift : $\varphi = kd'n'$

Experimental Observation of Dynamical Breakdown

heating by intense near-resonant optical molasses

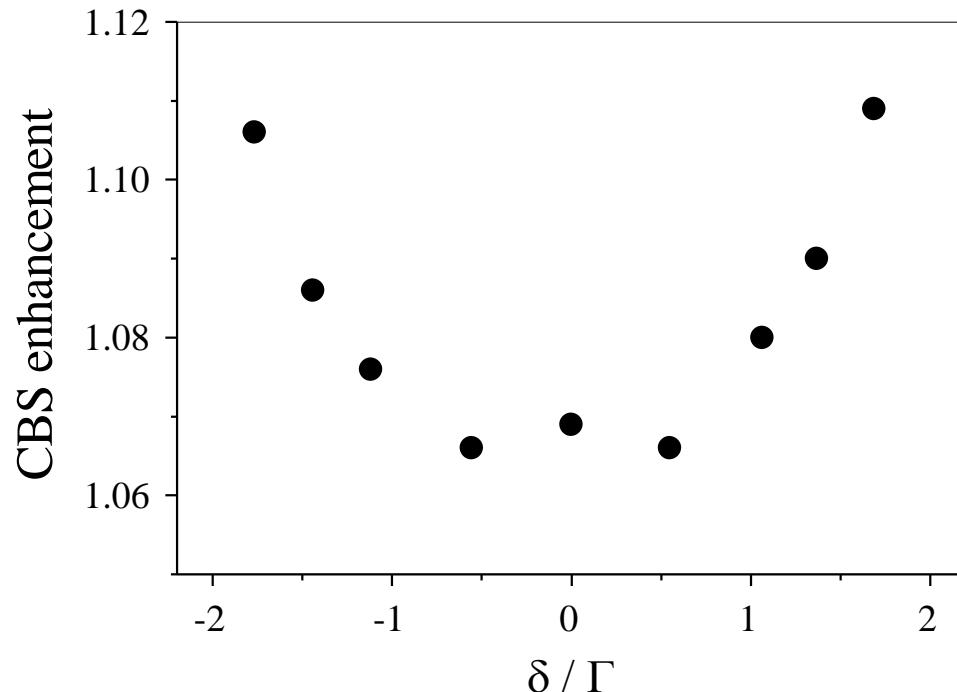


Strong temperature reduction for resonant CBS

$$c_N(v) = \exp \left[-\frac{N^3}{3} \left(\frac{k v}{\Gamma} \right)^2 \right]$$

Dynamical Breakdown of CBS

large detuning : $\delta \gg kv \Rightarrow$ partial restoration of interference contrast

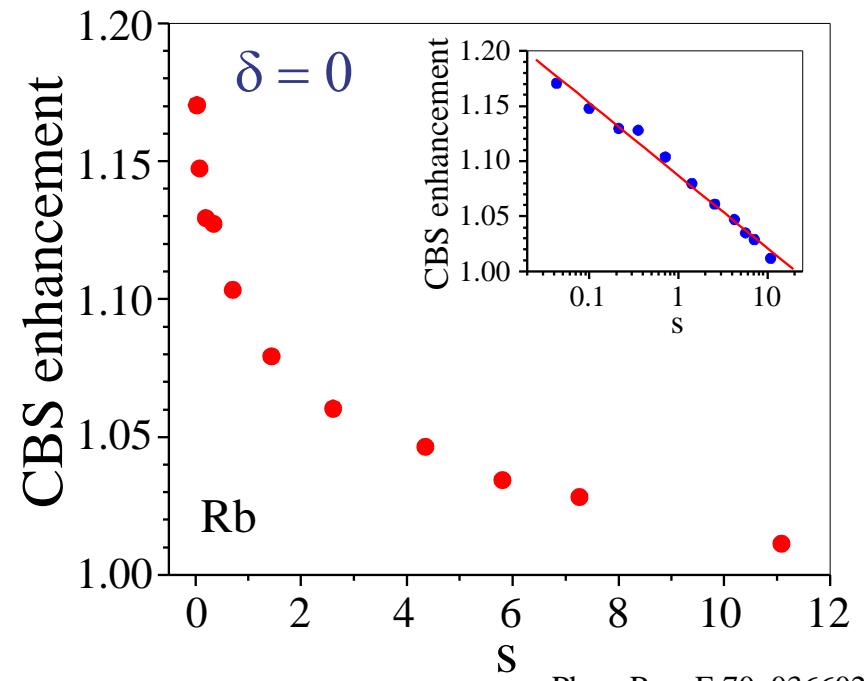
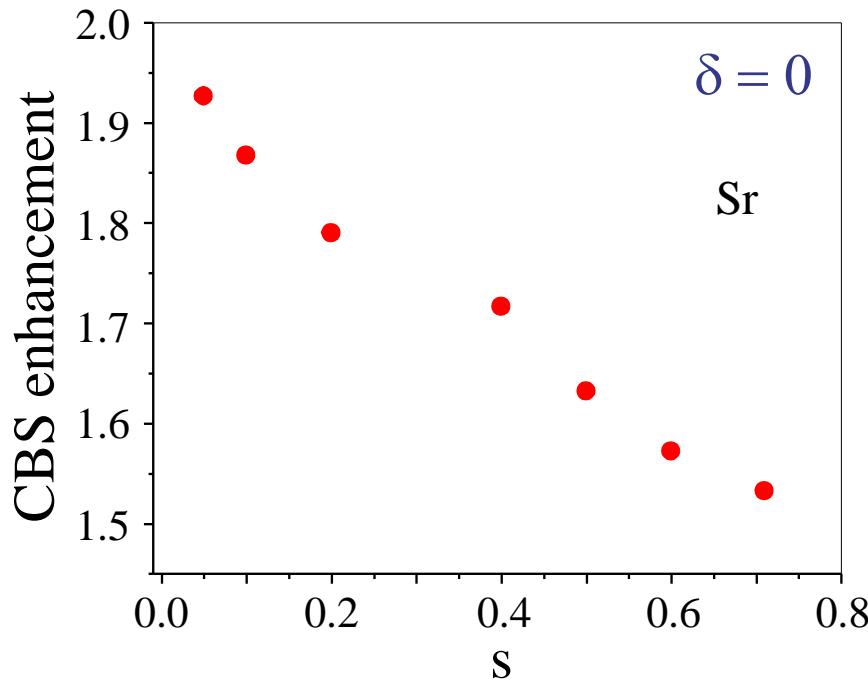


\Rightarrow room temperature CBS possible?

$$\text{CBS} \sim \Gamma/2kv$$

Using frequency approach
N. Cherroret: priv. comm.

Inelastic scattering : Mollow triplet yet another source of decoherence



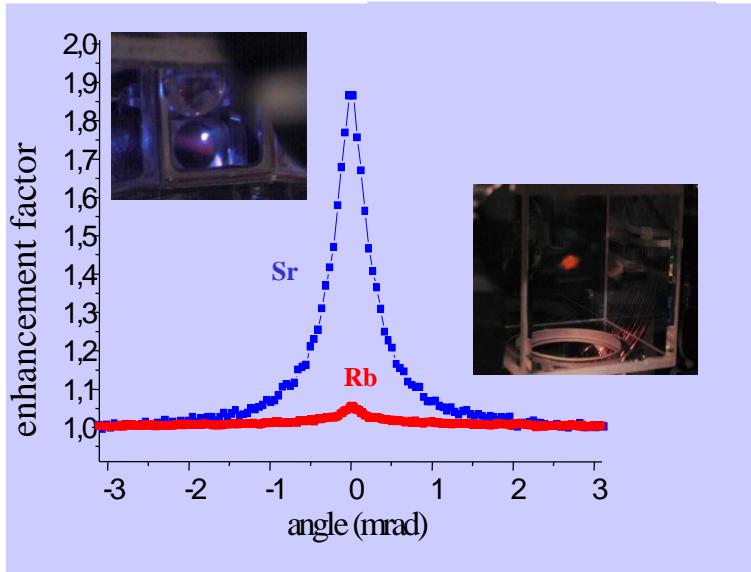
Phys. Rev. E 70, 036602 (2004).

Inelastic scattering effects similar to
Doppler induced frequency redistribution

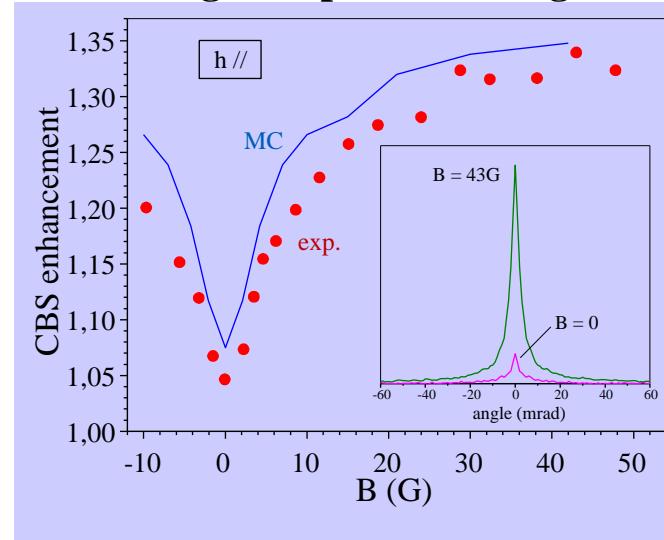
... theory : very hard and not very conclusive so far

In Summary : on the road towards localization
 many problems ☹ ... mostly/partially solved ☺

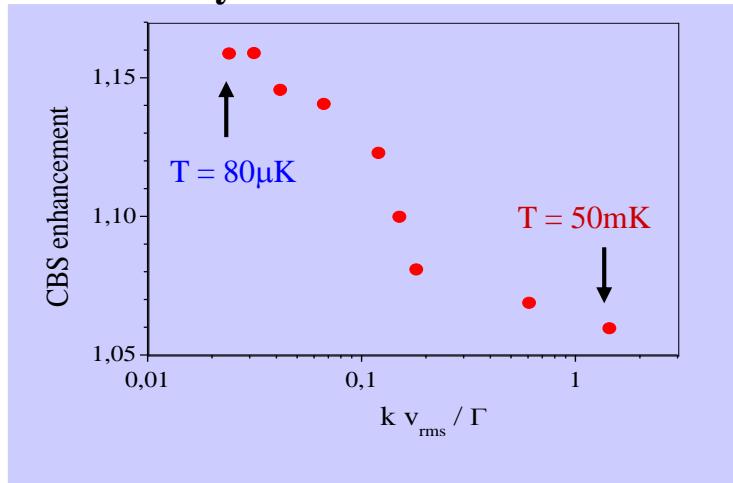
Coherence after atomic scattering



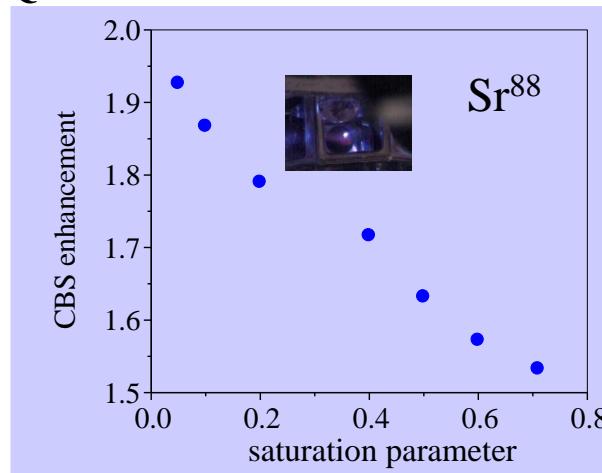
Magnetic path incoding



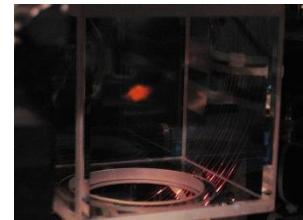
Dynamical breakdown



Quantum fluctuations breakdown



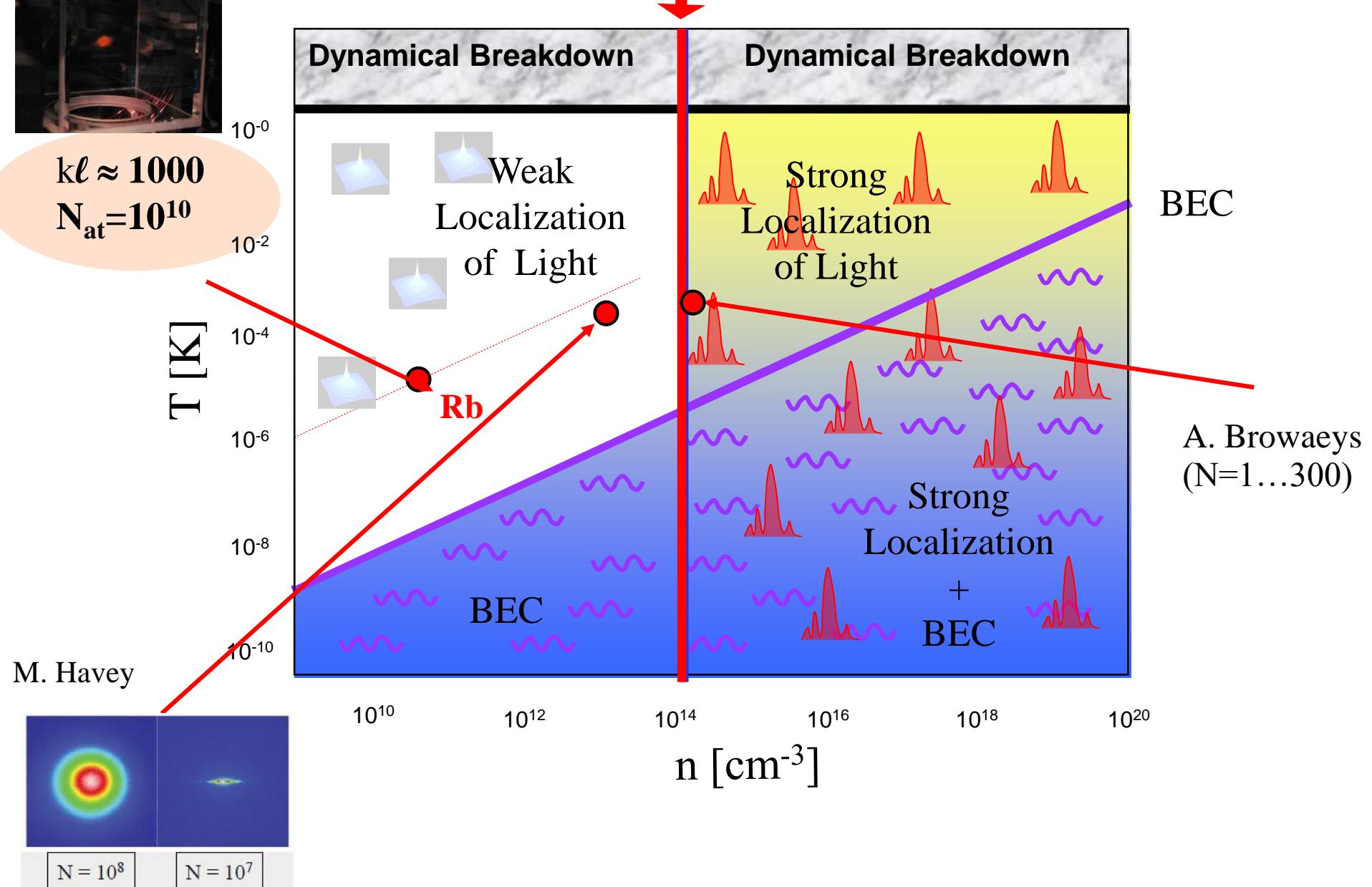
Perspectives : Towards strong localization of light



$k\ell \approx 1000$
 $N_{at} = 10^{10}$

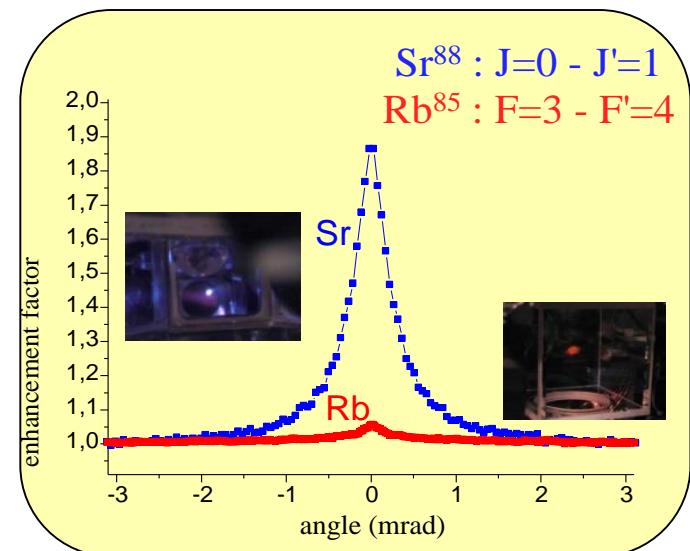
Ioffe-Regel :

$$k\ell \approx 1$$



Problems to face for strong localization

- High spatial densities : Ioffe-Regel criterion :
 $k \ell \approx 1$ $\rho \approx 10^{14} \text{ at/cc}$
- Internal structure of Rb ?
- Atomic motion?
& light induced forces



Lecture 2 : Interference Effects in Light Scattering by Cold Atoms

2.1 Coherent Backscattering of Light

Numerical Simulations

2.2 Dicke Super- and Subradiance

Coupled Dipoles

Cooperative effects in time dependant scattering

Numerical Simulations

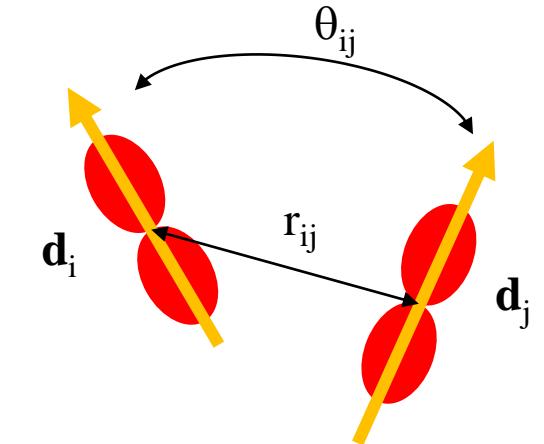
Cooperative Scattering and Dicke states

- **Ab initio model for light scattering**
- **Experiments**

Resonant dipole-dipole interactions :

induced dipoles = optical coherence

$$g_{\alpha\beta}(\mathbf{r}_{ij}) = \frac{3}{2} e^{ik_0 r_{ij}} \left[\left(\frac{1}{k_0 r_{ij}} + \frac{i}{(k_0 r_{ij})^2} - \frac{1}{(k_0 r_{ij})^3} \right) \delta_{\alpha\beta} - \left(\frac{1}{k_0 r_{ij}} + \frac{3i}{(k_0 r_{ij})^2} - \frac{3}{(k_0 r_{ij})^3} \right) \frac{r_{ij,\alpha} r_{ij,\beta}}{r_{ij}^2} \right],$$



Near field (static) term ($k_0 r_{ij} \ll 1$)

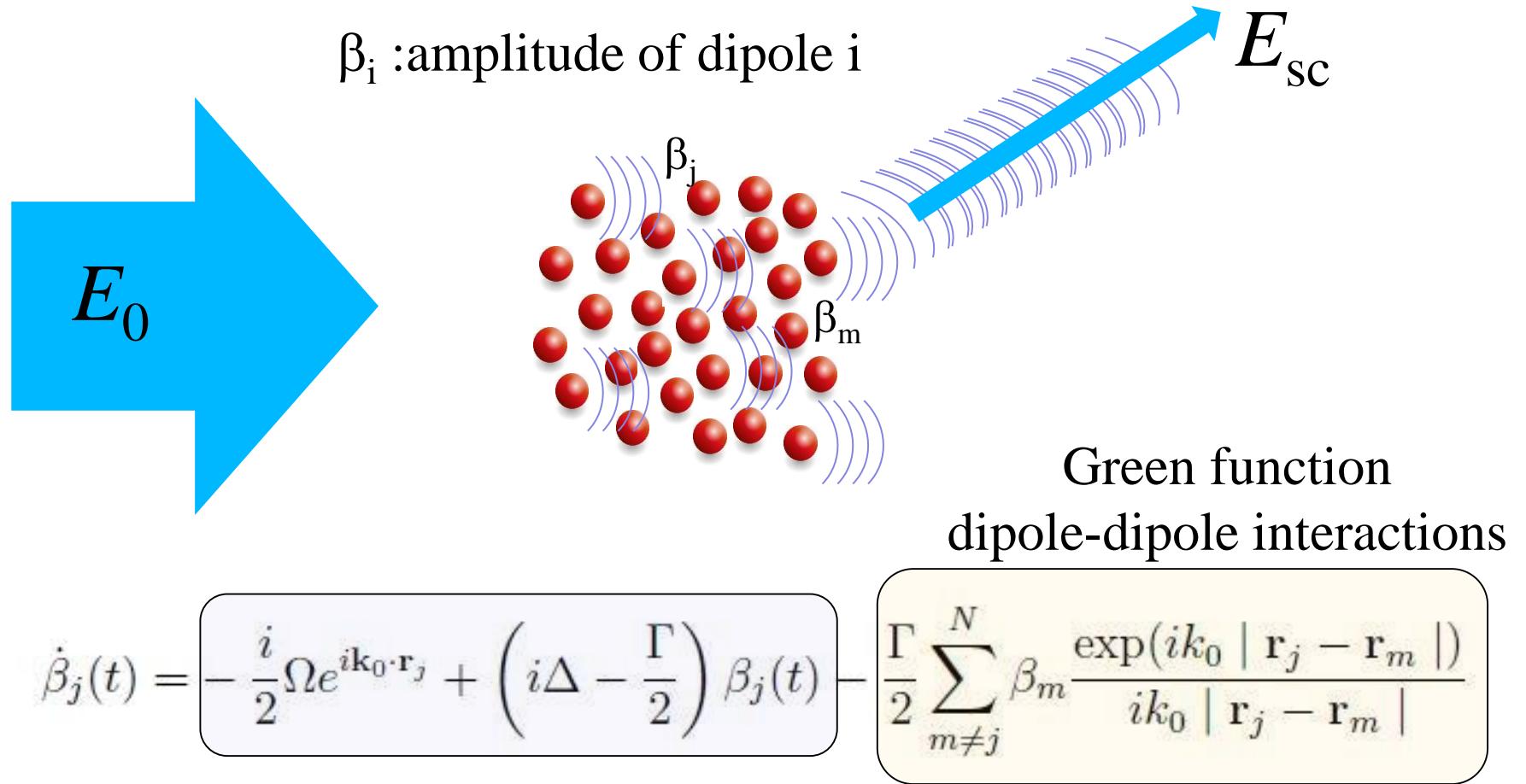
$$d_{ij} = \frac{1}{2} \frac{\gamma^2 \hbar^2}{r_{ij}^3} (1 - 3 \cos^2 \theta_{ij})$$

Widely used scalar approximation :

far field term only + neglecting polarisation effects

$$g(r_{ij}) = \frac{e^{ik_0 r_{ij}}}{k_0 r_{ij}}$$

Building up a refractive index « ab initio » (from individual atoms to macroscopic index)



$$E_{sc}(\mathbf{r}) = -\frac{\hbar\Gamma}{2d} \sum_{j=1}^N \beta_j \frac{e^{ik_0 |\mathbf{r} - \mathbf{r}_j|}}{k_0 |\mathbf{r} - \mathbf{r}_j|}$$

Microscopic derivation from full Hamiltonian

$$\hat{H} = \hat{H}_0 + \hat{H}_1$$

$$\frac{d\hat{a}_{\mathbf{k}}}{dt} = \frac{1}{i\hbar} [\hat{a}_{\mathbf{k}}, \hat{H}] = -ig_k \exp[i(\omega_k - \omega_0)t] \sum_{m=1}^N \hat{\sigma}_m \\ \times \exp(-i\mathbf{k} \cdot \mathbf{r}_m).$$

$$\frac{d\hat{\sigma}_{3j}}{dt} = \frac{1}{i\hbar} [\hat{\sigma}_{3j}, \hat{H}] = i\Omega_0 \hat{\sigma}_j \exp(-i\mathbf{k}_0 \cdot \mathbf{r}_j) \\ + 2i \sum_{\mathbf{k}} g_k \hat{a}_{\mathbf{k}}^\dagger \hat{\sigma}_j \exp[i(\omega_k - \omega_0)t - i\mathbf{k} \cdot \mathbf{r}_j] + \text{h.c.}$$

$$\frac{d\hat{\sigma}_j}{dt} = \frac{1}{i\hbar} [\hat{\sigma}_j, \hat{H}] = i\Delta_0 \hat{\sigma}_j + \frac{i\Omega_0}{2} \hat{\sigma}_{3j} \exp(i\mathbf{k}_0 \cdot \mathbf{r}_j) \\ + i \sum_{\mathbf{k}} g_k \hat{\sigma}_{3j} \hat{a}_{\mathbf{k}} \exp[-i(\omega_k - \omega_0)t + i\mathbf{k} \cdot \mathbf{r}_j]$$

Non RWA contributions

$$\sum_{\mathbf{k}} g_k^2 \exp(i\mathbf{k} \cdot \mathbf{R}) \int_0^\infty dt' \exp[-ic(k - k_0)t'] \\ \rightarrow \frac{\Gamma}{2ik_0|\mathbf{R}|} \exp(ik_0|\mathbf{R}|),$$

Recover classical coupled dipole model

$$\frac{d\hat{\sigma}_j(t)}{dt} = i\Delta_0 \hat{\sigma}_j(t) - \frac{i\Omega_0}{2} \hat{I}_j \exp(i\mathbf{k}_0 \cdot \mathbf{r}_j) - \frac{\Gamma}{2} \sum_{m=1}^N \gamma_{jm} \hat{\sigma}_m(t)$$

$$\hat{H}_0 = \hbar \sum_{j=1}^N \left\{ -\frac{\Delta_0}{2} \hat{\sigma}_{3j} + \frac{\Omega_0}{2} (\hat{\sigma}_j \exp(-i\mathbf{k}_0 \cdot \mathbf{r}_j) + \hat{\sigma}_j^\dagger \exp(i\mathbf{k}_0 \cdot \mathbf{r}_j)) \right\} \\ \hat{H}_1 = \hbar \sum_{j=1}^N \sum_{\mathbf{k}} g_k \left[\hat{a}_{\mathbf{k}}^\dagger \hat{\sigma}_j \exp[i(\omega_k - \omega_0)t - i\mathbf{k} \cdot \mathbf{r}_j] \right. \\ \left. + \hat{\sigma}_j^\dagger \hat{a}_{\mathbf{k}} \exp[-i(\omega_k - \omega_0)t + i\mathbf{k} \cdot \mathbf{r}_j] \right].$$

Trace over vacuum modes

$$\frac{d\hat{\sigma}_j}{dt} = i\Delta_0 \hat{\sigma}_j - \frac{i\Omega_0}{2} \hat{I}_j \exp(i\mathbf{k}_0 \cdot \mathbf{r}_j) \\ - \sum_{\mathbf{k}} g_k^2 \sum_{m=1}^N \exp[i\mathbf{k} \cdot (\mathbf{r}_j - \mathbf{r}_m)] \\ \times \int_0^t dt' \hat{\sigma}_m(t - t') \exp[-i(\omega_k - \omega_0)t']$$

Beyond linear optics:

Full quantum model :

$$\Delta_{jm} = \frac{\cos(k_0|\mathbf{r}_j - \mathbf{r}_m|)}{k_0|\mathbf{r}_j - \mathbf{r}_m|}$$

$$\gamma_{jm} = \frac{\sin(k_0|\mathbf{r}_j - \mathbf{r}_m|)}{k_0|\mathbf{r}_j - \mathbf{r}_m|}$$

$$\begin{aligned}\frac{d\sigma_j^-}{dt} &= \left(i\Delta_0 - \frac{\Gamma}{2}\right)\sigma_j^- + \frac{i\Omega_0}{2}e^{i\mathbf{k}_0 \cdot \mathbf{r}_j}\sigma_j^z \\ &\quad + \frac{\Gamma}{2}\sum_{m \neq j}^N \sigma_j^z\sigma_m^-(\gamma_{jm} - i\Delta_{jm}), \\ \frac{d\sigma_j^z}{dt} &= i\Omega_0 \left(e^{-i\mathbf{k}_0 \cdot \mathbf{r}_j}\sigma_j^- - \text{h.c.}\right) - \Gamma(1 + \sigma_j^z) \\ &\quad - \Gamma\sum_{m \neq j}^N (\sigma_m^+\sigma_j^-(\gamma_{jm} + i\Delta_{jm}) + \text{h.c.})\end{aligned}$$

Lindblad formalism :

$$\frac{d\hat{\rho}}{dt} = -\frac{i}{\hbar}[\hat{H}_0, \hat{\rho}] + \mathcal{L}_f[\hat{\rho}]$$

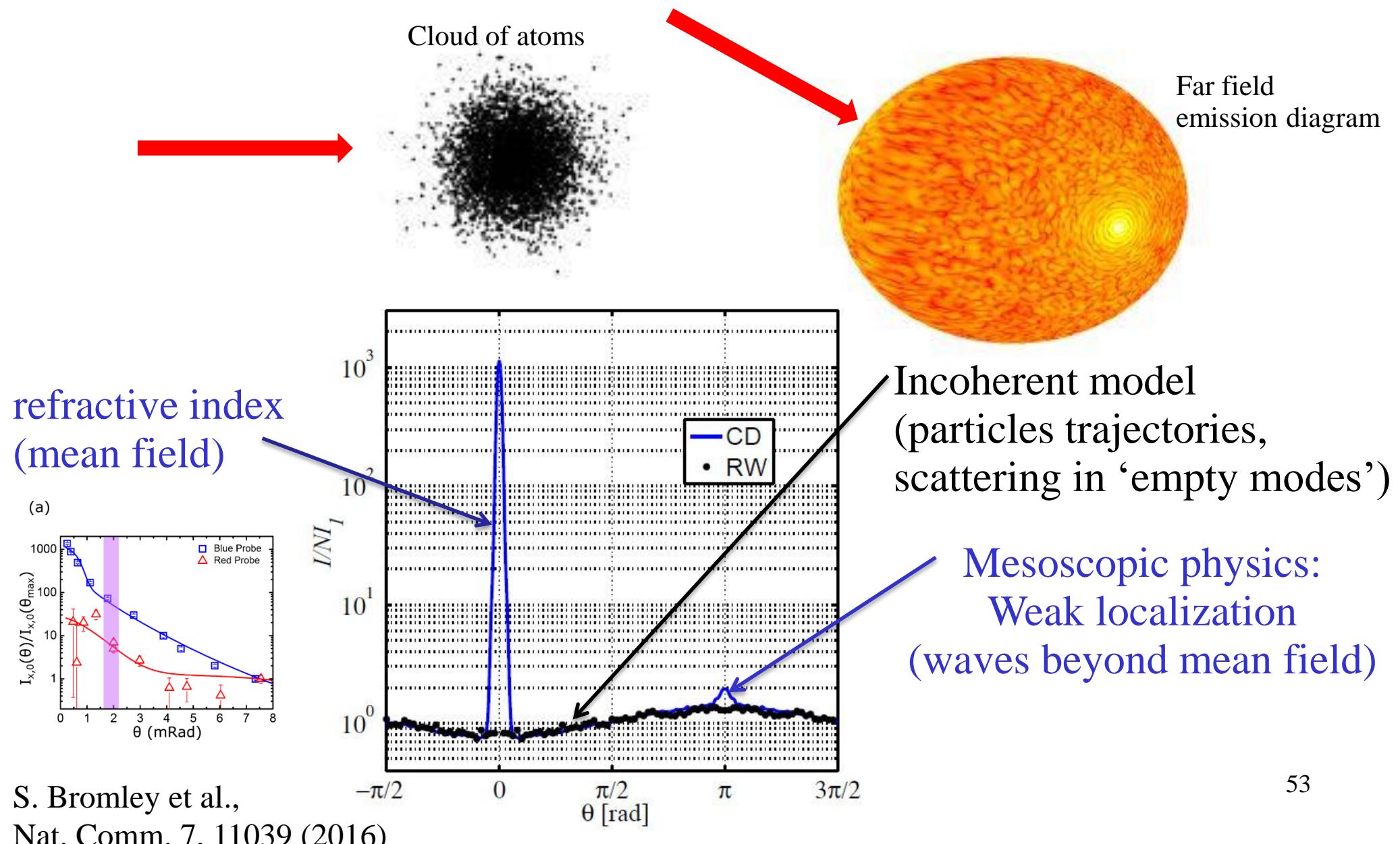
$$\hat{H}_0 = \frac{\hbar}{2}\sum_{a=1}^N \left[\delta_a \hat{\sigma}_a^z + \sum_{b=1, b \neq a}^N g(\mathbf{r}_{ab}) \hat{\sigma}_a^+ \hat{\sigma}_b^- \right] \quad \mathcal{L}_f[\hat{\rho}] = \frac{1}{2}\sum_{a,b} f(\mathbf{r}_{ab})(2\hat{\sigma}_b^- \hat{\rho} \hat{\sigma}_a^+ - \hat{\sigma}_a^+ \hat{\sigma}_b^- \hat{\rho} - \hat{\rho} \hat{\sigma}_a^+ \hat{\sigma}_b^-)$$

$$g(\mathbf{r}_{ab}) = -\frac{3\Gamma}{2} \left\{ \sin^2 \theta \frac{\cos \zeta_{ab}}{\zeta_{ab}} + (3\cos^2 \theta - 1) \left[\frac{\cos \zeta_{ab}}{(\zeta_{ab})^3} + \frac{\sin \zeta_{ab}}{(\zeta_{ab})^2} \right] \right\}$$

$$f(\mathbf{r}_{ab}) = \frac{3\Gamma}{2} \left\{ \sin^2 \theta \frac{\sin \zeta_{ab}}{\zeta_{ab}} + (3\cos^2 \theta - 1) \left[\frac{\sin \zeta_{ab}}{(\zeta_{ab})^3} - \frac{\cos \zeta_{ab}}{(\zeta_{ab})^2} \right] \right\},$$

- R. H. Lehmberg, Phys. Rev. A 2, 883 (1970)
J. Ott et al., Phys. Rev. A 87, 061801(R), (2013)
L. Pucci et al., Phys. Rev. A 95, 053625 (2017)
B. Zhu et al., New J. Phys. 17, 083063 (2015)

Spherical gaussian cloud: steady state emission diagram



Resonant dipole-dipole interactions :

Scalar approximation captures :

- 1) Continuous ‘index of refraction’ feature
- 2) Multiple Scattering
(each dipole radiates to the outside modes)
- 3) Speckle & mesoscopic coherent backscattering

**To be used with care even for large interatomic distances
(no polarisation effect)**

Nevertheless : VERY useful (hope for analytical results)

Time resolved light emission : Dicke 1954

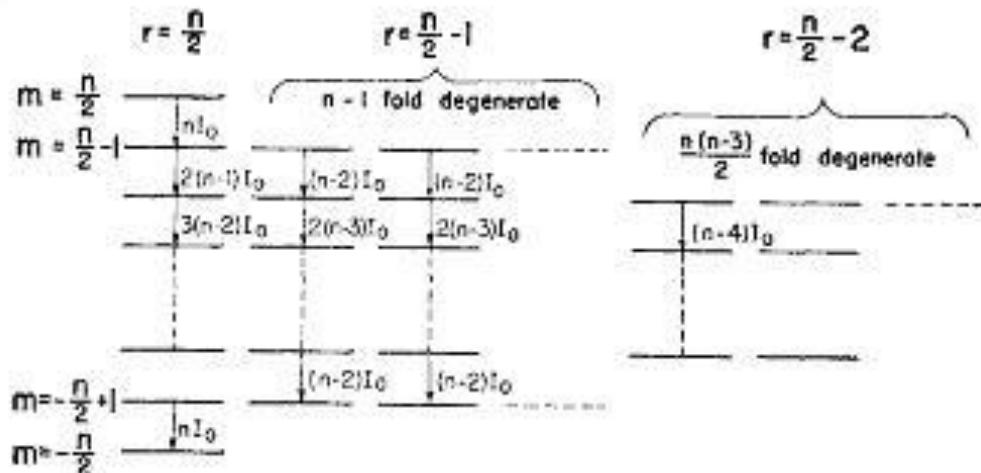
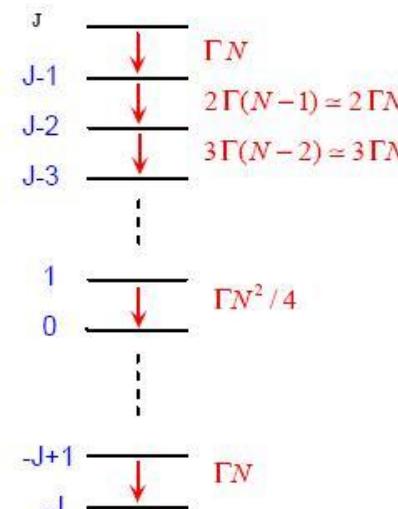
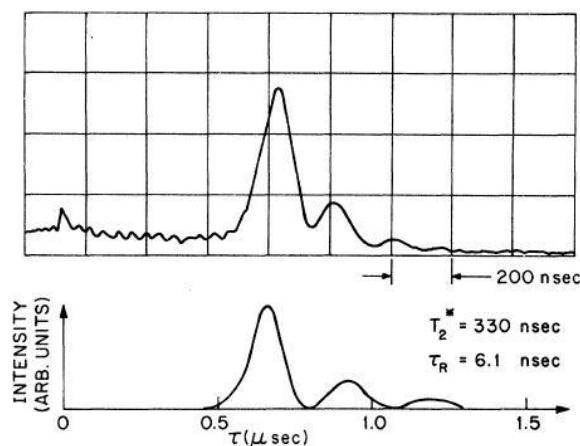


FIG. 1. Energy level diagram of an n -molecule gas, each molecule having 2 nondegenerate energy levels. Spontaneous radiation rates are indicated. $E_m = mE$.



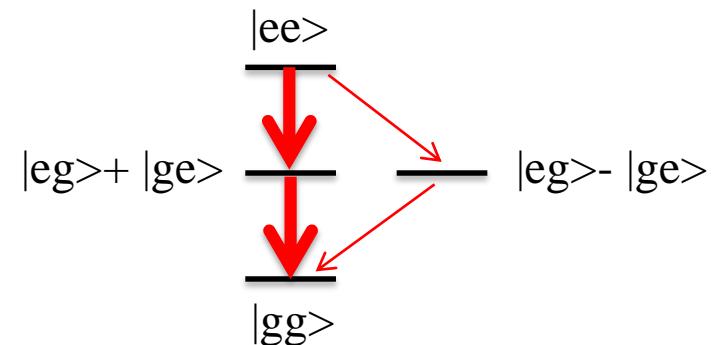
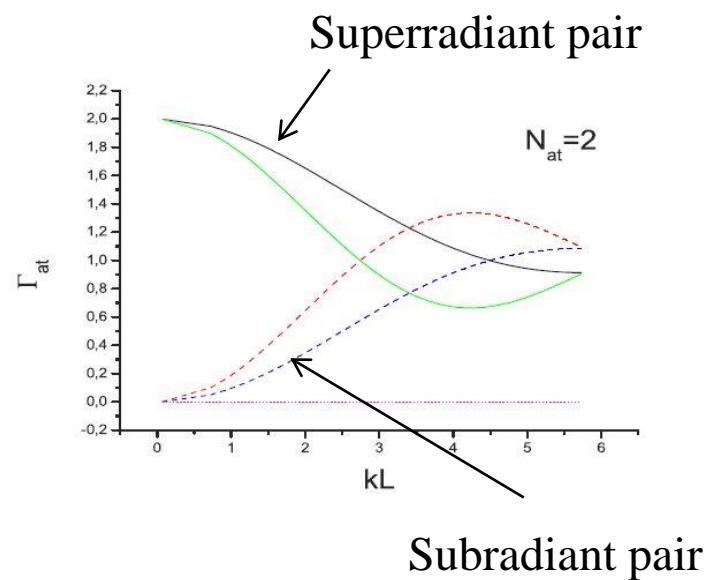
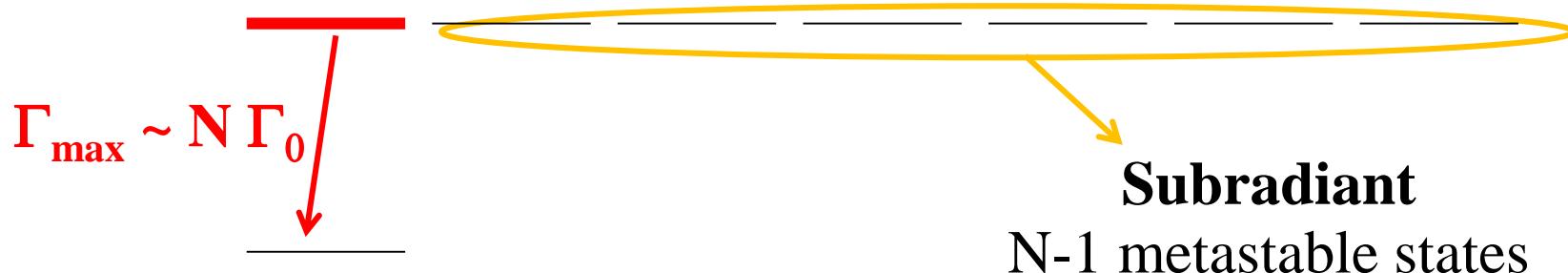
R. Dicke 1954



First experimental observation of ‘superradiance’

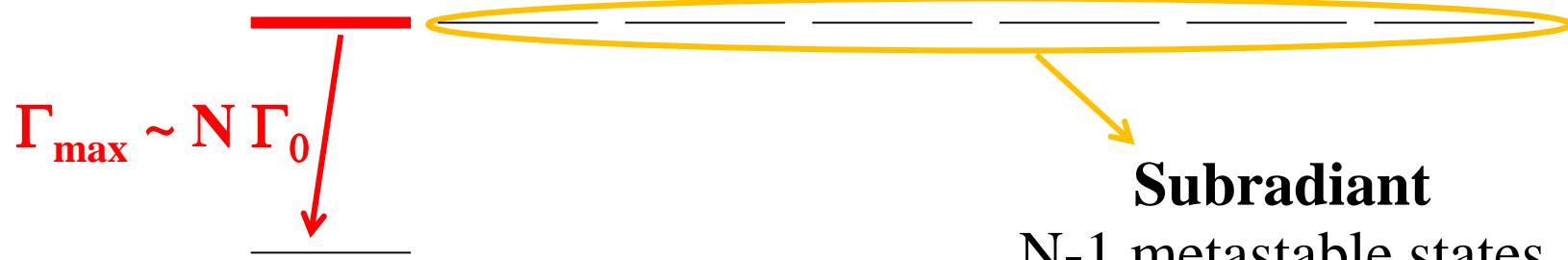
Feld et al. 1973

Single photon excitation / low intensity limit Coupled classical oscillators



Single photon excitation / low intensity limit

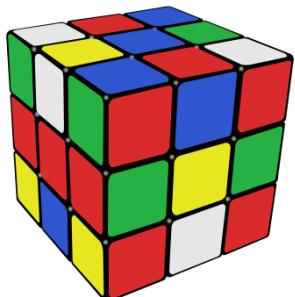
Small Volume ($L \ll \lambda$)



Fast and slow collective time scales emerging from **long range** coupling

Extended Volume ($L \gg \lambda$)

$$\Gamma_{\max} \sim C \Gamma_0 \sim b_0 \Gamma_0$$



$$b_0 = N_{\text{at}} / N_{\text{modes}}$$

Cooperativity
without cavity

$$N_{\text{modes}} \sim (L/\lambda)^2$$

$$b_0 \sim N_{\text{at}} / (kL)^2$$

Spontaneous emissions vs coherent scattering

Single atom :

$$\frac{d\rho_{ee}}{dt} = -\Gamma\rho_{ee} + i\frac{\Omega_L}{2} (\rho_{eg} - \rho_{ge})$$

$$\frac{d\rho_{ge}}{dt} = -(i\delta + \frac{\Gamma + \Gamma^*}{2})\rho_{ge} - i\frac{\Omega_L}{2} (\rho_{ee} - \rho_{gg})$$

$$\frac{d\rho_{gg}}{dt} = +\Gamma\rho_{ee} - i\frac{\Omega_L}{2} (\rho_{eg} - \rho_{ge})$$

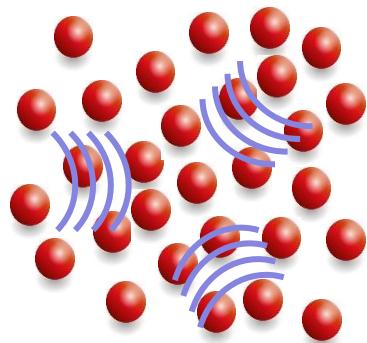
Dephasing collisions

No impact on excited state lifetime

Broadening of spectrum for scanning lasers

Important in buffer gaz cells

Many atoms :



Oscillation of dipole ‘i’ perturbed by other dipoles

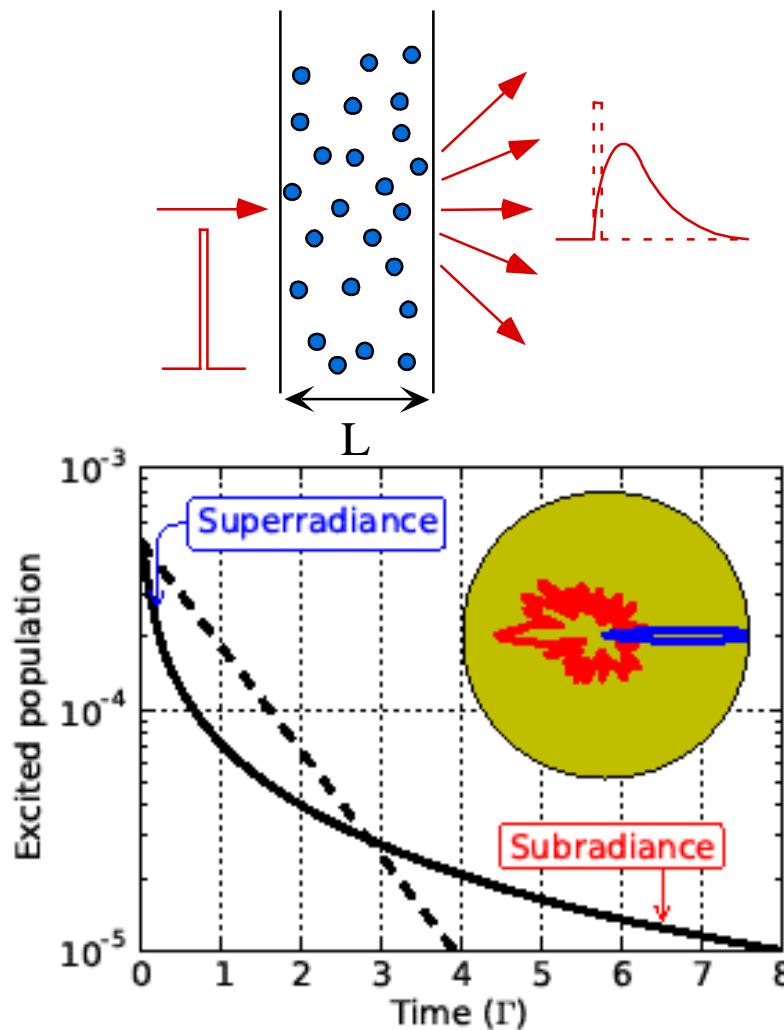
Possibility of different lifetimes for coherences and populations

Evolution of excited state populations NOT included in coupled dipole model

Homework 1 :

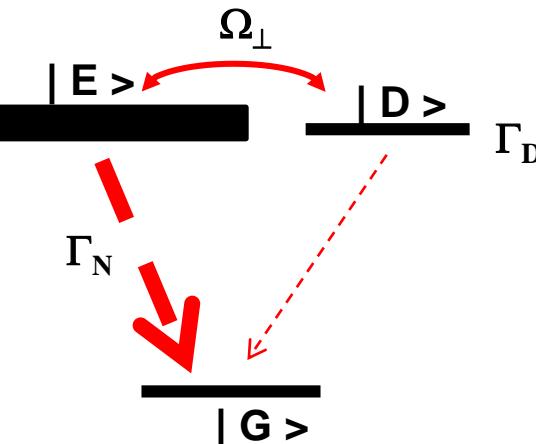
Start from coupled dipole equations and
derive effective dephasing term Γ^* in a single atom evolution

Time dependent experiments : coherent scattering

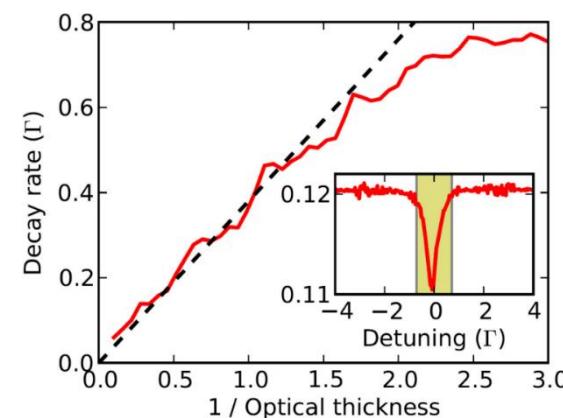


Numerical Simulation of N driven coupled dipoles

Superradiance = bright state
Subradiance = metastable ‘dark’ states



Late decay time $\propto b_0$



Numerical Simulations



Link to Matlab codes of Robin Kaiser's lecture in Les Houches 2019

Random Walk based codes :

- [RandomWalk LH2019.m](#)
- [Ohm LH2019.m](#)
- [TransmissionProfile N100000.m](#)
- [RT LH2019.m](#)
- [RandomWalk CBS.m](#)

Coupled Dipole based codes :

- [CoupledDipoles LH2019.m](#)

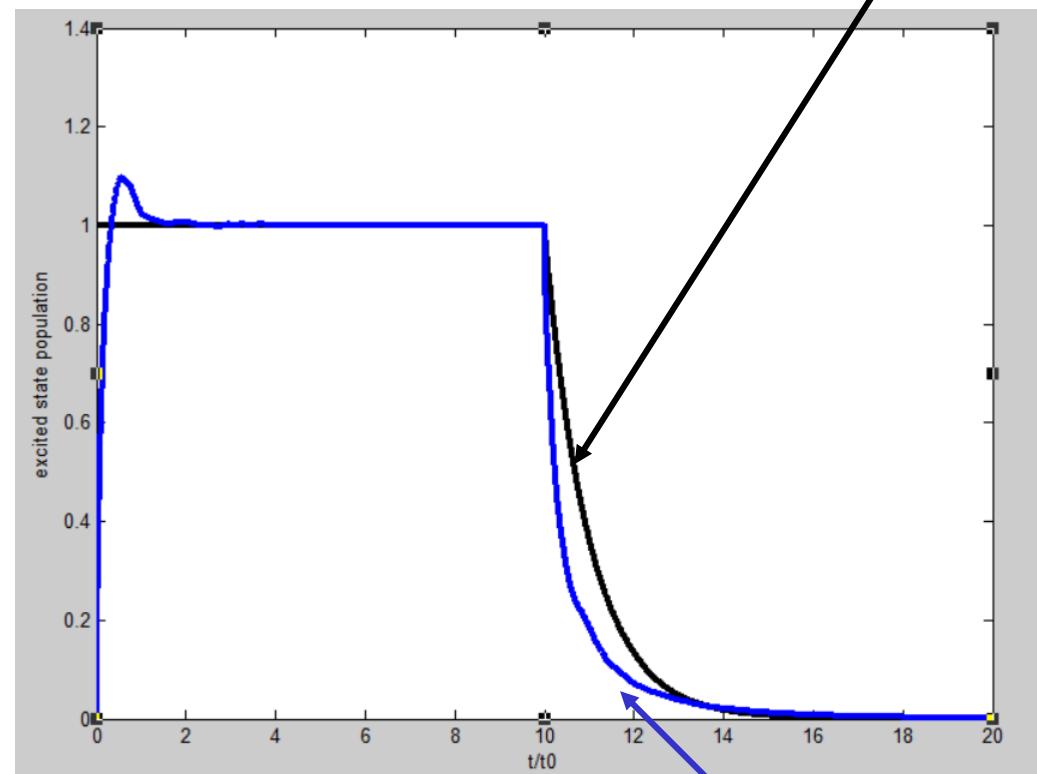
Contact : Robin Kaiser
e-mail: Robin.Kaiser@inphyni.cnrs.fr
<http://www.kaiserlux.eu/coldatoms/>

```

1 % Coupled Dipole Model
2 clear all;
3
4 % Simulation parameters (42 seconds computing time with parameters below)
5 Gamma=1; %inverse lifetime
6 Delta=-20; % Detuning (in units of Gamma)
7 Omega=0.1; %Rabi frequency of laser drive (in units of Gamma)
8 Nat=1000; % atom number
9 kR=5; % cloud radius in units of 1/k
10 dt=1/200; % time step for simulation
11 Tpulse=10; % duration of laser pulse (in units of Gamma-1)
12 Tmax=20; % duration of simulation(in units of Gamma-1)
13
14 % Homogeneous cloud (for gaussian cloud replace 'rand' by 'randn')
15 X=(rand(Nat,1)-0.5)*kR;
16 Y=(rand(Nat,1)-0.5)*kR;
17 Z=(rand(Nat,1)-0.5)*kR;
18 % computing relative distances
19 xjm=X*ones(1,Nat)-ones(Nat,1)*X';
20 yjm=Y*ones(1,Nat)-ones(Nat,1)*Y';
21 zjm=Z*ones(1,Nat)-ones(Nat,1)*Z';
22 rjm=sqrt(xjm.^2+yjm.^2+zjm.^2)+diag(ones(Nat,1)); % diagonal term added to avoid divergence of the Green function
23
24 % Green function of dipole dipole coupling
25 Vjm=exp(li*rjm)./(li*rjm); % 1/R long range dipole dipole coupling
26 V=Vjm-diag(diag(Vjm)); %removing artificially non-zero diagonal terms
27 % Adding free single atom evolution with dipole dipole coupling
28 M=(-Gamma/2+li*Delta)*eye(Nat)-Gamma/2*Vjm; % 'eye' = ones on the diagonal
29 % Plane wave excitation
30 Om=li*Omega*exp(li*Z)/2; %plane wave excitation
31 Beta_SteadyState=M\Om; % steady-state solution for d beta / dt = M beta +Om
32
33
34 % time dynamics with all atoms intially in the ground state
35 beta=zeros(Nat,Tmax/dt); % initialization of time dependent dipole amplitudes
36 %Switch-on
37 for it=2::Tpulse/dt %even very small time steps with this simple time solver is not good
38 beta(:,it)=beta(:,it-1)+dt*(M*beta(:,it-1)+Om);
39 pop1(it)=sum(abs(beta(:,it)));
40 end
41 for it=2:1:Tpulse/dt %Kunge Kutta : a well known better solution
42 k1=M*beta(:,it-1)+Om;
43 k2=M*(beta(:,it-1)+k1*dt/2)+Om;
44 k3=M*(beta(:,it-1)+k2*dt/2)+Om;
45 k4=M*(beta(:,it-1)+k3*dt)+Om;
46 beta(:,it)=beta(:,it-1)+(dt/6)*(k1+2*k2+2*k3+k4);
47 popRK(it)=sum(abs(beta(:,it)));
48 end
49
50 %Switch-off
51 for it=Tpulse/dt+1:1:Tmax/dt %Kunge Kutta : a well known better solution
52 k1=M*beta(:,it-1); %same equation but with Om=0
53 k2=M*(beta(:,it-1)+k1*dt/2); %same equation but with Om=0
54 k3=M*(beta(:,it-1)+k2*dt/2);%same equation but with Om=0
55 k4=M*(beta(:,it-1)+k3*dt);%same equation but with Om=0
56 beta(:,it)=beta(:,it-1)+(dt/6)*(k1+2*k2+2*k3+k4);
57 popRK(it)=sum(abs(beta(:,it)));
58 end
59
60 % Single atom decay reference
61 betal=ones(1,Tmax/dt);
62 for it=Tpulse/dt+1:1:Tmax/dt
63 betal(it)=exp(-(it-Tpulse/dt)*dt);
64 end
65 plot((1:Tmax/dt)*dt, betal,'DisplayName','betal','YDataSource','betal', 'Color', 'Black');figure(gcf) % Single Atom decay for reference
66 hold on;
67 plot((1:Tmax/dt)*dt, popRK/popRK(Tpulse/dt),'DisplayName','popRK','YDataSource','popRK');figure(gcf)

```

Single atom dynamics



Cooperative dynamics

Single Photon Superradiance for $N \gg 2$

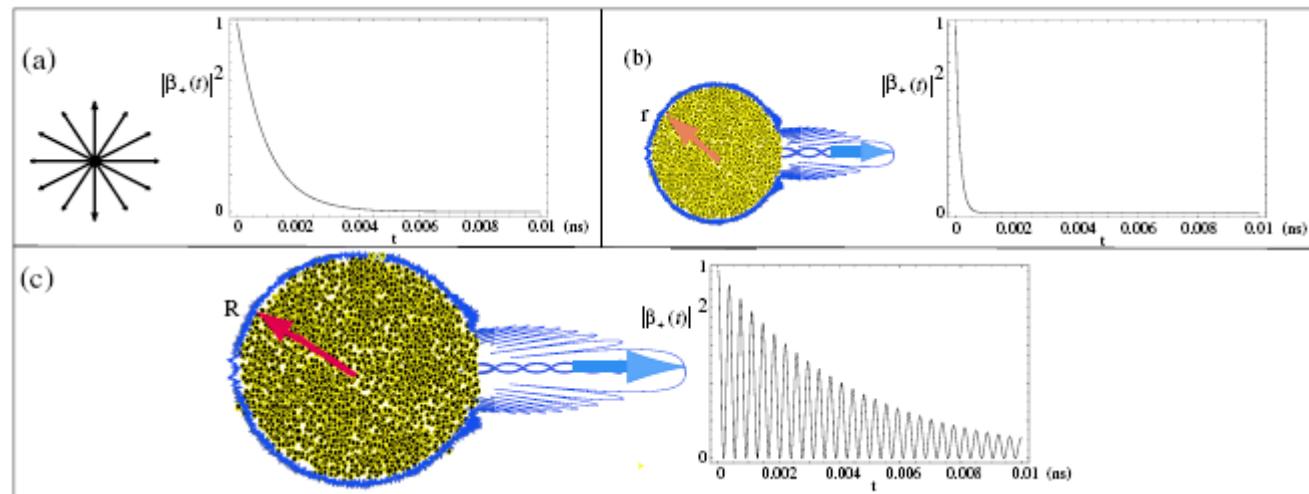
PRL 100, 160504 (2008)

PHYSICAL REVIEW LETTERS

week ending
25 APRIL 2008

Dynamical Evolution of Correlated Spontaneous Emission of a Single Photon from a Uniformly Excited Cloud of N Atoms

Anatoly A. Svidzinsky, Jun-Tao Chang, and Marlan O. Scully



Timed Dicke State

$$|+\rangle_{\mathbf{k}_0} = \frac{1}{\sqrt{N}} \sum_{j=1}^N e^{i\mathbf{k}_0 \cdot \mathbf{r}_j} |g_1, g_2, \dots, e_j, \dots, g_N\rangle$$

+ many other papers by Scully, Glauber, Friedberg, ...

Time dependant experiments : Dicke superradiance

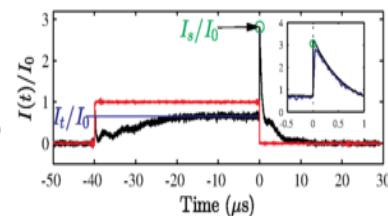
Optical precursors experiments :

H. Jeong, A. M. C. Dawes, D. J. Gauthier, Phys. Rev. Lett. 96, 143901 (2006).

J. F. Chen, S. Wang, D. Wei, M. M. T. Loy, G. K. L. Wong, S. Du, Phys. Rev. A 81, 033844 (2010)

Forward single photon superradiance

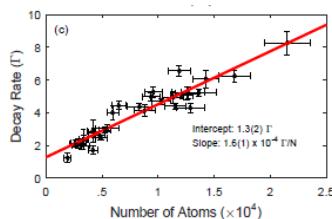
M. Chalony et al., Phys. Rev. A 84, 011401(R)



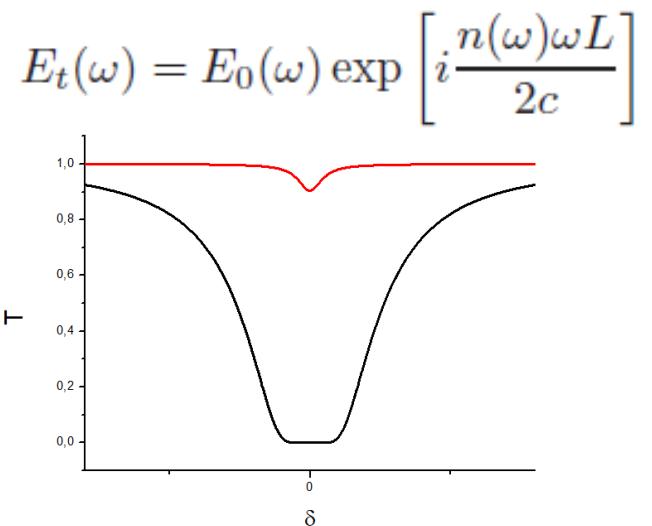
C. C. Kwong, et al., Phys. Rev. Lett. 113, 223601 (2014)

S. J. Roof, et al., Phys. Rev. Lett. 117, 073003 (2016)

S. Jennewein et al., Phys. Rev. A 97, 053816 (2018)



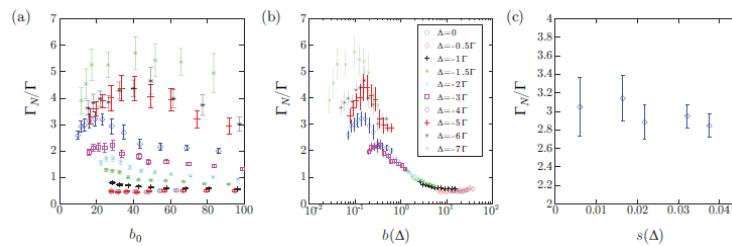
Mean field ‘index’ effect



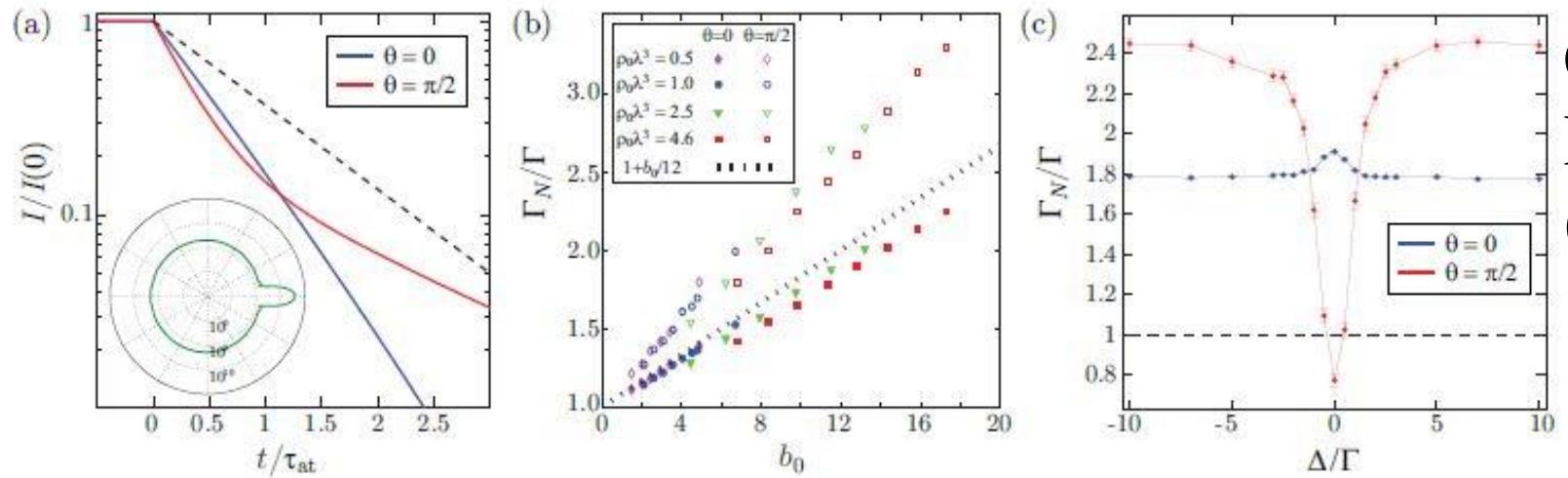
Off-axis single photon superradiance

M. O. Araujo et al., PRL 117, 073002 (2016),

A. Kuraptsev et al., Phys. Rev. A 96, 023830 (2017).

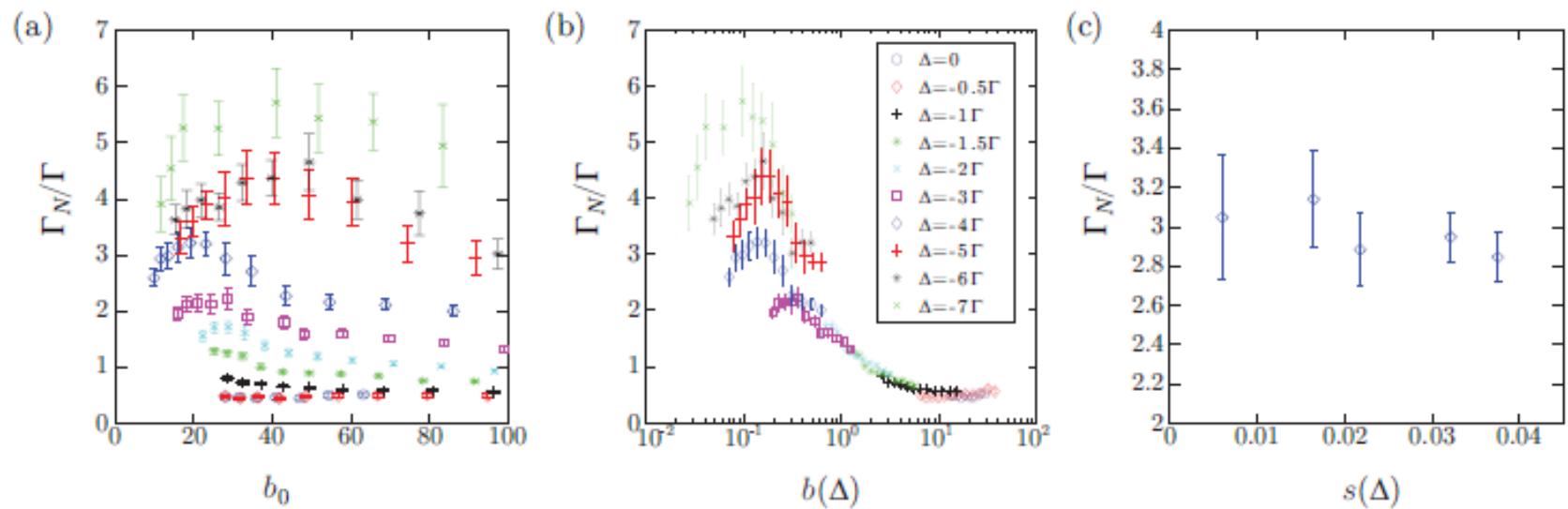


Superradiance in dilute and large cloud of cold atoms



Coupled
Dipoles
(Numerics)

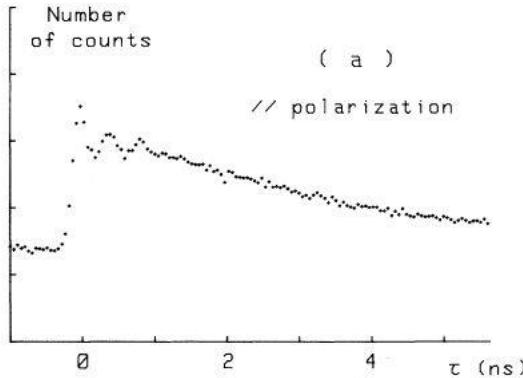
Off-axis Superradiance \neq forward superradiance



Experiments

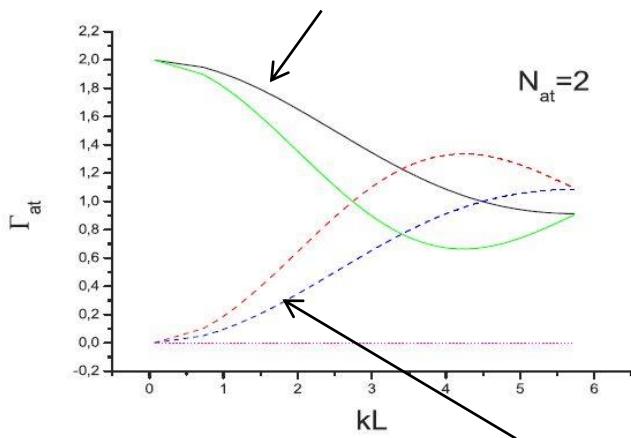
Dicke subradiance

Two atom interference



P. Grangier, A. Aspect, and J. Vigue,
PRL 54, 418 (1985).

Superradiant pair



Subradiant pair

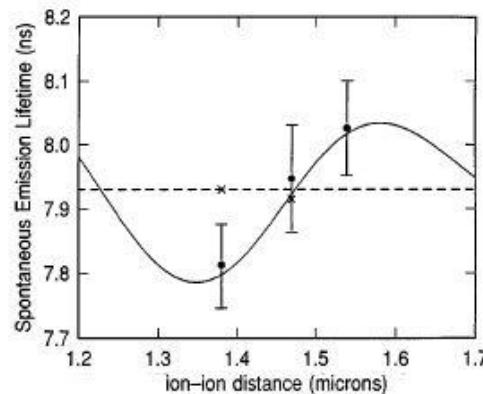
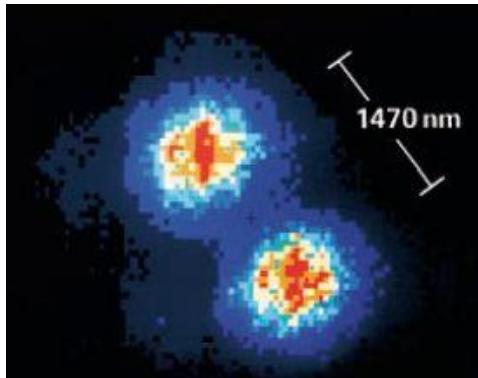
$$p_{\theta, \phi}(\tau) = f(\theta, \phi)(1 + \epsilon \cos \Omega \tau) e^{-D(\tau)}$$

The difference between $\exp\{-D(\tau)\}$ and the decay factor $\exp\{-\Gamma\tau\}$ for a free atom is due to the resonant radiative coupling between both atoms.⁶ In our experimental conditions, this difference gives an effect completely negligible⁷ compared with the one due to the interference term $\epsilon \cos \Omega \tau$.

$$d=v\tau \quad v=500\text{m/s} \quad t=4.7\text{ns} \quad \lambda=407\text{nm}$$

$$kd = 2\pi d/\lambda = 36$$

Subradiant pairs : N=2 : distance controls

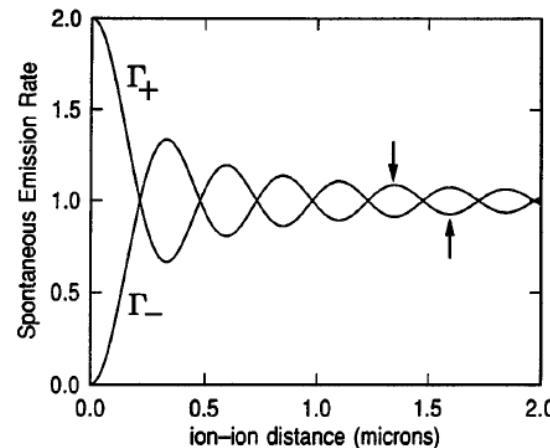


R. G. DeVoe and R. G. Brewer,
PRL 76, 2049 (1996).

Subradiance for N>>1 and long range ?

$\lambda=493\text{nm}$ $d=1500\text{nm}$

$$kd = 2\pi d/\lambda = 19$$



Forward ‘subradiance echo’ from inverted system

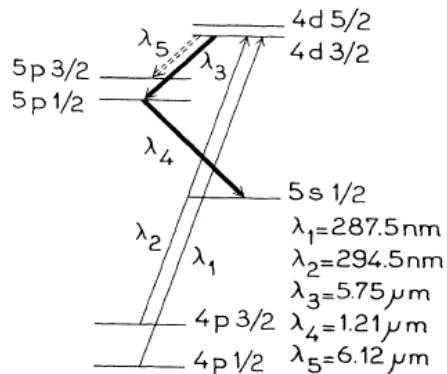
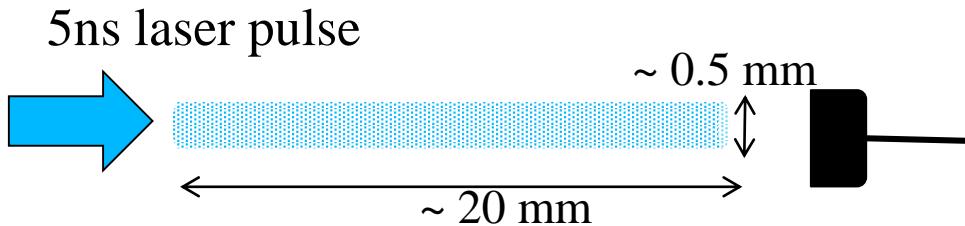


FIG. 1. Relevant level diagram of gallium.



Pencil shape excitation

... contains subradiant pairs created from an inverted system ...

‘A subradiant state can thus exist only after the emission, with a delay time T_D , of the superradiant light pulse which creates it and before its destruction by ordinary spontaneous emission: **its duration cannot be longer than T_{sp} .**’

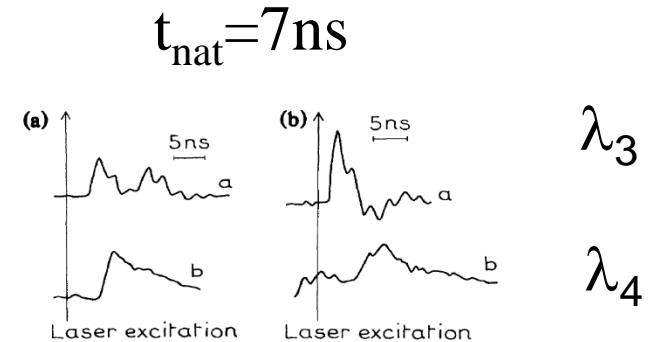


FIG. 2. Typical superradiant signals. (a) corresponds to the excitation with saturated linearly polarized light from the $4p_{3/2}$ level; (b), the $4p_{1/2}$ level. In both cases, traces *a* and *b* correspond respectively to $4d \rightarrow 5p$ and to $5p \rightarrow 5s$ transitions. The visible oscillations of the signals are most likely due to the hyperfine structure of the $5p_{1/2}$ level.

D. Pavolini et al. , Phys. Rev. Lett. 54, 1917 (1985)

A. Crubellier et al., J. Phys. B: At. Mol. Phys. 18 3811 (1985)

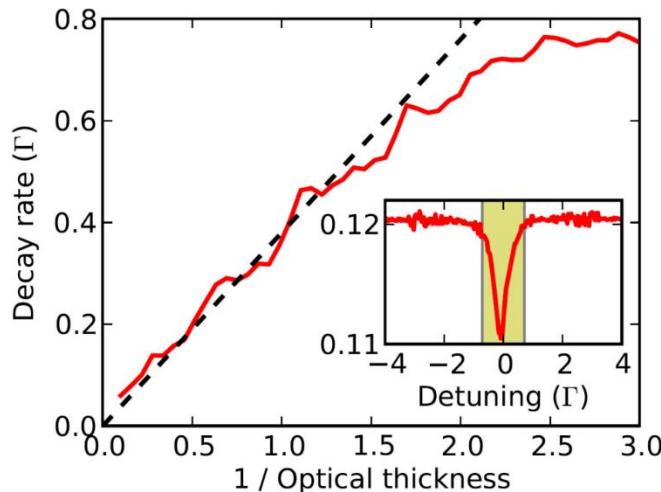
Fragile subradiance

Single Photon Dicke subradiance for N two level systems
(in 3D free space, $N \gg 2$) had **not been observed**

- Does **not** require large spatial densities
(near field effect maybe even bad : Gross&Haroche 1982)
- Fragile to decoherence
- Requires large optical densities in **all** directions ($b_0 \gg 1$)
- Exploits the $1/r$ **long range** dipole-dipole interaction

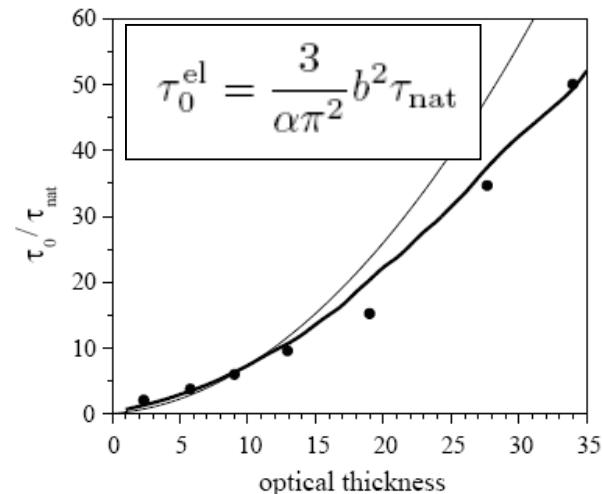
Subradiance vs incoherent scattering

$$t_{\text{sub}} \propto b_0$$



- Does not require large spatial densities
- Requires large optical densities

$$t_{\text{Rad.Trap.}} \propto b(\delta)^2$$

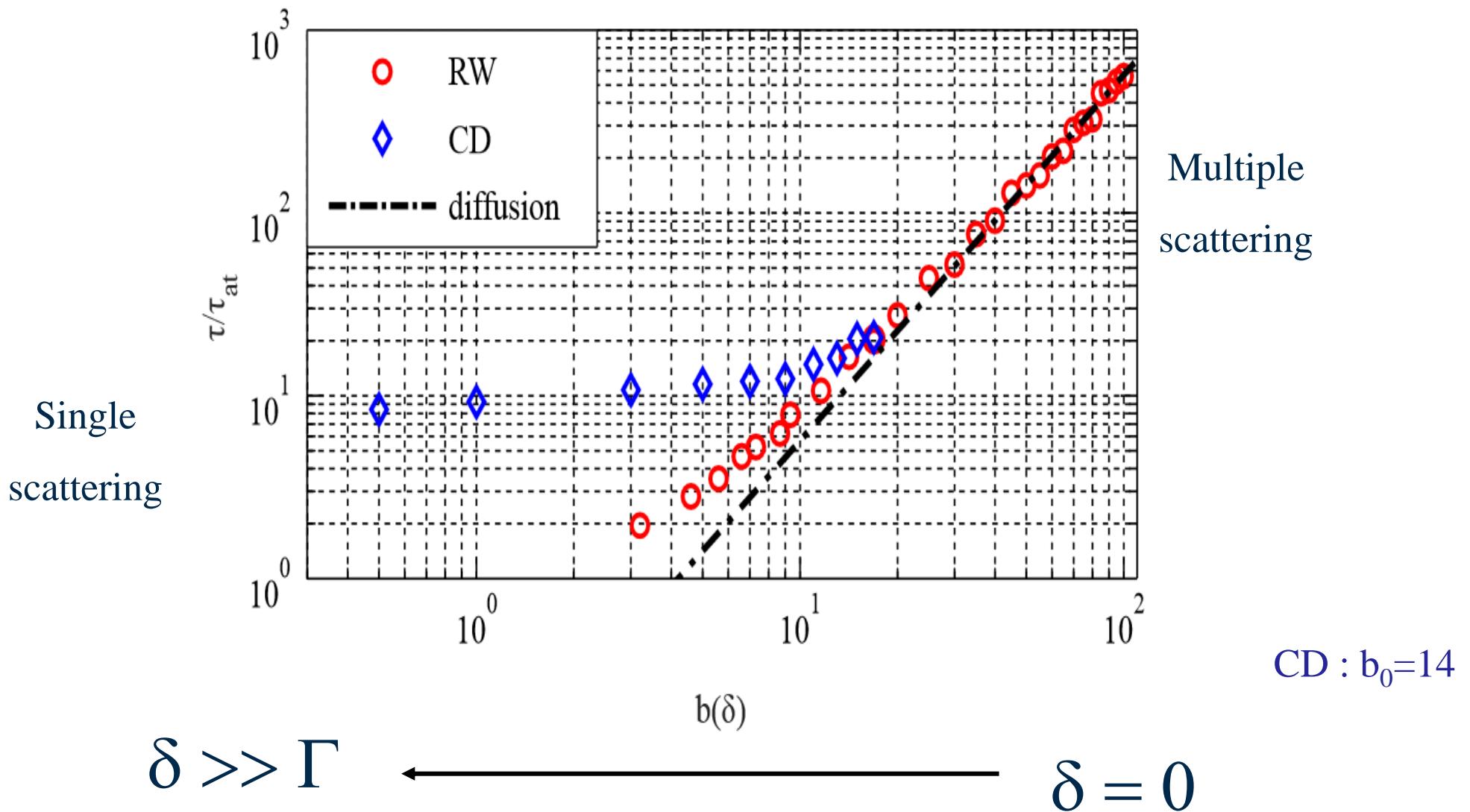


- Random walk of photons (without interference)
- Diffusion equation

$$t_{\text{Anderson}} \propto \exp\{b(\delta)\} ?$$

- Density Threshold ?

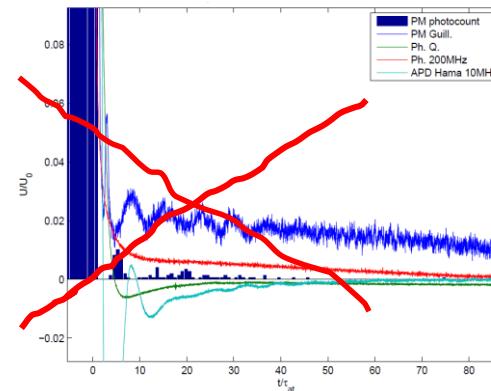
Numerical simulations : random walk vs coupled dipoles



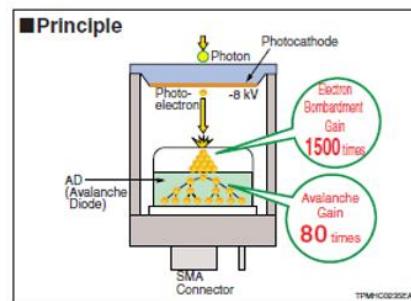
Technical requirements:

- 1) To switch off the probe beam fast and completely ✓ → ~15 ns with AOMs
- 2) A fast, sensitive and low-noise detector
- 3) A detector with a clean response

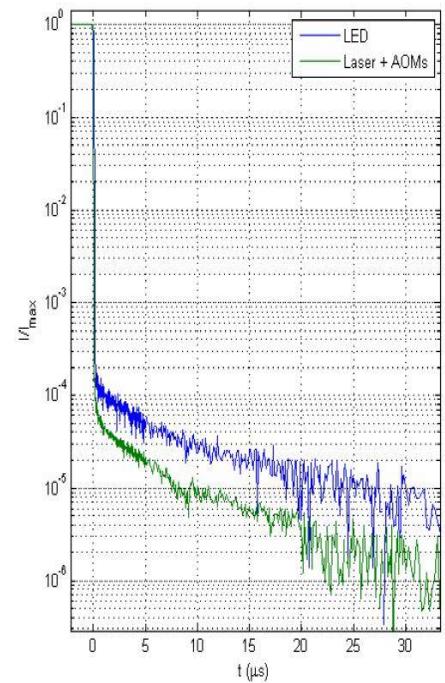
No so easy !



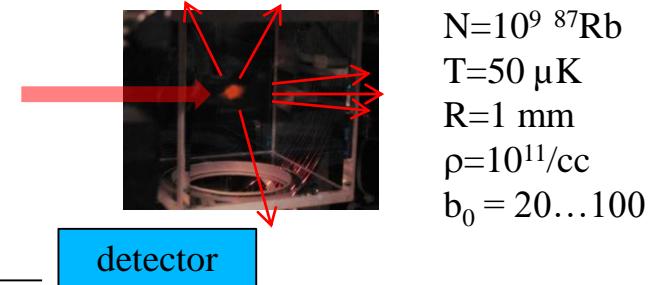
- ‘Hybrid’ photodetector by Hamamatsu, without afterpulsing
- Photocounting regime (low signal)



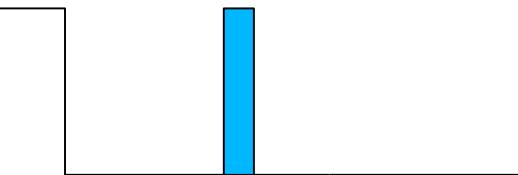
Integration of ~ 500 000 cycles (~ 14h, at night)
→ 4 decades of dynamics ☺



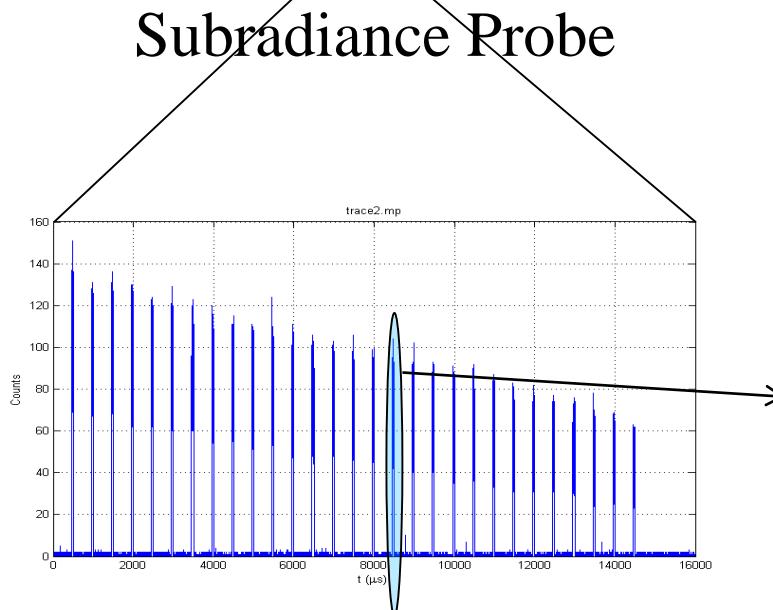
Experiment



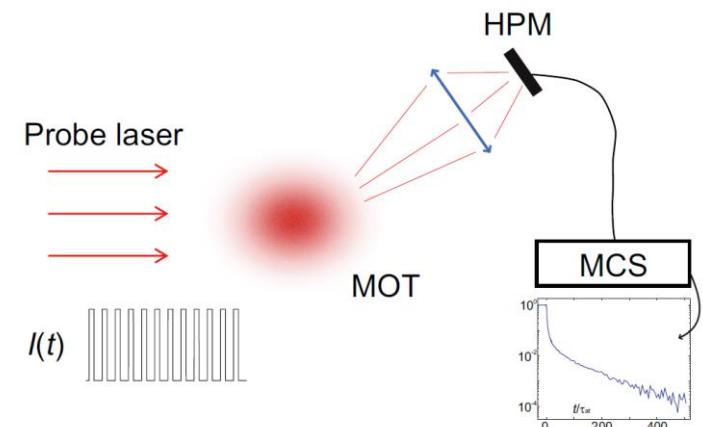
MOT + Dark MOT
(50+30 ms)



Data average :
500 000 cycles
(1 curve/night)



12 pulses of 30μs



Incoming probe beam with:

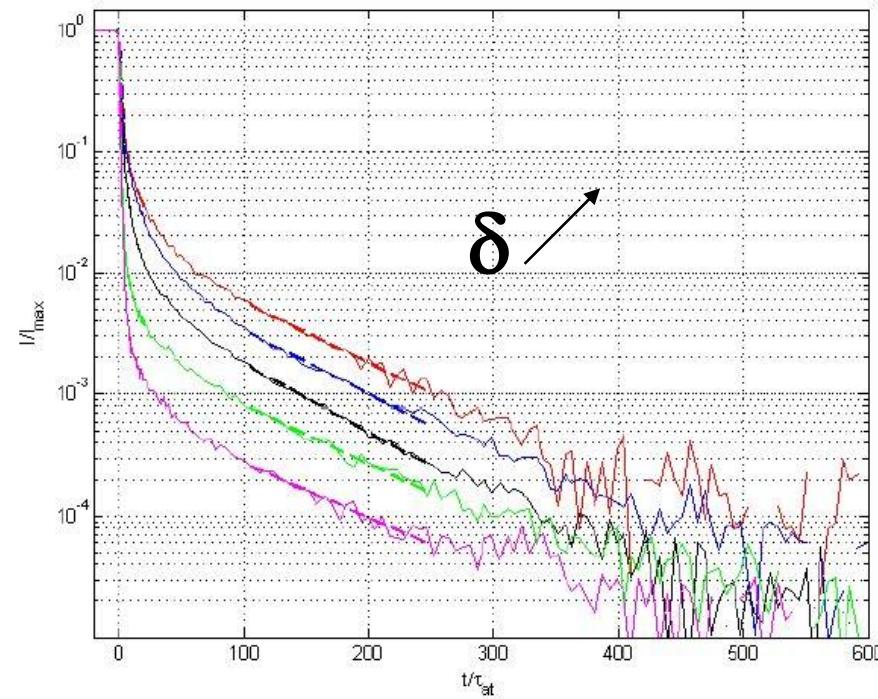
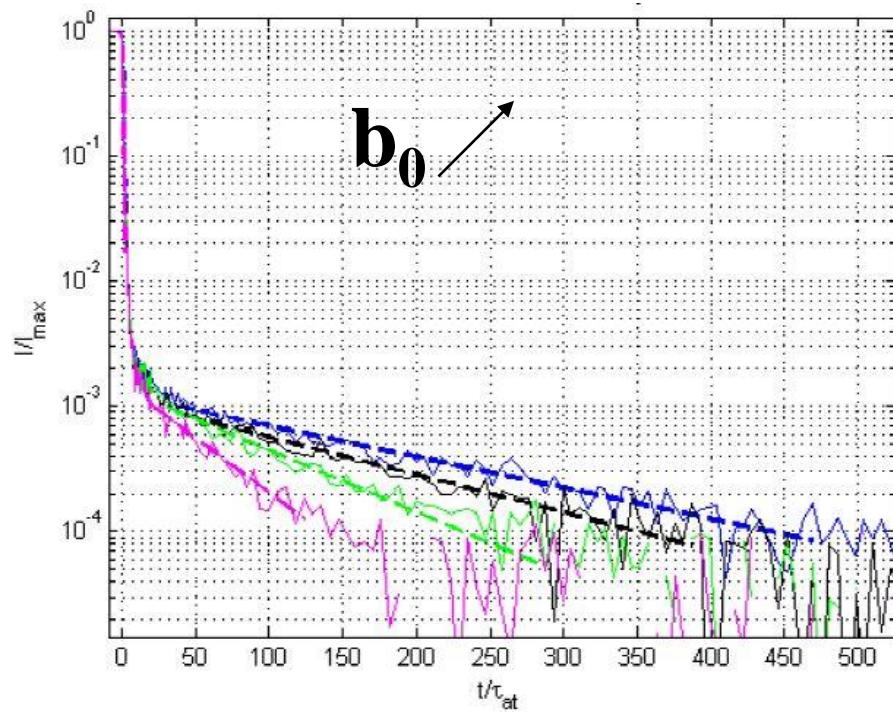
- large beam size
- large detuning δ

All atoms are driven by the same field
(negligible absorption)

No multiple scattering

Experimental results

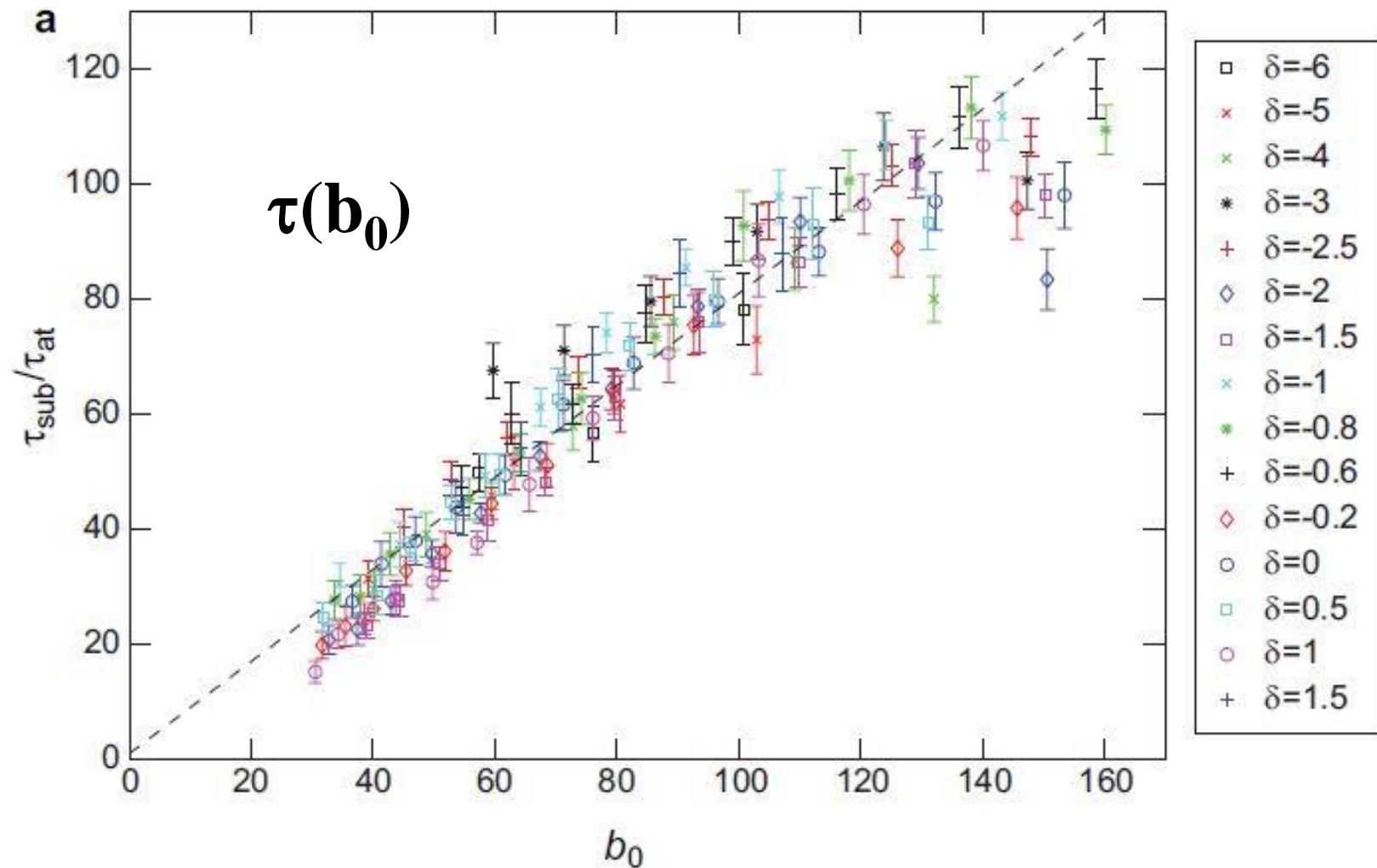
Long decay at $b(\delta) < 1$ ☺



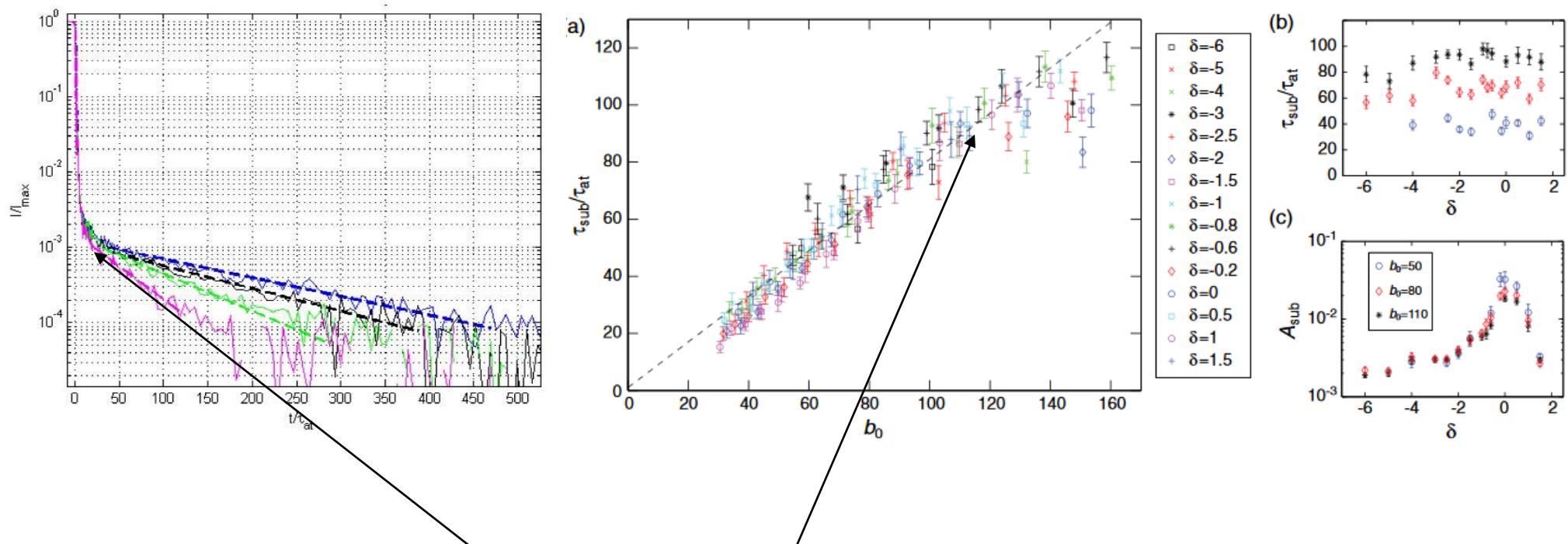
Fit: Exponential decay to zero with two parameters: amplitude and time constant.

Range: One decade above the noise floor → the slowest measurable time constant.

Increases as b_0 ☺



Energy stored in subradiant manifold



$10^{-3} \times 100 = 10\% \text{ Not so bad } \smiley$

Population of collective modes

Coupled dipole model $\dot{\beta}_i = \left(i\Delta - \frac{\Gamma_0}{2} \right) \beta_i - \frac{i\Omega(r)}{2} + \frac{i\Gamma_0}{2} \sum_{i \neq j} V_{ij}(r_{ij}) \beta_j$

Compact form

$$\dot{\mathbf{B}} = \mathbf{M} \times \mathbf{B} + \boldsymbol{\Omega}$$

$$\mathbf{B}_0 = \sum_k \alpha_k \mathbf{V}_k$$

eigenmodes of M

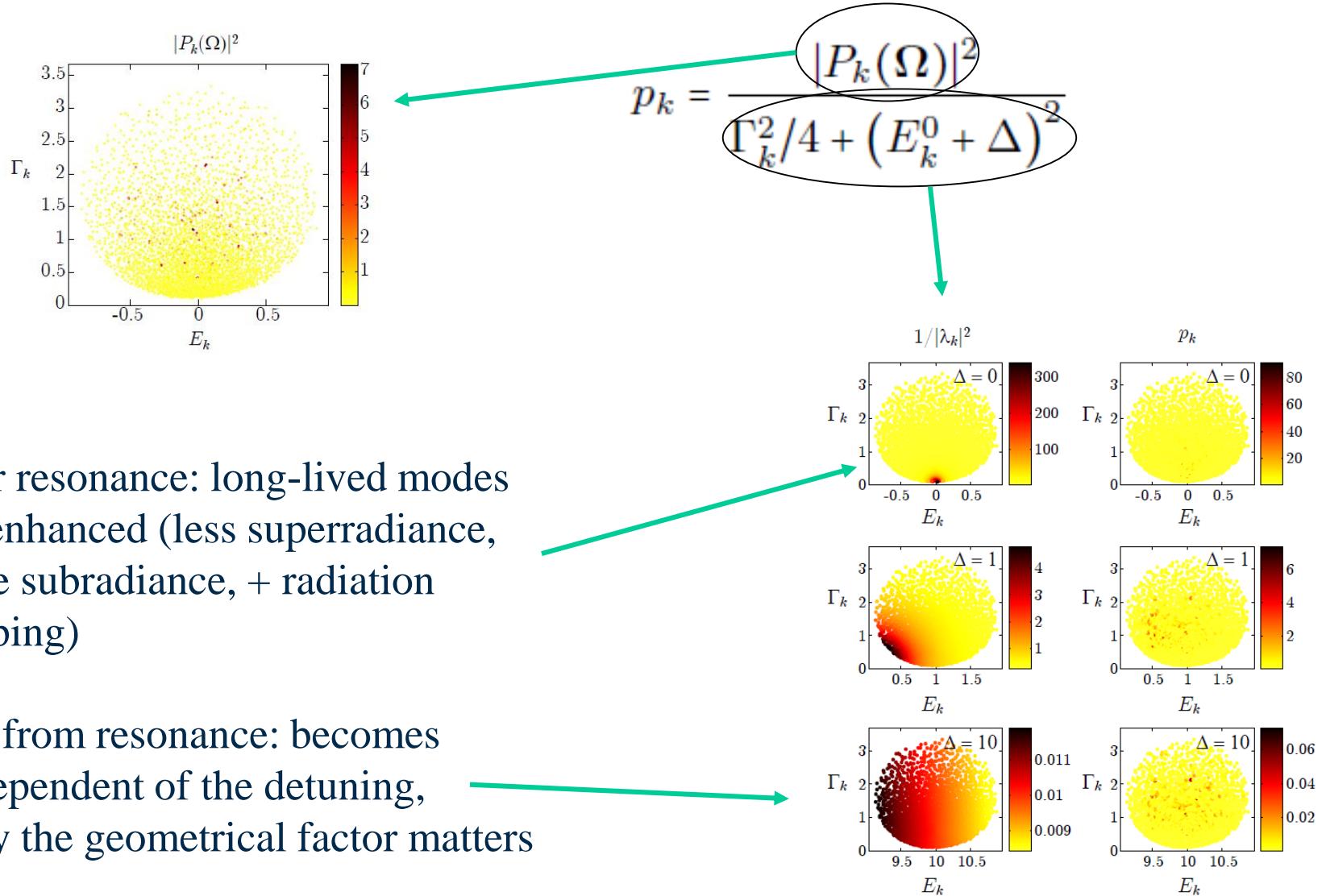
$$\alpha_k = -\frac{(\mathbf{V}^{-1} \boldsymbol{\Omega})_k}{\lambda_k} = -\frac{P_k(\boldsymbol{\Omega})}{\lambda_k}$$

Eigenvalues of M

Population of collective modes : $p_k = |\alpha_k|^2$

$$p_k = \frac{|P_k(\boldsymbol{\Omega})|^2}{\Gamma_k^2/4 + (E_k^0 + \Delta)^2}$$

Population of collective modes

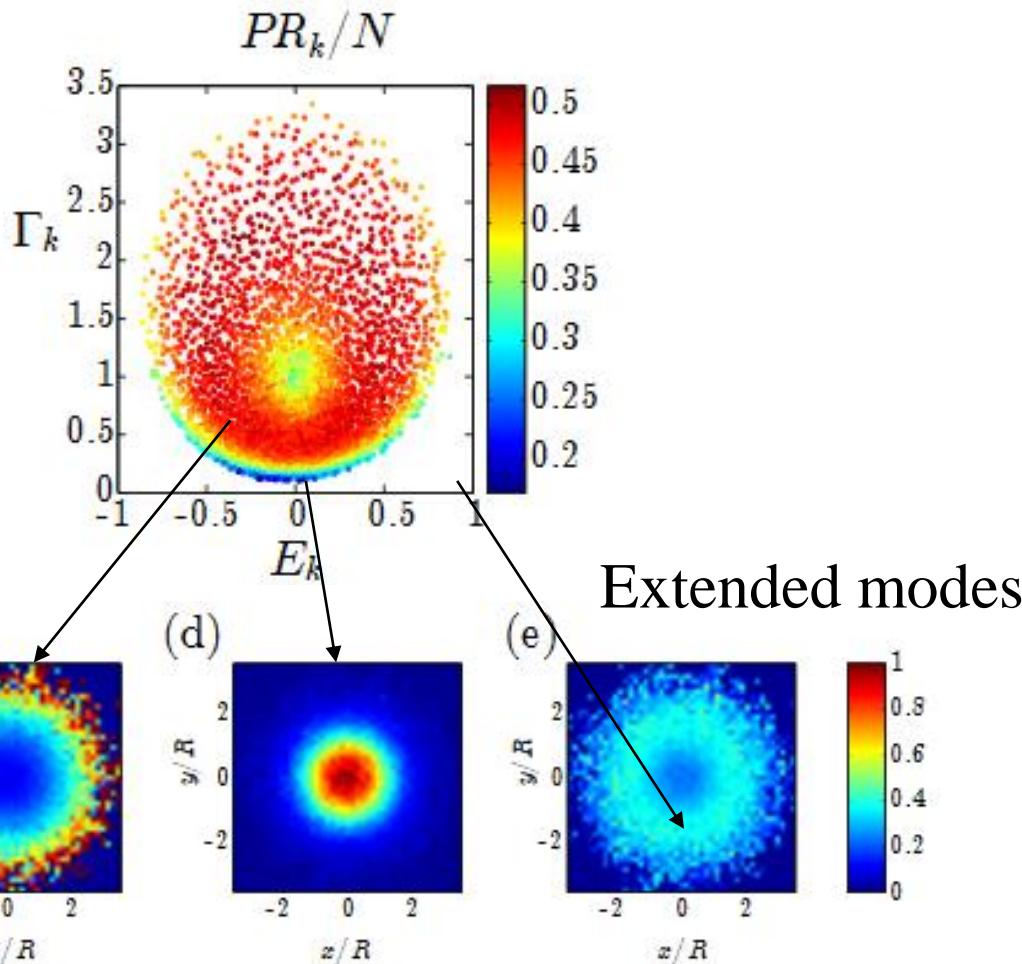


Qualification of collective modes

Partition ratio :

number of atoms participating in a mode

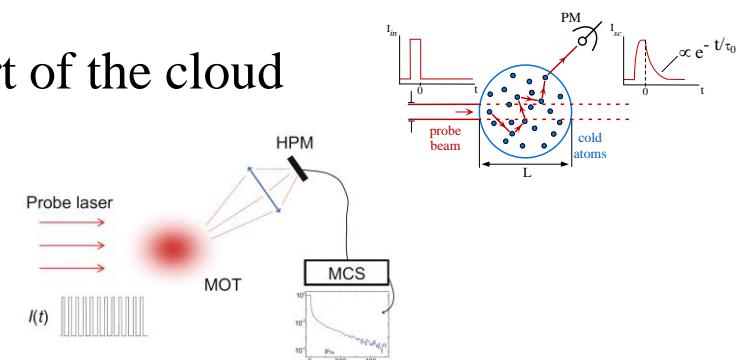
$$PR_k = \frac{\sum_i |V_{ki}|^2}{\sum_i |V_{ki}|^4}$$



Long lived ‘radiation trapping’ modes

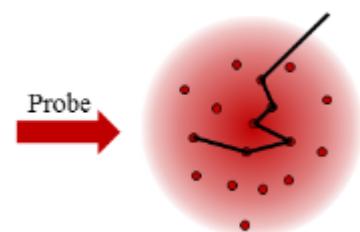
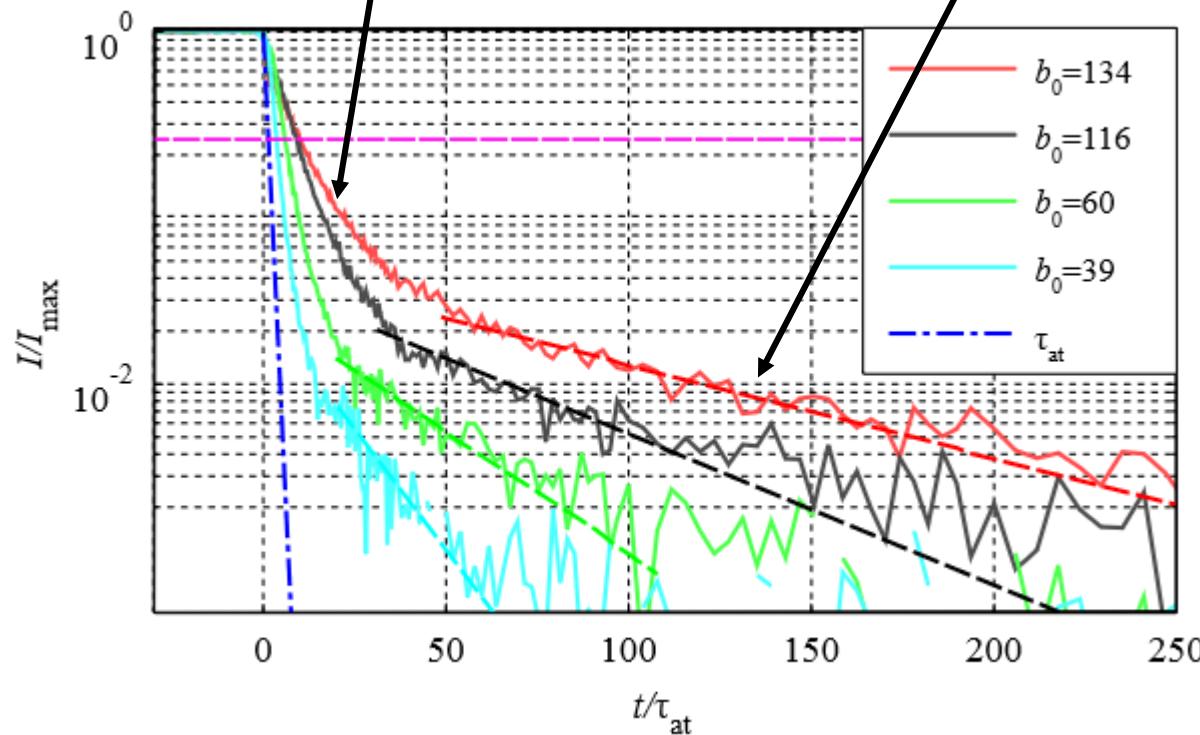
From Radiation Trapping to Subradiance

Radiation Trapping favored when excited central part of the cloud

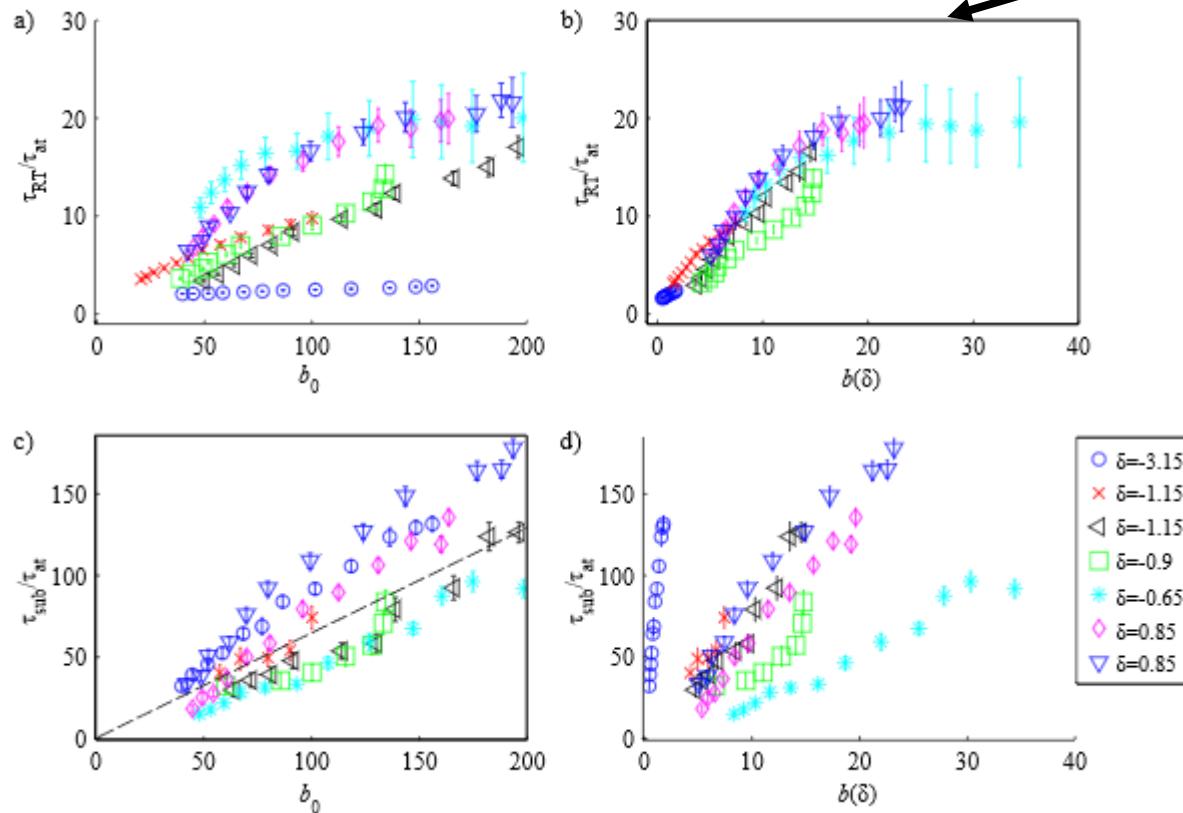


Subradiance favored when excited extended modes

Coexistence of **Radiation Trapping** and **Subradiance**



Scaling laws for Radiation Trapping : $b(\delta)$



Scaling laws for subradiance: b_0

Subradiance and atomic motion

Subradiant states = **collective mode** / superposition of many single atom excitations

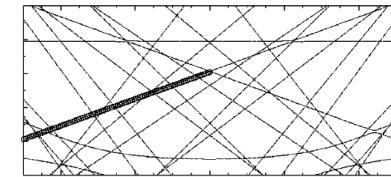
How sensitive is subradiance to decoherence?

Role of atomic motion :

kv vs Γ_{sub} ?

adiabatic following of eigenstates?

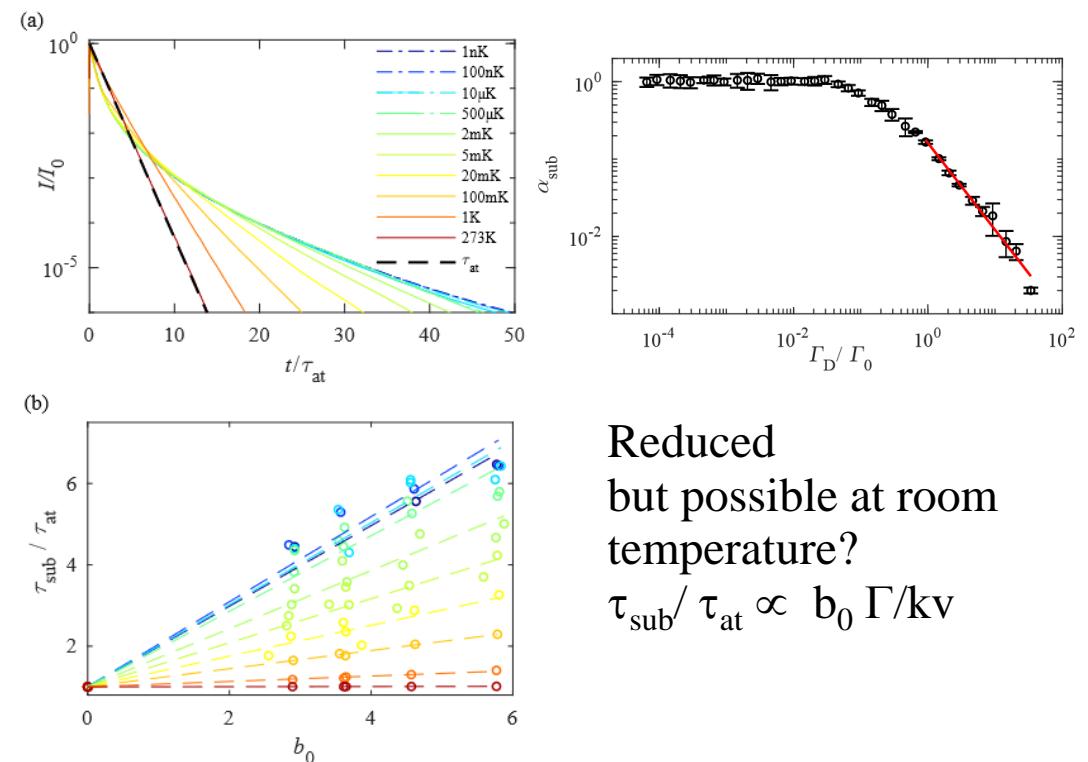
Mixing between many long lived eigenstates



Coupled dipole model with **moving** atoms

$$\dot{\beta}_i = \left[-\frac{\Gamma}{2} + i(\Delta_0 - \mathbf{k}_0 \cdot \mathbf{v}_i) \right] \beta_i - \frac{i\Omega_0}{2} + \frac{i\Gamma}{2} \sum_{j \neq i} V_{ij} \beta_j$$

$$V_{ij}(t) = \frac{e^{ik|\mathbf{r}_i - \mathbf{r}_j + (\mathbf{v}_i - \mathbf{v}_j)t|}}{k|\mathbf{r}_i - \mathbf{r}_j + (\mathbf{v}_i - \mathbf{v}_j)t|} e^{-i\mathbf{k}_0 \cdot [\mathbf{r}_i - \mathbf{r}_j + (\mathbf{v}_i - \mathbf{v}_j)t]}$$



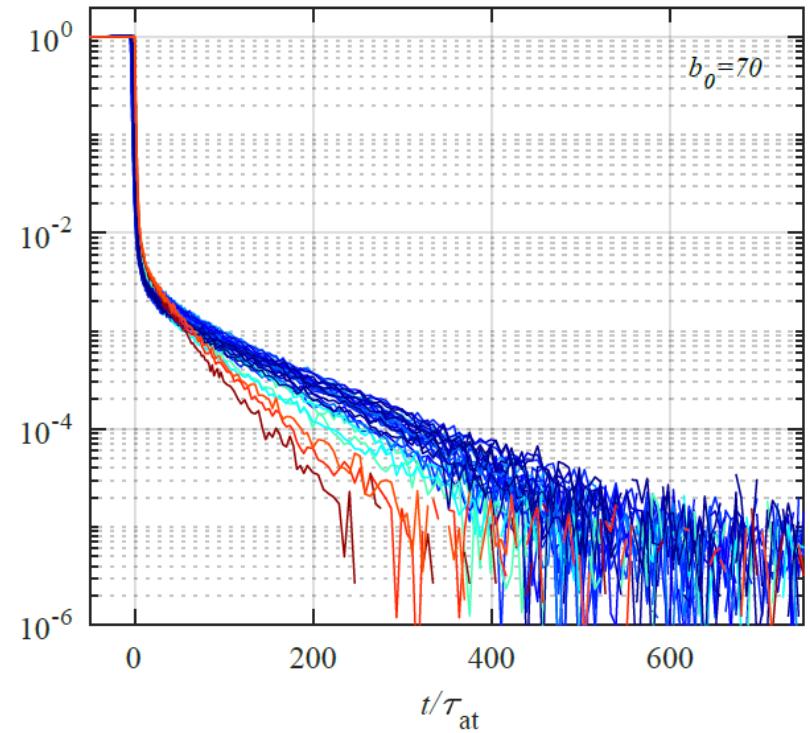
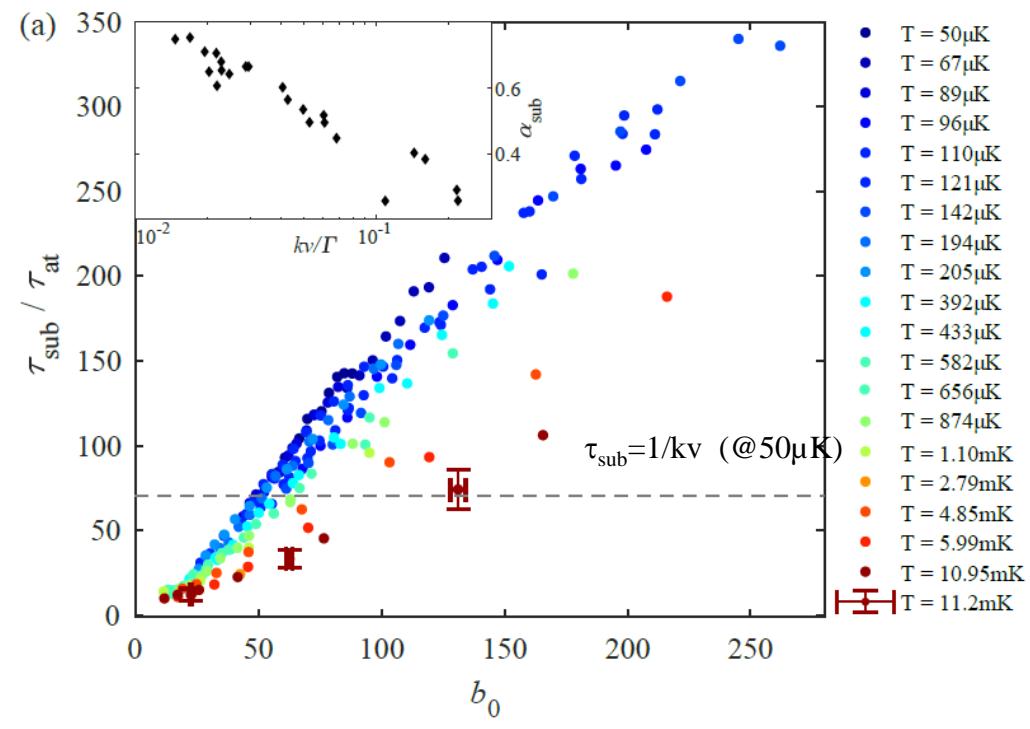
Reduced
but possible at room
temperature?

$$\tau_{\text{sub}}/\tau_{\text{at}} \propto b_0 \Gamma/kv$$

T. Bienaime et al., Phys. Rev. Lett, 108, 123602 (2012)

P. Weiss et al., Phys. Rev. A 100, 033833 (2019)

Subradiance is ± robust against temperature



Beyond linear optics with single photons?

Light scattering in the linear optics limit

$$\mathbf{d} = \alpha \mathbf{E}_L$$

$$\langle \mathbf{d} \rangle = \text{Tr}(\rho \mathbf{d}) = \|\mathbf{d}\|(\rho_{ge} - \rho_{eg})$$

Photon emission from single excitation sector : $\mathbf{d} = \mathbf{0}$

Cannot be treated in a classical optics framework

Need Lindblad formalism or equivalent full quantum treatment

Homework 2 :

Use Lindblad formalism (or equivalent full quantum treatment) to derive decay curve from initial single excitation **population**

From Coherent backscattering to cooperative scattering

Coherent backscattering / mesoscopic approach

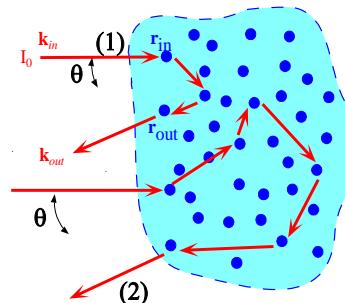
diagrammatic approach

Assumes ‘independent scattering Ansatz’

excellent for analytical results 😊

does not describe subradiance 😞

$$R \approx L = \begin{array}{c} \otimes \\ \otimes \end{array} + \begin{array}{c} \otimes \\ \otimes \end{array} \begin{array}{c} \otimes \\ \otimes \end{array} + \begin{array}{c} \otimes \\ \otimes \end{array} \begin{array}{c} \otimes \\ \otimes \end{array} \begin{array}{c} \otimes \\ \otimes \end{array} + \dots$$
$$C = \begin{array}{c} \otimes \\ \otimes \end{array} \begin{array}{c} \otimes \\ \otimes \end{array} + \begin{array}{c} \otimes \\ \otimes \end{array} \begin{array}{c} \otimes \\ \otimes \end{array} \begin{array}{c} \otimes \\ \otimes \end{array} + \dots$$



Population of collective modes : $p_k = |\alpha_k|^2$

driving collective modes

More ‘exact’ 😊

no analytic result so far 😞

does not describe coherent backscattering 😞

$$p_k = \frac{|P_k(\Omega)|^2}{\Gamma_k^2/4 + (E_k^0 + \Delta)^2}$$

