Interaction and disorder :

Light scattering by cold atoms:

Localization and Cooperative Scattering

Robin KAISER

Nice, France

Lecture 1 : **Multiple Scattering of Light in Cold Atoms**

1.1 Steady State Results : Ohm's Law for Photons
+ Numerical Random Walk Simulations
1.2 Time dependent scattering : radiation trapping
+ Numerical Random Walk Simulations
1.3 Random laser

Lecture 2 : **Interference Effects in Light Scattering by Cold Atoms**

2.1 Coherent Backscattering of Light by Cold Atoms
+Numerical Simulations with Weak Localization Corrections
2.2 Dicke Super- and Subradiance
+Numerical Simulations with Coupled Dipoles

Lecture 3 : **Anderson Localisation of Light**

3.1 Anderson Lattice Model
3.2 Effective Hamiltonian Approach
3.3 Scalar vs vectorial light : red light for Anderson localization
3.4 Outlook : towards localization of light in cold atoms

Link to Mathlab codes :

http://www.kaiserlux.de/coldatoms/LesHouches2019Kaiser.html

Lecture 1 : Multiple Scattering of Light in Cold Atoms

1.1 Steady State Results : Ohm's Law for Photons

Single Atom response: optical Bloch equations
Random Walk : Beer-Lambert's Law, diffusion, Ohm's law, Transmission profile
Numerical Simulations

1.2 Time dependant Scattering : radiation trapping

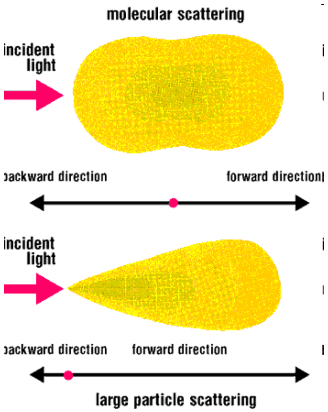
Phase, group and transport velocity
Slow diffusion
Numerical Simulations

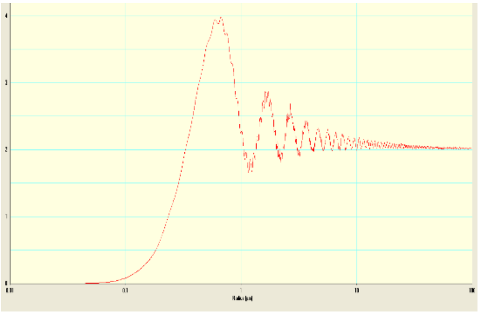
1.3 Multiple Scattering + gain : random lasers

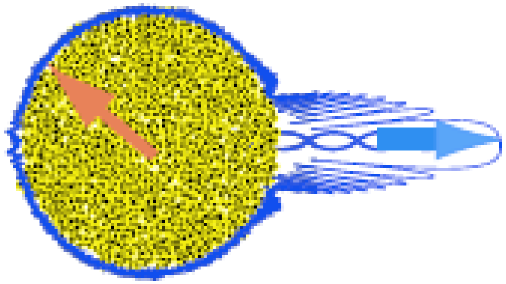
Scattering Properties of Atoms

• Rayleigh scattering (elastic process) $\propto \omega^4$

• Mie scattering (elastic process)

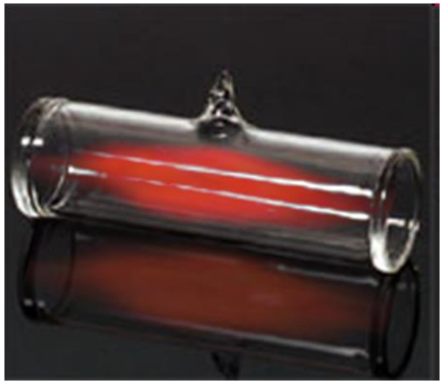


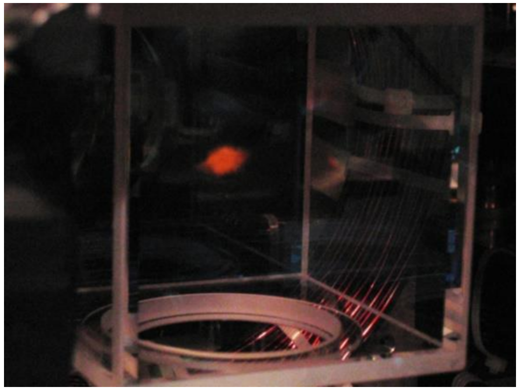




• **Resonant scattering** (elastic and inelastic)

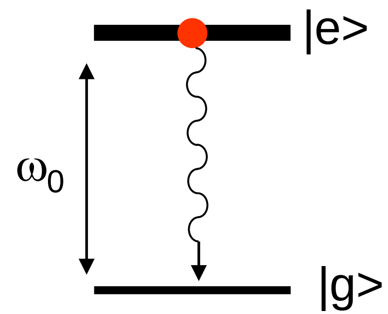
matter waves and ultracold atoms
cold atoms (☺)
moving atoms (Doppler)
multilevel atoms (Raman)





Single Atom response : Optical Bloch Equations

1. Spontaneous Emission

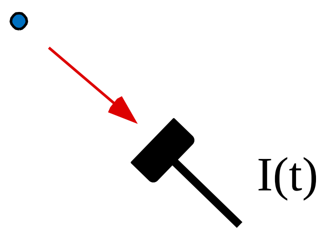


$\Gamma = 1/\tau_{nat}$

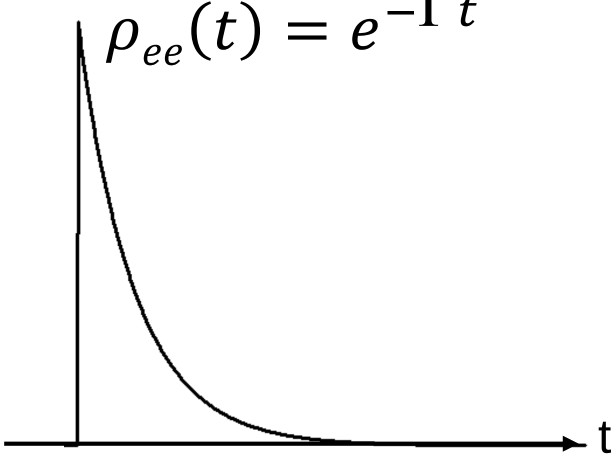
$$\boxed{\frac{d\rho_{ee}}{dt} = -\Gamma\rho_{ee}}$$

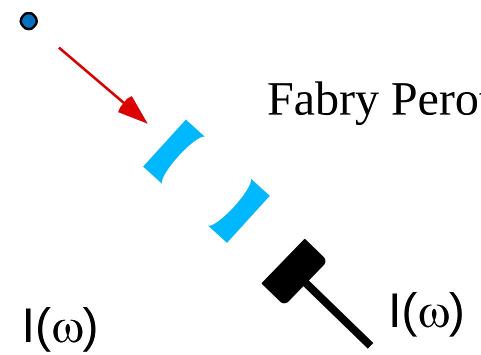
initial condition: t=0 : excited state

$$\rho_{ee}(0) = 1$$

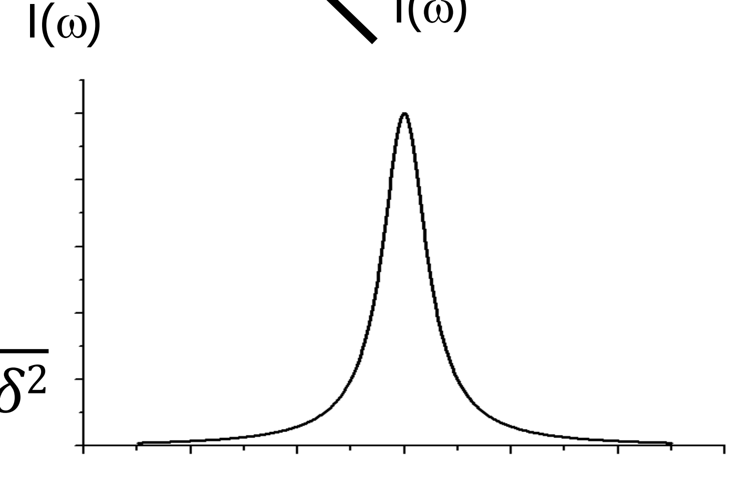


$$\rho_{ee}(t) = e^{-\Gamma t}$$





$$\rho_{ee}(\omega) \propto \frac{1}{1 + 4\delta^2}$$



$\delta = \omega_{cav} - \omega_0$

Single Atom response : Optical Bloch Equations

2. Stimulated Emission

$$\frac{d\rho_{ee}}{dt} = -\Gamma\rho_{ee} + i\frac{\Omega_L}{2}\left(\rho_{eg} - \rho_{ge}\right)$$

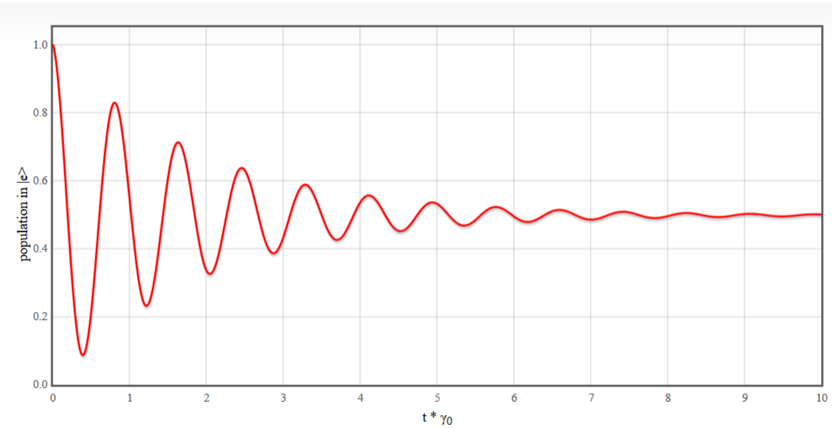
$$\frac{d\rho_{ge}}{dt} = -(i\delta + \frac{\Gamma}{2})\rho_{ge} - i\frac{\Omega_L}{2}\left(\rho_{ee} - \rho_{gg}\right)$$

$$\frac{d\rho_{gg}}{dt} = +\Gamma\rho_{ee} - i\frac{\Omega_L}{2}\left(\rho_{eg} - \rho_{ge}\right)$$

$$s_0 = \frac{2\Omega_L{}^2}{\Gamma^2} = \frac{I}{I_{sat}}$$

$\hbar\Omega_L$**=-dE**

initial condition: t=0 : excited state $\rho_{ee}(0) = 1$



[Rabi Oscillations (courtesy of J. Evers)]

Single Atom response : Optical Bloch Equations

3. Photon Absorption

$$\frac{d\rho_{ee}}{dt} = -\Gamma\rho_{ee} + i\frac{\Omega_L}{2}\left(\rho_{eg} - \rho_{ge}\right)$$

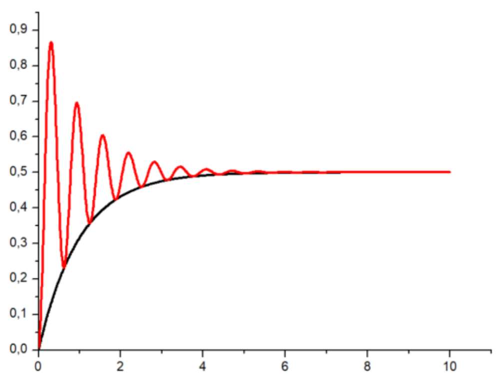
$$\frac{d\rho_{ge}}{dt} = -(i\delta + \frac{\Gamma}{2})\rho_{ge} - i\frac{\Omega_L}{2}\left(\rho_{ee} - \rho_{gg}\right)$$

$$\frac{d\rho_{gg}}{dt} = +\Gamma\rho_{ee} - i\frac{\Omega_L}{2}\left(\rho_{eg} - \rho_{ge}\right)$$

initial condition: t=0 : ground state

$$\rho_{gg}(0) = 1$$

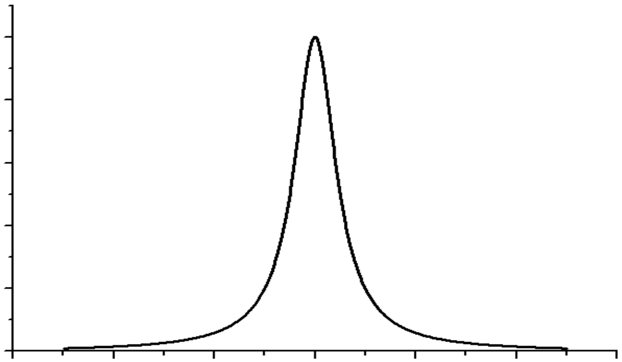
$\rho_{ee}(t)$



t

$$\rho_{ee}^{st} = \frac{\Omega_L^2/2}{\delta^2 + \Omega_L^2/2 + \Gamma^2} = \frac{1}{2}\frac{s}{1+s}$$

$\rho_{ee}^{st}(\delta)$

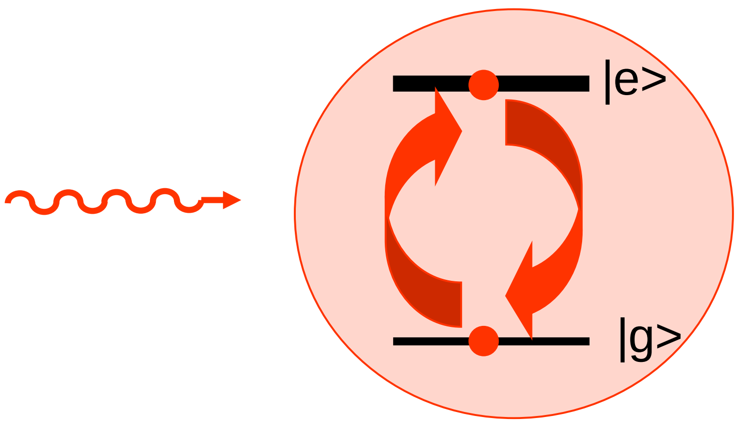


δ

s<<1 : absorption spectrum = emission spectrum

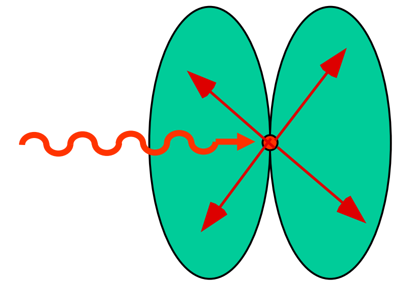
Optical Bloch Equations

Photon Scattering



Coherent oscillation between ground and excited state:

optical coherence = dipole
population ≠ dipoles



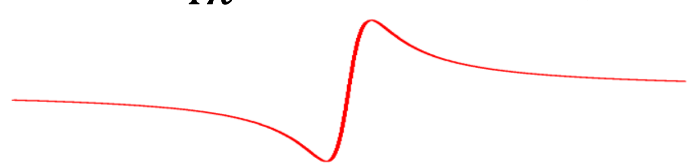
Dipole emission diagram

Induced dipole $\boldsymbol{d} = \alpha \boldsymbol{E}_L$

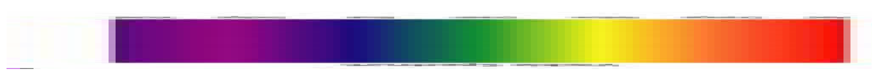
$$< d >= \mathrm{Tr}(\rho d) = ||d||(\rho_{ge} - \rho_{eg})$$

index of refraction of dilute cloud :

$$n = 1 - \frac{3}{4\pi^2}\rho\lambda^3 \frac{\delta/\Gamma}{1+4(\delta/\Gamma)^2} + i\,\frac{3}{4\pi^2}\rho\lambda^3 \frac{1}{1+4(\delta/\Gamma)^2}$$



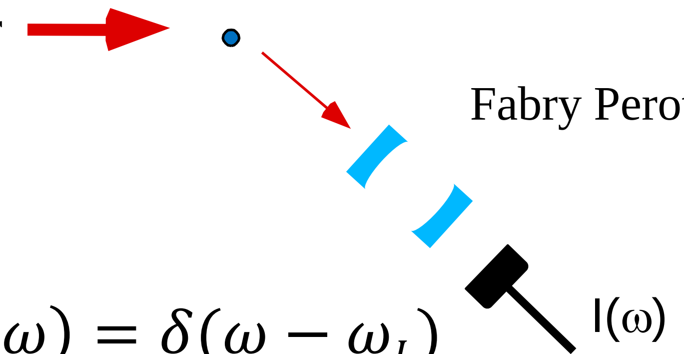
$n - 1 \approx 10^{-4}$ (MOT)
$n - 1 \approx 0.1$ (BEC)



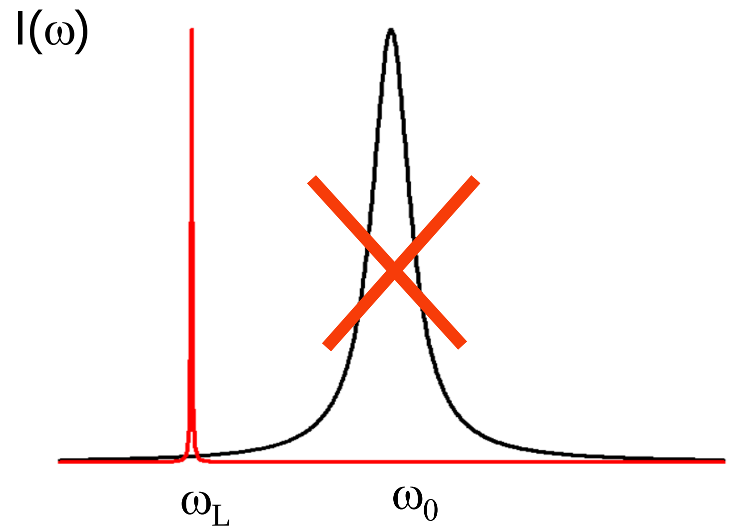
Scattered field

induced dipole with precise phase relation to incident field
scattered field with precise phase relation to induced dipole

$\Rightarrow$ elastic scattering

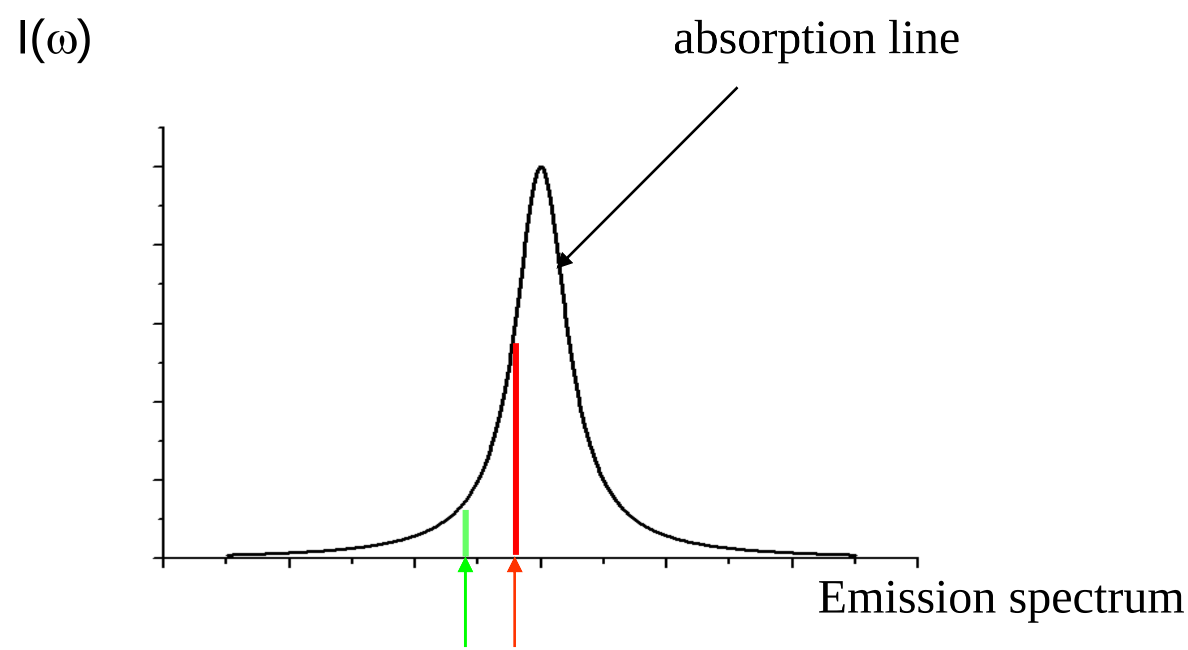


$$I(\omega) = \delta(\omega - \omega_L)$$



scattering of photon ≠ absorption + spont. emission !!!

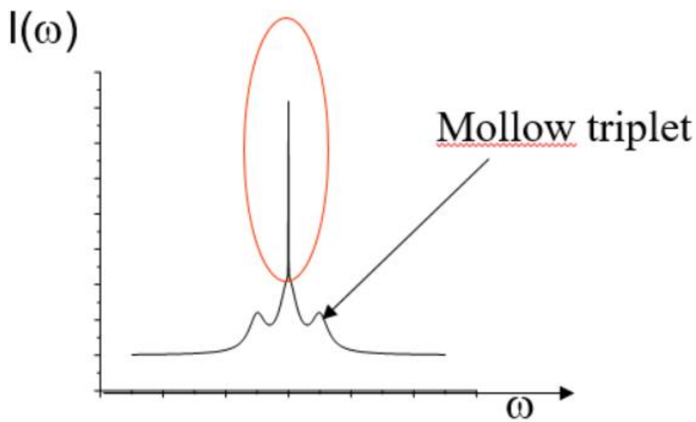
Spectrum of Scattered Field



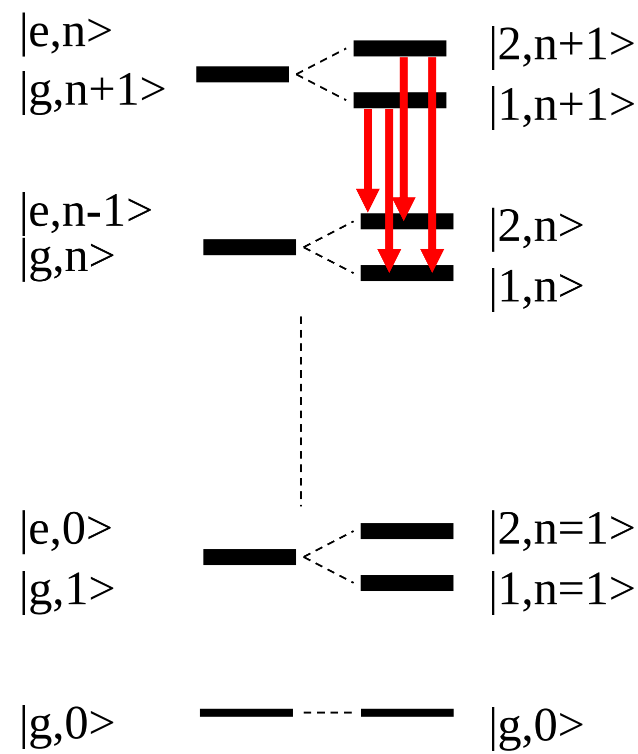
elastic scattering for : cold atoms ($kv \ll \Gamma$), low incident intensity ($I \ll I_{sat}$), no internal structure (Raman scattering)

Inelastic Scattering

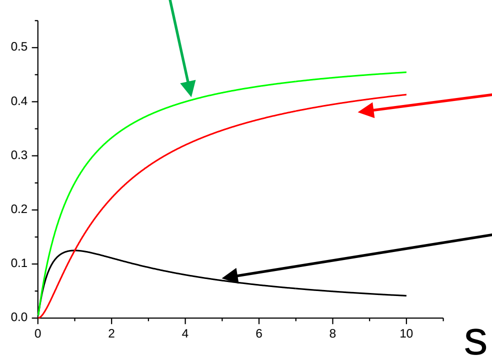
Emission spectrum



Dressed atom



$$\Gamma_{sc} = \Gamma_{coh} + \Gamma_{incoh} = \frac{\Gamma}{2}\frac{s}{1+s} = \Gamma\varrho_{ee}^{st}$$



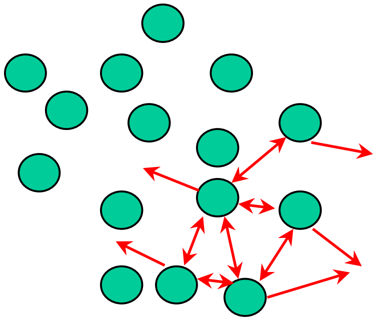
$$\Gamma_{\text{incoh}} = \frac{\Gamma}{2}\frac{s^2}{(1+s)^2}$$

$$\Gamma_{\text{coh}} = \frac{\Gamma}{2}\frac{s}{(1+s)^2} = \Gamma\left|\varrho_{eg}^{st}\right|^2$$

C. Cohen-Tannoudji, S. Reynaud, J. Phys.B10 345 (1977).

Multiple Scattering

Photons ...



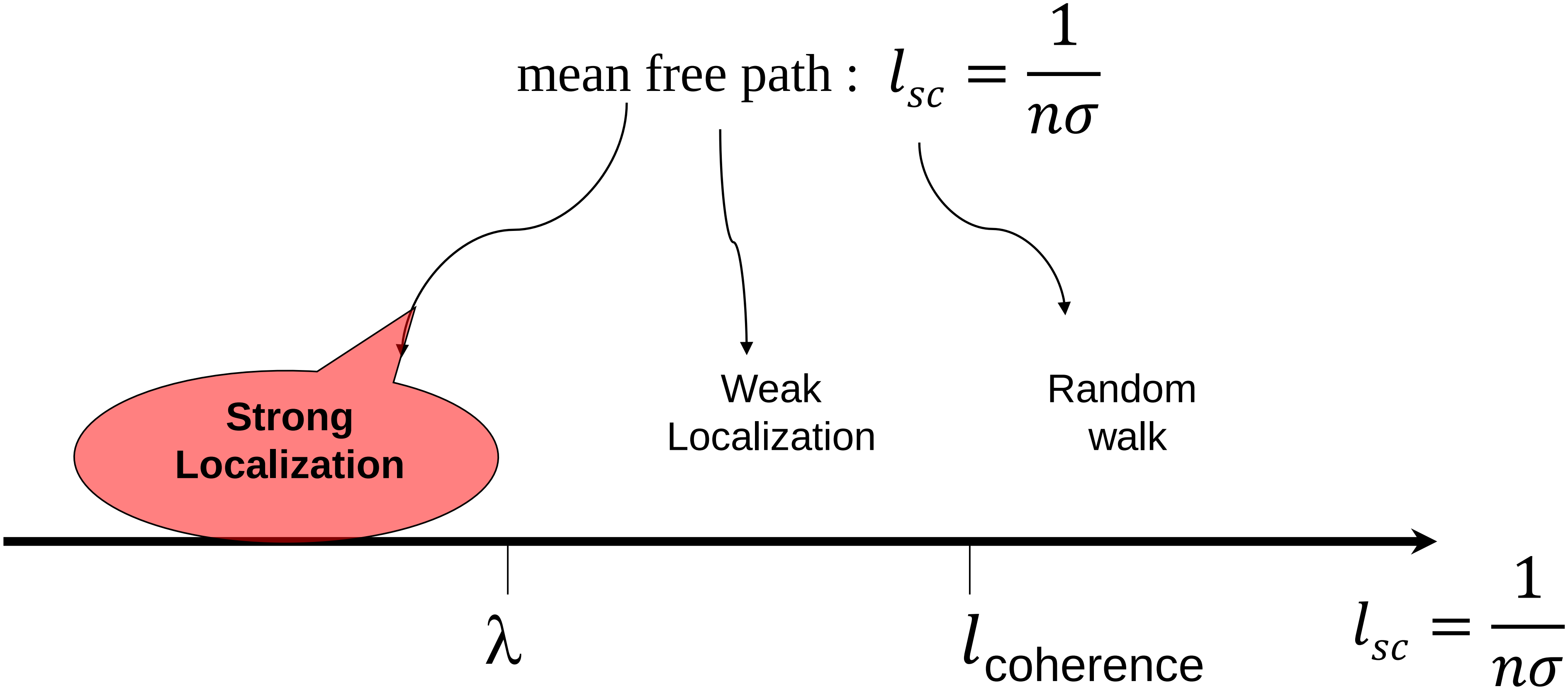
Random walk :

Diffusion

coefficient $D_0 \approx \ell^2 / \tau$

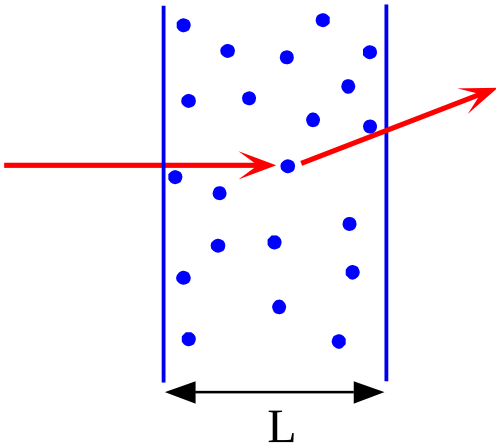
$\ell = 1/n\,\sigma$

Coherence, Disorder and Interferences



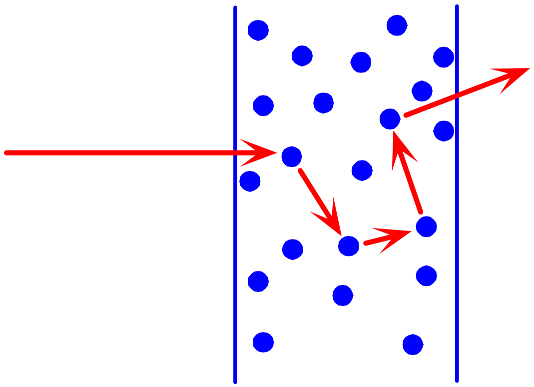
Single scattering (b=L/l_{sc}<<1)

Mean free path : $$l_{sc} = \frac{1}{n\sigma}$$



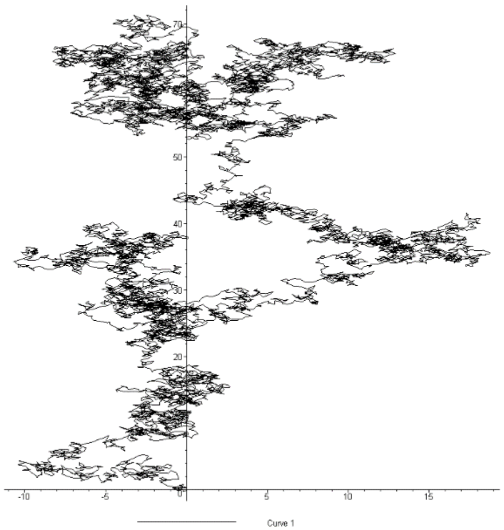
Beer-Lambert law : $$T_{coh} = e^{-\frac{L}{l_{sc}}}$$

Multiple scattering (b=L/l_{sc}>>1)

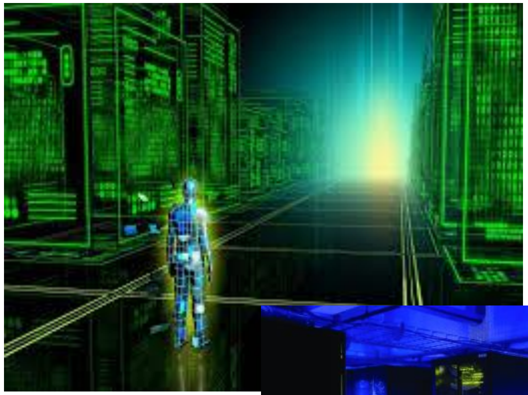


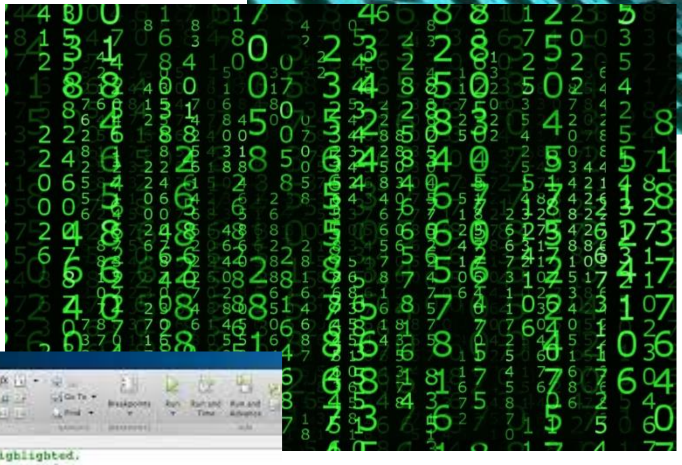
Ohm's law : $$T_{diff} \propto \frac{l_{sc}}{L}$$

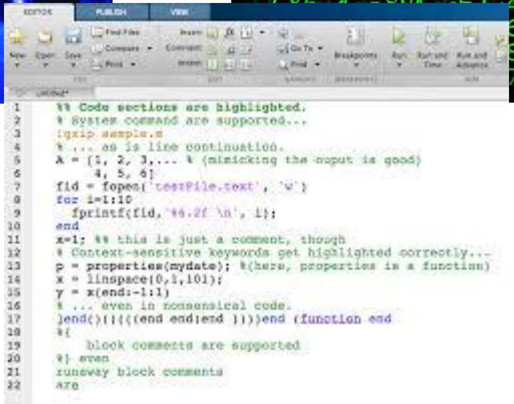
Random Walk



Numerical Simulations







www.kaiserlux.de/coldatoms/LesHouches2019Kaiser.html

Rechercher

Link to Mathlab codes of Robin Kaiser's lecture in Les Houches 2019

Random Walk based codes :

- RandomWalk_LH2019.m
- Ohm_LH2019.m
- TransmissionProfile_N100000.m
- RT_LH2019.m
- RandomWalk_CBS.m

Coupled Dipole based codes :

- CoupledDipoles_LH2019.m

Contact : Robin Kaiser
e-mail: Robin.Kaiser@inphyni.cnrs.fr
http://www.kaiserlux.eu/coldatoms/

Editor - C:\Users\Kaiser\Documents\speaches\2019\LesHouches2019\NumericalSimulations\1RandomWalk\RandomWalk_LH2019.m

```matlab
% Random walk (RW) in a finite slab
% All lengths (Lz, R, ell, x0, y0) have to be given in the same unit: e.g in 1/k.
% The outputs are: the Trajectory, the number of scattering events Nsc (0 for transmitted photons), the output angles theta, phi, the output positions x, y

clear all;
% Parameters of the slab
Lz=5; % length of the slab (e.g. Lz=5)
R=10; %transverse size     (e.g. R=10)
ell=1; %scattering mean free path   (e.g. ell=1)

Nph=10; % number of photons
for iph=1:1:Nph  %loop on different photons injected in the slab
 clear X Y Z

% Initial Conditions
Nsc=1;  % number of scattering events already done
cos_phi = 1; theta=0; % direction of inicident propagation
x=0; %initial position along x
y=0; %initial position along y
z=0; % Initial impact with the vapor (NOT the 1st scattering event!)
r=sqrt(x^2+y^2);

% length of the 1st step using an exponential distribution (Beer-Lambert)
step = -ell*log(rand);  % use the inverse function f^(-1) to draw the random number following a distribution f: exp -> log

% The first step is ballistic in forward direction
z = z+step;
    % register position of the photon after this first step
    X(Nsc) = x;
    Y(Nsc) = y;
    Z(Nsc) = z;


while all(z<=Lz  && z>0  && r<R) % Next scattering event inside the slab

    % draw a new random direction
    theta = 2*pi*rand;
    cos_phi = -1+2*rand;  % uniform between -1 and 1 (always  check that the distribution is uniform: mistakes often happen with cos, sin etc)
    sin_phi = sqrt(1-cos_phi^2);  % always >0 since phi is between 0 and pi

    % length of the next step in an exponential ditribution (Beer-Lambert)
    step = -ell*log(rand);

    % new position
    x = x + step*sin_phi*cos(theta);
    y = y + step*sin_phi*sin(theta);
    z = z + step*cos_phi;
    r = sqrt(x^2+y^2);

    % register the next steps (trajectory)
    Nsc = Nsc+1;  % add 1 to number of scattering events
    X(Nsc) = x;
    Y(Nsc) = y;
    Z(Nsc) = z;

end
phi = acos(cos_phi); %outgoing angle


% plotting the trajectory (for illustration)


 %plot an indication of the contour of the slab
 Rc=[R R];
 N=1000;
 [Xc,Yc,Zc] = cylinder(Rc,N);
 surf(Xc,Yc,Lz*Zc, 'edgecolor','none'); alpha(0.1); %indiction of slab
 hold on;
 %plot trajectory
 plot3([0 X], [0 Y], [0 Z]), xlabel('x/l'), ylabel('y/l'), zlabel('z/l'), axis([-R R -R R -2 Lz+2]);
 pause(1);
 close(gcf);

end;
```

script   Ln 54   Col 16   OVR

www.kaiserlux.de/coldatoms/LesHouches2019Kaiser.html

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- **CoupledDipoles_LH2019.m**

Contact : Robin Kaiser
e-mail: Robin.Kaiser@inphyni.cnrs.fr
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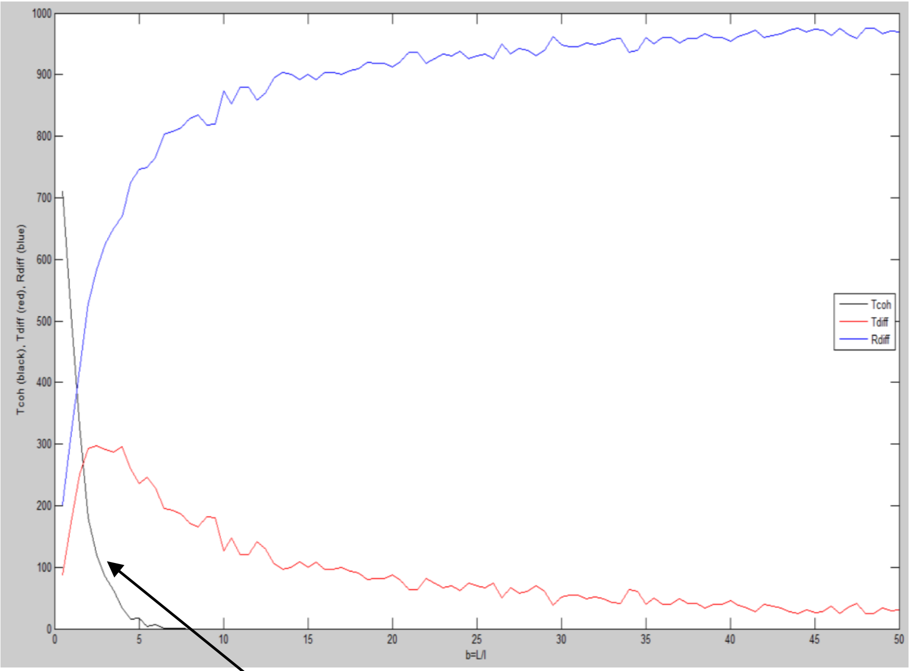
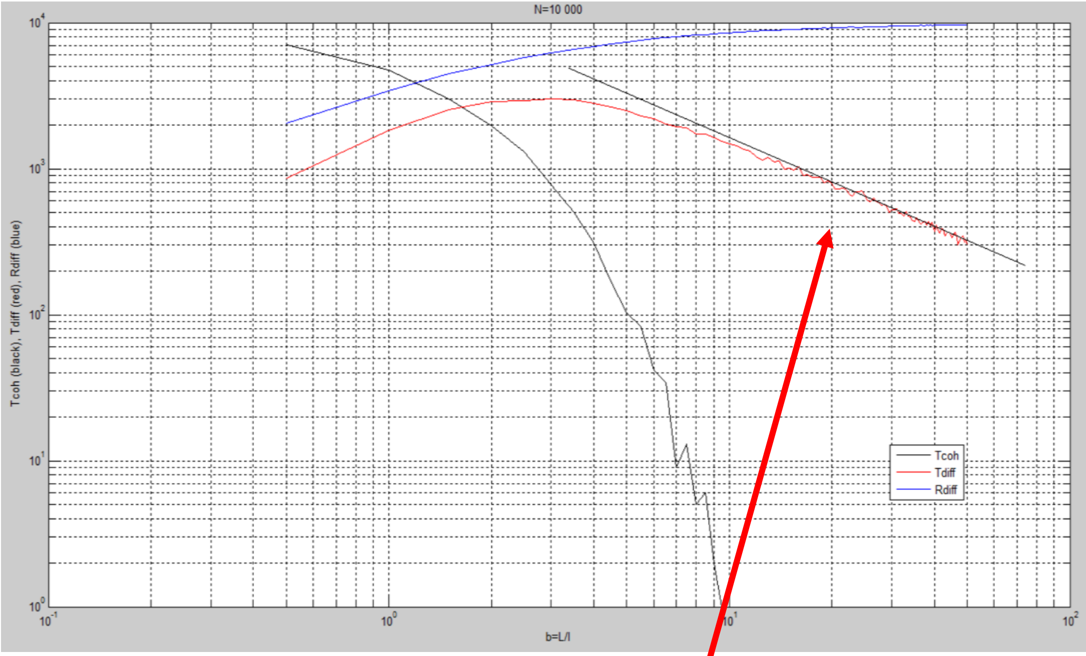
Random walk through a slab:

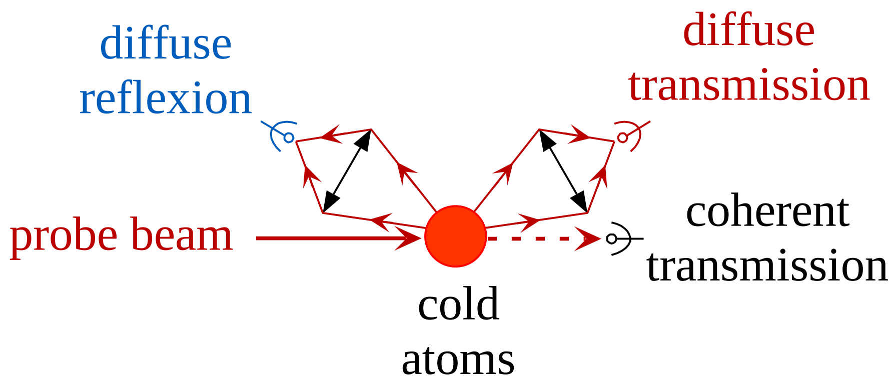




Beer-Lambert law : $T_{coh} \propto e^{-L/\ell}$

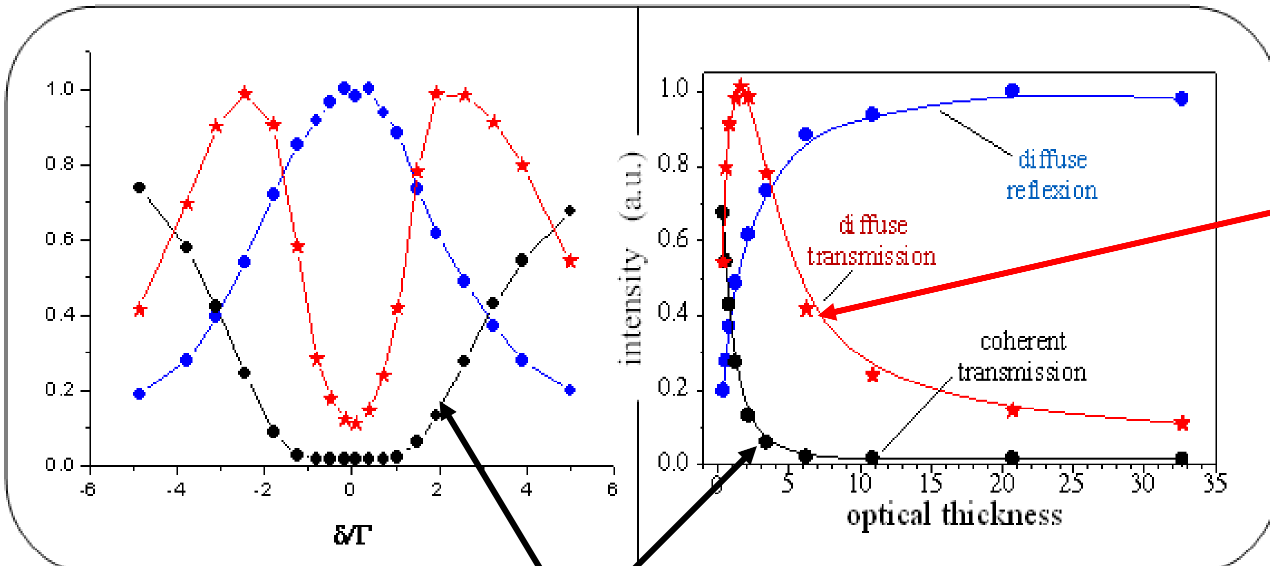
Ohm's law for photons

Scattering Experiments with Cold Atomic



MOT parameters :

- MOT diameter ≈ few mm
- N ≈ few 10^{11}
- b ≈ 200
- velocities ≈ 0.1m/s



Ohm's law for photons

$\boxed{T_{diff} \propto 1/L}$

Beer-Lambert law : $T_{coh} \propto e^{-L/\ell}$

Optical thickness :

$b(\delta)=b_0/(1+4\delta^2/\Gamma^2)$

$$T_{coh} = e^{-b_0/_{1+4\delta^2/\Gamma^2}}$$

A short digression ☺

Mean free path : $l_{sc} = \dfrac{1}{n\sigma}$

$$P(x) \propto e^{-x/lsc}$$

```
step = -ell*log(rand);
```

$$< x > = \int xP(x)dx = l_{sc}$$

$$\int P(x)dx = 1$$

$$D = < x^2 > = \int x^2 P(x)dx = l_{sc}$$

If $<x>$ and $<x^2>$ are finite,
the distribution of a large number of steps Σx_i
converges to a Gaussian distribution
(central limit theorem)

What if … ‘heavy tail’ step size distribution . :

$$P(x) \propto \frac{1}{x^{\alpha}}$$

non stationnary $\int P(x)dx=\infty$	‘ballistic’ $\langle \mathbf{x} \rangle = \infty$	‘super-diffusive’ $\langle \mathbf{x}^2 \rangle = \infty$	normal $\langle \mathbf{x}^2 \rangle < \infty$

1 2 3 α

Lévy Flights

how to measure $\langle x \rangle \pm \sqrt{\langle x^2 \rangle}$: weak ergodicity breaking

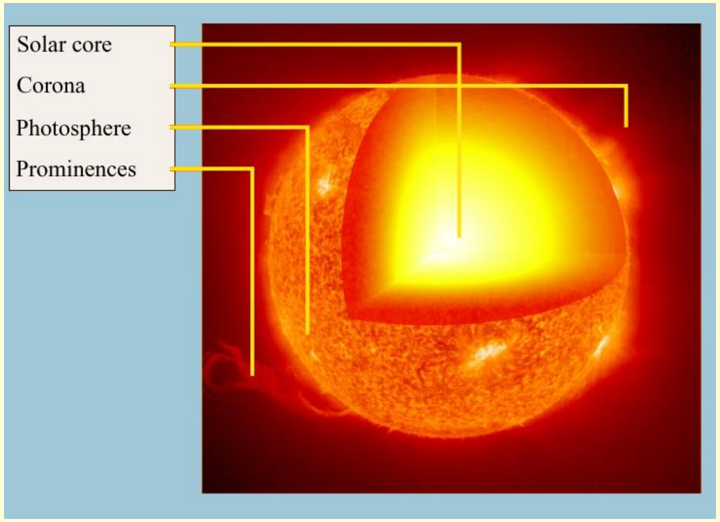
On Radiation Diffusion and the Rapidity of Escape of Resonance Radiation from a Gas

By CARL KENTY
General Electric Vapor Lamp Company, Hoboken, N. J.

(Received August 25, 1932)

The radiation diffusion process is considered from the standpoint of the free paths of the diffusing resonance quanta as influenced by the Doppler and other line broadening effects. Abnormally long free paths are found to be of such importance as to enable resonance radiation to escape from a body of gas faster than has usually been supposed. It is assumed that a large concentration of diffusing resonance quanta will, on the basis of Doppler broadening only, give rise to a characteristic excitation of atoms, as dependent on their speeds, which can be represented by a distribution function which will lie between two limiting distribution functions, namely (1) Maxwell's distribution function and (2) a distribution function expressing a lower relative excitation of the high speed atoms than that of Maxwell, based on the excitation of all atoms as if by absorption of the core of the line. On the basis of (1) and (2), limiting expressions are derived for: (a) the fraction of emitted quanta traversing at least a given distance before absorption, (b) the diffusion coefficient, (c) the average square free path, (d) the average free path. A fundamental difference between radiation diffusion and molecular diffusion appears in that whereas (a) decreases exponentially with the distance in the latter case it is found to decrease only linearly (roughly) with the distance in the former case. For this reason very long free paths are found to be of relatively great importance in radiation diffusion. It is found that, for a gas container of infinite size, (b), (c), and (d) are all infinite. For a gas container of finite size, esti-

Multiple Scattering of Light in Hot Atomic vapors :



Random walk of photons / Radiation trapping in

- dense atomic vapours
- discharge
- hot plasmas
- gas lasers
- stars
- intergalactic scattering
- GPS/ Galileo (hot Rb atom clock)

Milne Equation (diffusion)

$$\frac{\partial n(r,t)}{\partial t} - D\Delta n(r,t) = S(r,t)$$

WRONG ! A random walk of photons in hot atomic vapours is NOT correctly described by a diffusion equation

Holstein Equation

$$\frac{\partial n(\vec{r},t)}{\partial t} = S(\vec{r},t) - \frac{1}{\tau_n} n(\vec{r},t) + \frac{1}{\tau_n} \int_{\text{volume}} n(\vec{r}\,',t) G(\vec{r}\,',\vec{r}) dv'$$

Important ingredient G(r’,r) = P(x) : how far does a photon travel

PHYSICAL REVIEW VOLUME 72, NUMBER 12 DECEMBER 15, 1947

Imprisonment of Resonance Radiation in Gases

T. HOLSTEIN
Research Laboratories, Westinghouse Electric Corporation, East Pittsburgh, Pennsylvania

(Received September 8, 1947)

VOLUME 93, NUMBER 12 PHYSICAL REVIEW LETTERS week ending 17 SEPTEMBER 2004

Photon Trajectories in Incoherent Atomic Radiation Trapping as Lévy Flights

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Universidade do Minho, Escola de Ciências, Departamento de Física, 4710-057 Braga, Portugal

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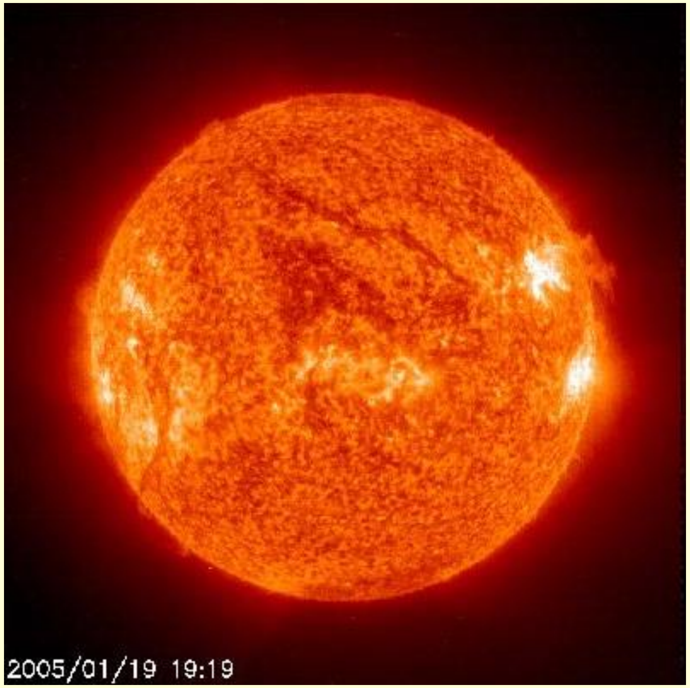
(Received 19 November 2003; published 13 September 2004)

Photon trajectories in incoherent radiation trapping for Doppler, Lorentz, and Voigt line shapes under complete frequency redistribution are shown to be Lévy flights. The jump length (r) distributions display characteristic long tails. For the Lorentz line shape, the asymptotic form is a strict power law $r^{-3/2}$, while for Doppler the asymptotic is $r^{-2}(\ln r)^{-1/2}$. For the Voigt profile, the asymptotic form always has a Lorentz character, but the trajectory is a self-affine fractal with two characteristic Hausdorff scaling exponents.

How to measure P(x) ?

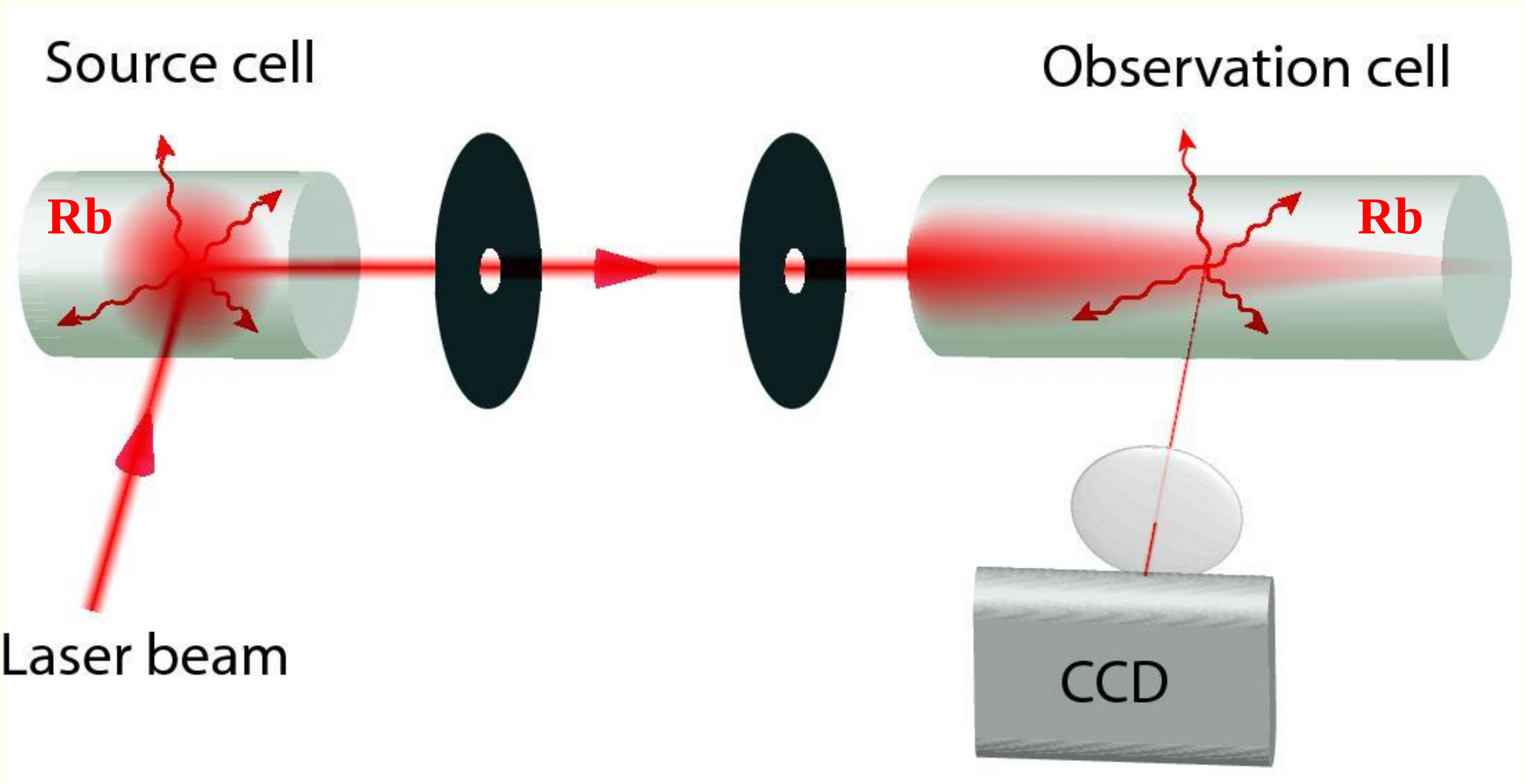
$$\frac{\partial n(\vec{r},t)}{\partial t} = S(\vec{r},t) - \frac{1}{\tau_n} n(\vec{r},t) + \frac{1}{\tau_n} \int_{\text{volume}} n(\vec{r}\,',t) G(\vec{r}\,',\vec{r}) dv'$$



How to track a Photon ????

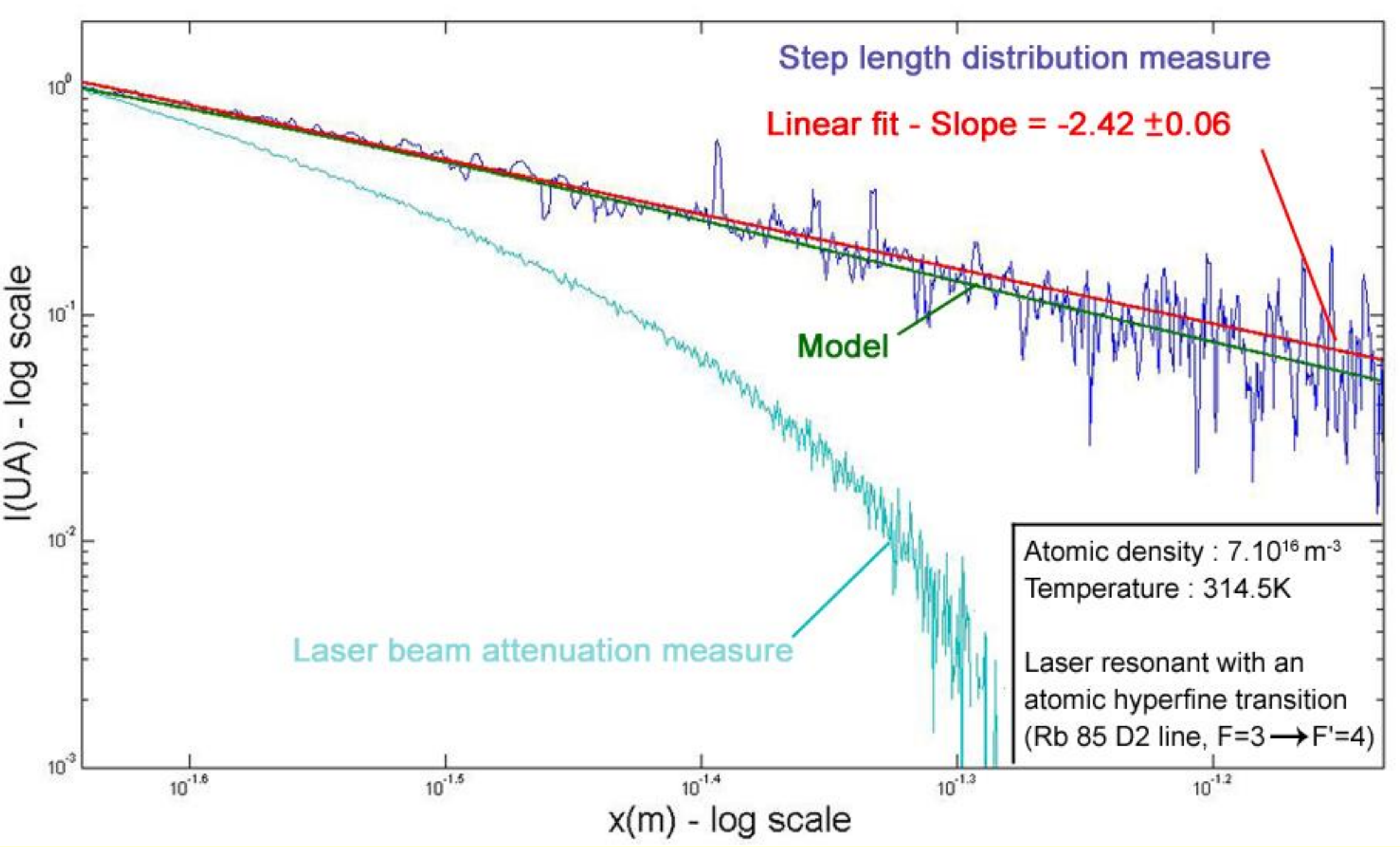
In Stars … ☹

in the lab ….



No correlation x_0 – P(x) : annealed disorder

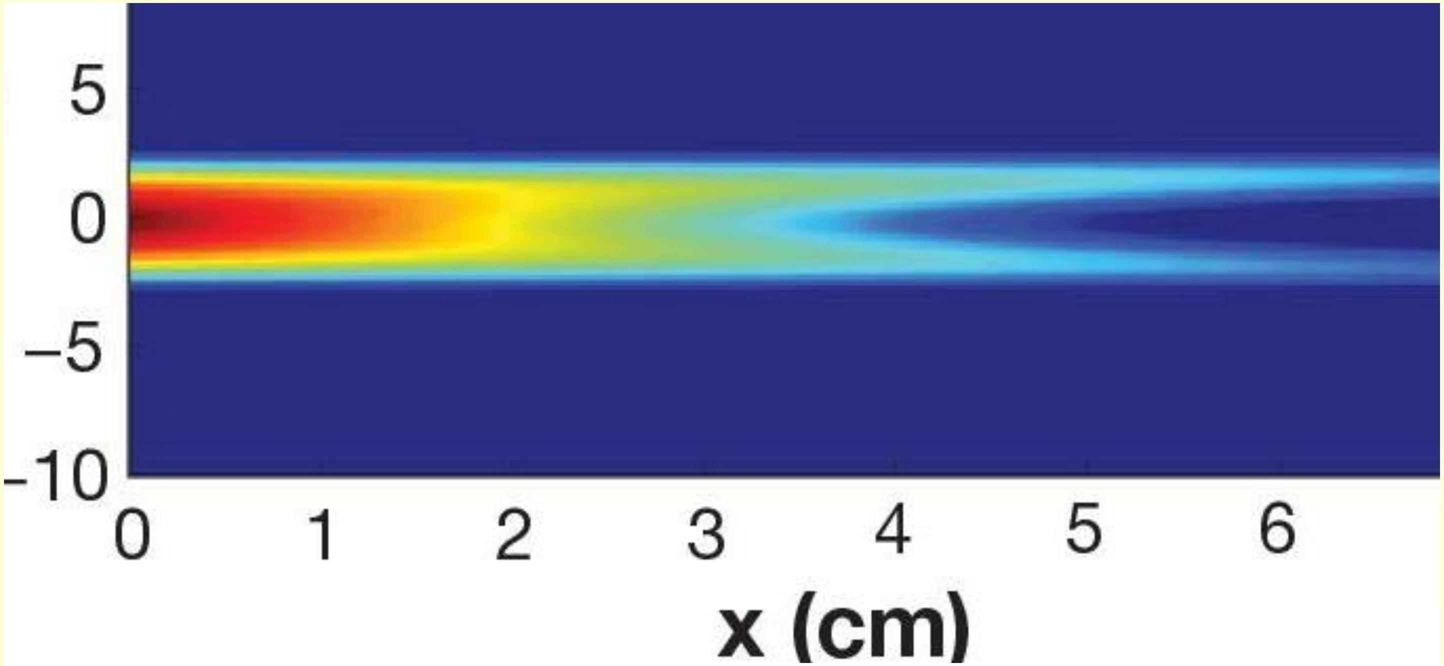
Step length distribution



neglecting the natural width of the atoms : $\mu=2$

N. Mercadier et al., Nature Physics **5**, 602 (2009)

Spatial evolution of the spectrum



$$P(x) \propto \int d\omega f(\omega) e^{-n\sigma(\omega)x} \propto \frac{1}{x^{\mu}}$$

Lévy flight usually **not** important for cold atoms…

As long as ;

$kv \ll \Gamma$: so be careful for ultranarrow lines

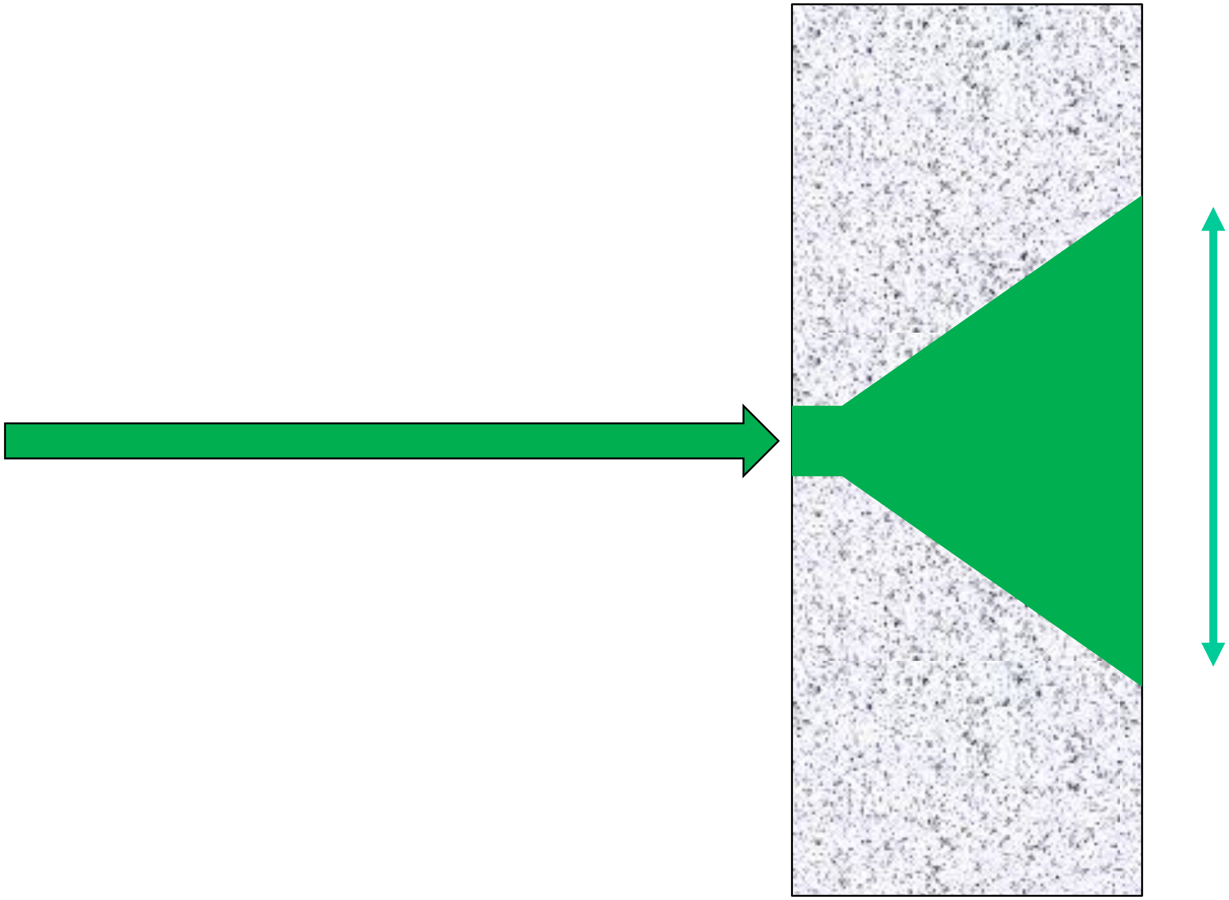
Or other inelastic broadening mechanisms

Random walk through a slab:

$$< x^2 > \propto Nstep l^2$$

$$L \propto \sqrt{N} \quad \leftrightarrow \quad Nstep \propto \frac{L^2}{l^2}$$

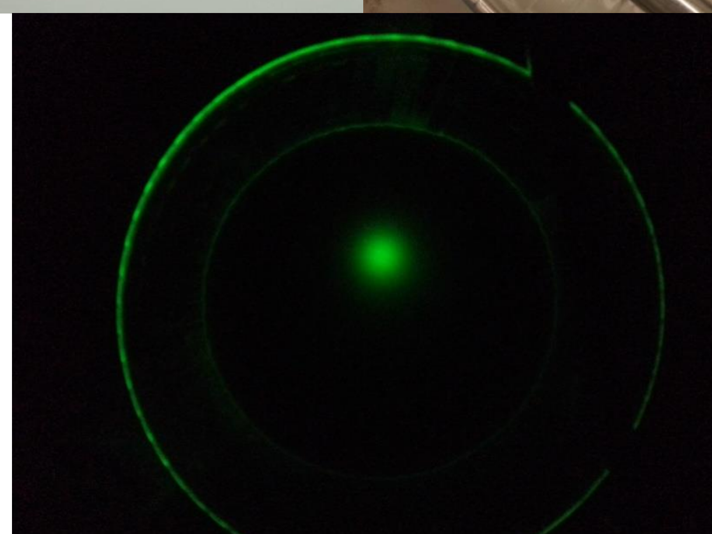
$$< x^2 > \propto L^2$$



Size independent on l_{sc}

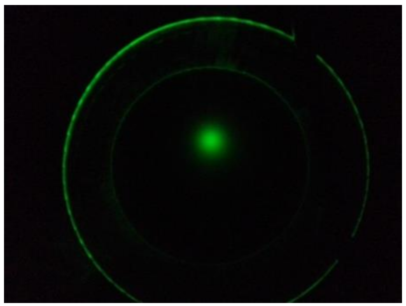
Many small steps vs fewer large step

Light diffusion in milk

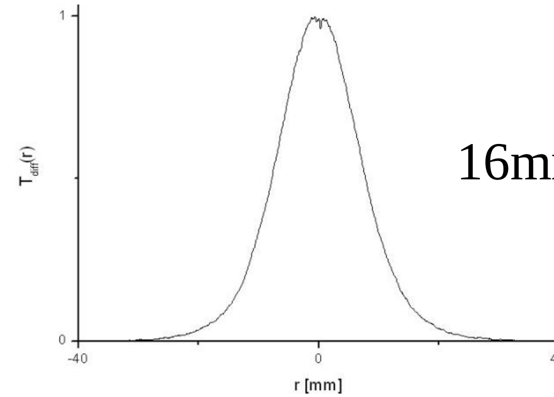


Diffusion equation + boundary conditions

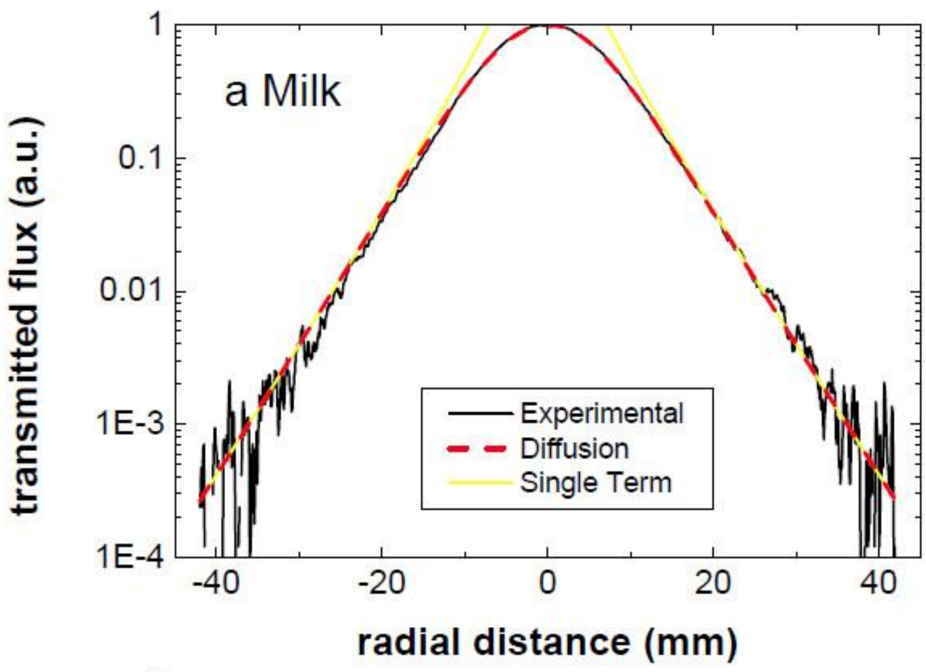
$$T_{\text{diff}}(r) = \frac{\pi}{4}\left(\frac{1}{L_e}\right)^2 i \left\{\sum_{n=1}^{+\infty} (-1)^{n+1}\, n \cos\left(n\pi \frac{z_e}{L_e}\right) \sin\left(n\pi \frac{\ell + z_e}{L_e}\right) \mathrm{H}_0^{(1)}\left(in\pi \frac{r}{L_e}\right)\right\}$$

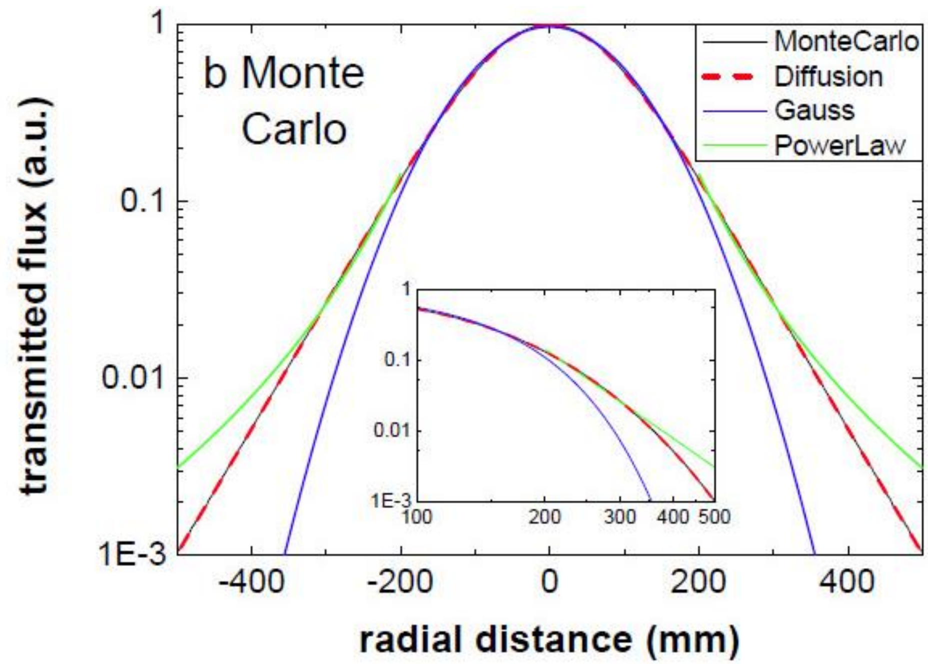


Radial average

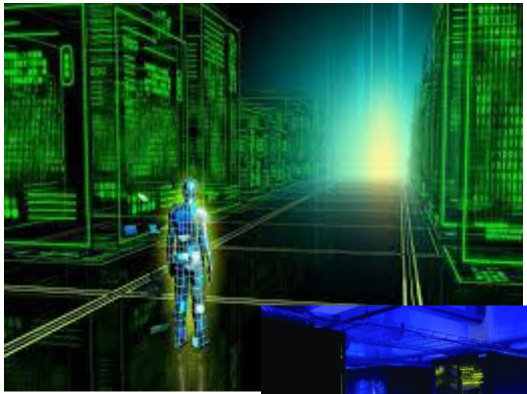


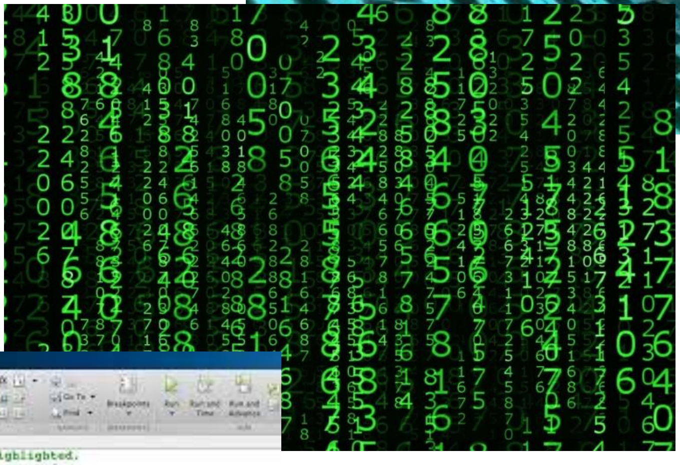
16mm milk

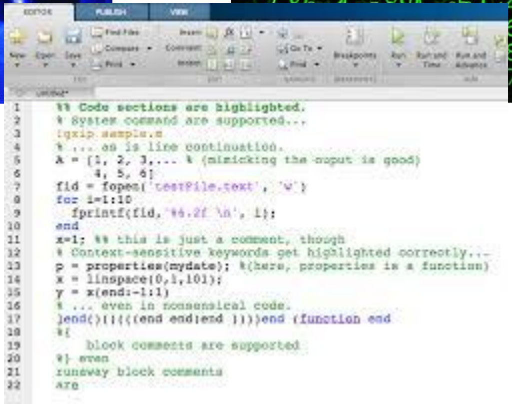




Numerical Simulations







www.kaiserlux.de/coldatoms/LesHouches2019Kaiser.html

Rechercher

Link to Mathlab codes of Robin Kaiser's lecture in Les Houches 2019

Random Walk based codes :

- RandomWalk_LH2019.m
- Ohm_LH2019.m
- TransmissionProfile_N100000.m
- RT_LH2019.m
- RandomWalk_CBS.m

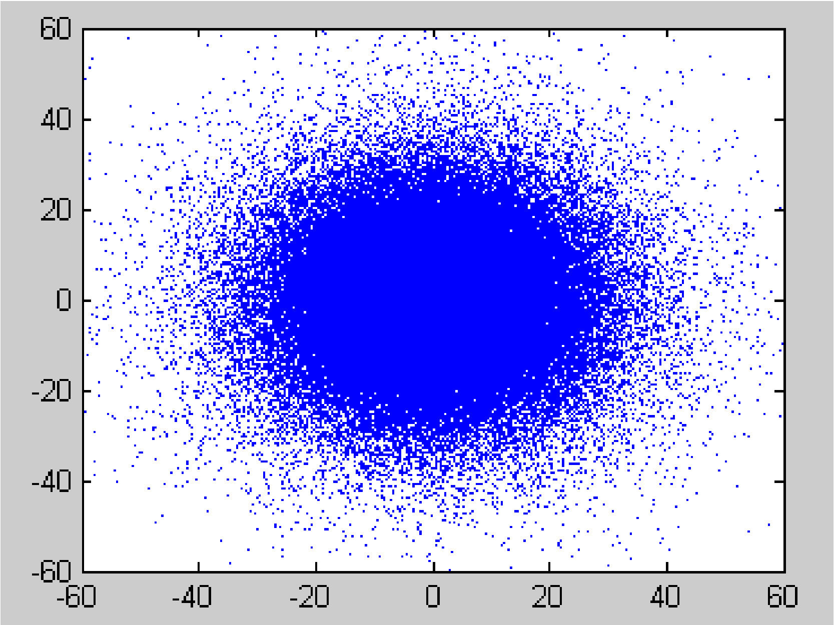
Coupled Dipole based codes :

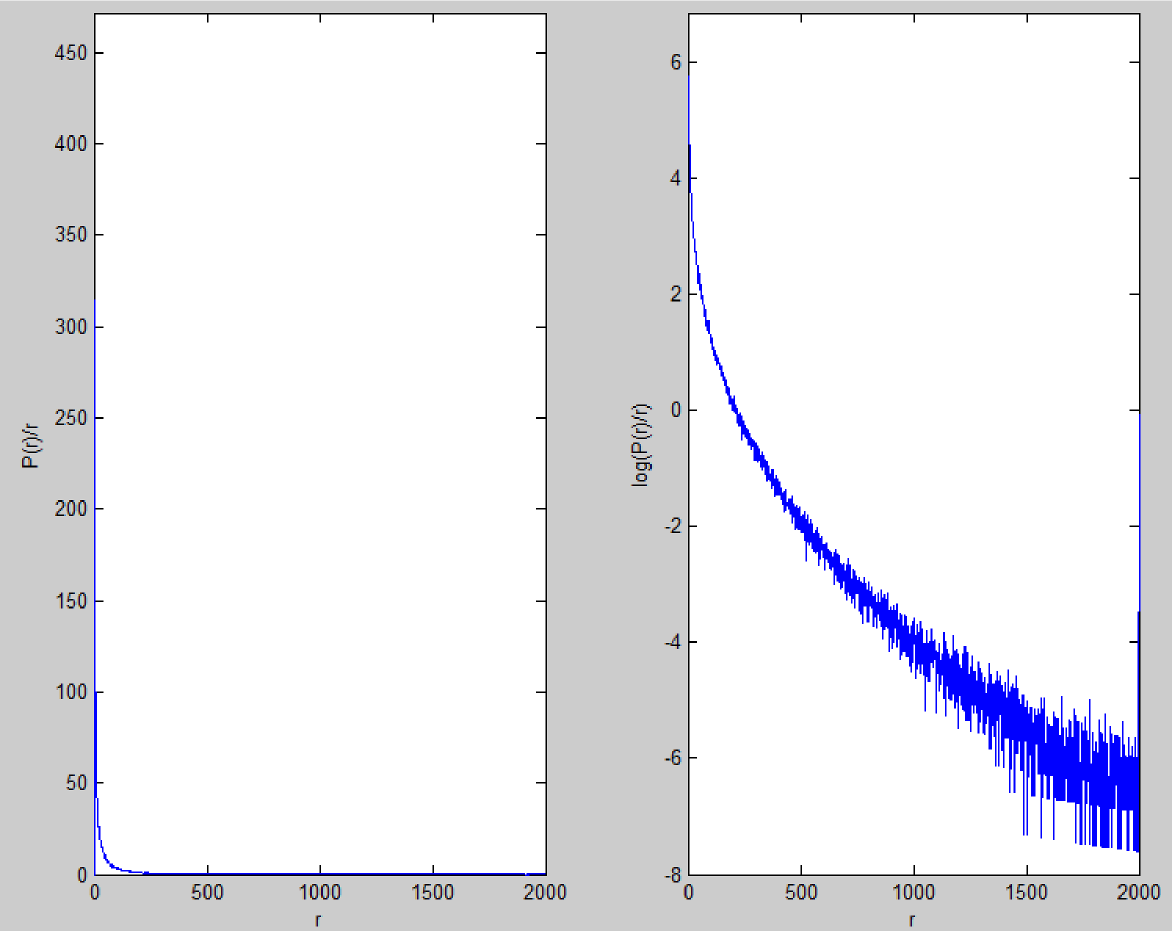
- CoupledDipoles_LH2019.m

Contact : Robin Kaiser
e-mail: Robin.Kaiser@inphyni.cnrs.fr
http://www.kaiserlux.eu/coldatoms/

```matlab
% Transmission profile through a finite slab

clear all;
% Parameters of the slab
Lz=20; % length of the slab
R=2000; %transverse size
ell=2; %scattering mean free path

tic; %useful to estimate the computation time : toc gives the elapsed time
Nph=100000; % number of photons (100 000 photons take 20 seconds computing time)
for iph=1:1:Nph

% Initial Conditions
Nsc=0;  % number of scattering events already done
cos_phi = 1; theta=0; % direction of incident propagation
x=0; %initial position along x
y=0; %initial position along y
z=0; % Initial impact with the vapor (NOT the 1st scattering event!)
r=sqrt(x^2+y^2);

% length of the 1st step using an exponential distribution (Beer-Lambert)
step = -ell*log(rand);  % use the inverse function f^(-1) to draw the random number following a distribution f: exp -> log

% The first step is ballistic in forward direction
z = z+step;
z1=z;

while all(z<=Lz  && z>0  && r<R) % Next scattering event inside the slab

    Nsc = Nsc+1;  % add 1 to number of scattering events

%     % record the trajectory (optional)
%     X(Nsc) = x;
%     Y(Nsc) = y;
%     Z(Nsc) = z;

    % draw a new random direction
    theta = 2*pi*rand;
    cos_phi = -1+2*rand;  % uniform between -1 and 1 (always  check that the distribution is uniform: mistakes often happen with cos, sin etc)
    sin_phi = sqrt(1-cos_phi^2);  % always >0 since phi is between 0 and pi

    % length of the next step in an exponential ditribution (Beer-Lambert)
    step = -ell*log(rand);

    % new position
    x = x + step*sin_phi*cos(theta) ;
    y = y + step*sin_phi*sin(theta);
    z = z + step*cos_phi;
    r = sqrt(x^2+y^2);

    % record the trajectory
    X(Nsc) = x;
    Y(Nsc) = y;
    Z(Nsc) = z;

end
if z>Lz
Nfinx(iph)=x;
Nfiny(iph)=y;
Nfinz(iph)=z;
Nfinr(iph)=x^2+y^2;
end;

end;

rr=(1:1:R);
pr=hist(Nfinr, rr);
pr(1)=0;
figure('Position', [450 550 400 300]);
plot(Nfinx, Nfiny, 'o', 'MarkerSize', 1)
axis([-3*Lz 3*Lz -3*Lz 3*Lz])


figure('Position', [100 50 1300 400]);
subplot(1,2,1), plot(rr, pr./rr), xlabel('r'), ylabel('P(r)/r'), axis([0 10*Lz 0 1.5*pr(2)/rr(2)])
subplot(1,2,2), plot(rr, log(pr./rr)), xlabel('r'), ylabel('log(P(r)/r)'), axis([0 5*Lz -2.5 log(3*pr(2)/rr(2))])
%subplot(1,3,3), plot(log(rr), log(pr./rr)), xlabel('log(r)'), ylabel('log(P(r)/r)'),

toc
```

Transmission profile





Lecture 1 : **Multiple Scattering of Light in Cold Atoms**

1.1 Steady State Results : Ohm's Law for Photons

Single Atom response: optical Bloch equations
Random Walk : Beer-Lambert's Law, diffusion, Ohm's law, Transmission profile
Numerical Simulations

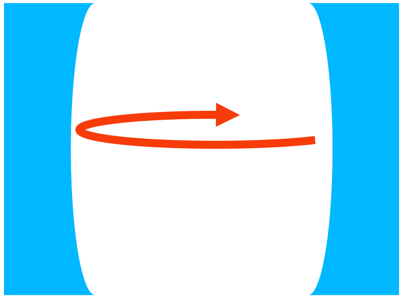
1.2 Time dependant Scattering : radiation trapping

Phase, group and transport velocity
Slow diffusion
Numerical Simulations

1.3 Multiple Scattering + gain : random lasers

How long takes scattering ?

<u>Resonant Single scattering</u>

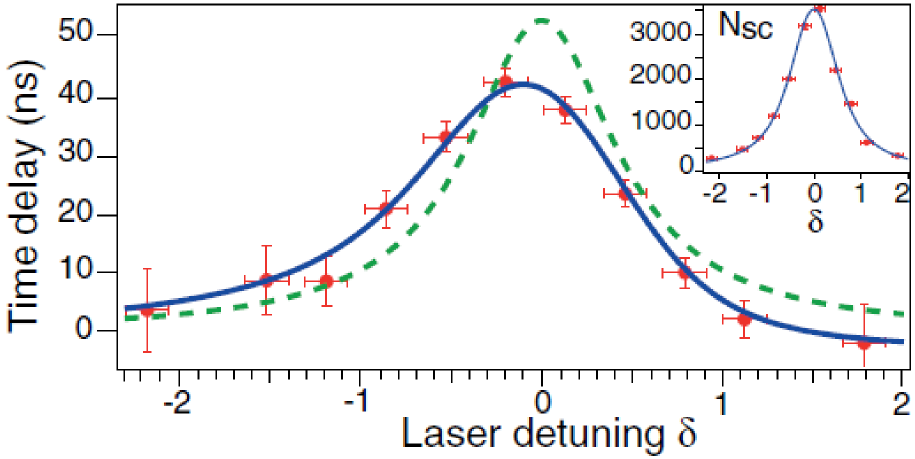


Resonant scattering delay $\tau_W = \frac{\partial\phi}{\partial\omega}$

$$\alpha(\omega) = \frac{6\pi}{k_0^3} \times \frac{-2(\omega-\omega_0)/\Gamma + i}{1 + 4(\omega-\omega_0)^2/\Gamma^2}$$

$$\phi(\omega) = \arctan\left(\frac{-\Gamma/2}{\omega-\omega_0}\right)$$

$$\tau_W = \frac{2}{\Gamma}\mathcal{L}(\omega)$$



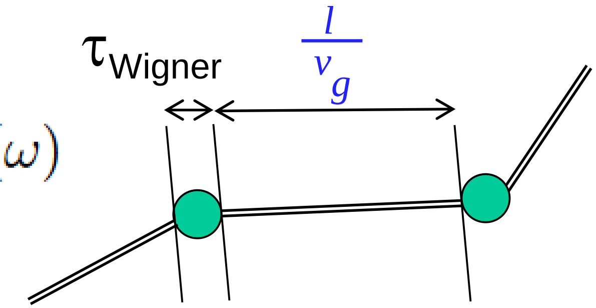
R. Bourgain et al., Opt. Lett. 38, 1963 (2013)

How long takes scattering ?

Resonant Multiple scattering

$$\boxed{\tau_{\mathrm{tr}} = \tau_{\mathrm{W}} + \frac{\ell_{\mathrm{sc}}}{v_{\mathrm{g}}}}$$

$$\tau_{\mathrm{W}} = \frac{2}{\Gamma}\mathcal{L}(\omega)$$



A. Lagendijk, B. v. Tiggelen, Phys. Rep. **270**, 143 (1996)

$$v_{\mathrm{g}} = \frac{\partial\omega}{\partial k}$$

$$v_g = \frac{c}{n + \omega\dfrac{dn}{d\omega}}$$

$$\frac{1}{v_{\mathrm{g}}} = \cancel{\frac{n}{c}} + k_0\frac{\partial n}{\partial\omega}$$

$$n - 1 = \frac{\rho}{2}\mathrm{Re}(\alpha)$$

$$\ell_{\mathrm{sc}} = \frac{1}{\rho\sigma_{\mathrm{sc}}} = \frac{1}{\rho\sigma_0\mathcal{L}(\omega)}$$

$$\begin{aligned}\tau_{\mathrm{tr}} &= \frac{1}{\Gamma}\left[2\mathcal{L}(\omega) - 1 + 8(\omega-\omega_0)^2\mathcal{L}(\omega)\right] \\ &= \frac{1}{\Gamma}\left\{2\mathcal{L}(\omega)\left[1 + 4(\omega-\omega_0)^2\right] - 1\right\} \\ &= \tau_{\mathrm{at}}\ .\end{aligned}$$

$$\boxed{\tau_{tr} = \tau_{at}}$$

G. Labeyrie et al., Phys. Rev. Lett. 91, 223904 (2003)
P. Weiss et al., New J. Phys. 20, 063024 (2018)

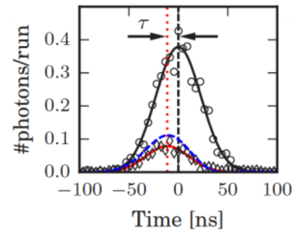
Time Resolved Experiments

- Phase velocity : $c = \dfrac{c_0}{n}$ propagation of phase for a monochromatic wave

$$c \lessgtr c_0$$

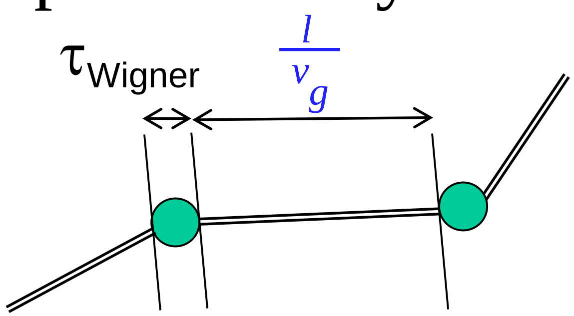
- Group velocity : $v_g = \dfrac{\partial \omega}{\partial k}$ propagation of transmitted gaussian pulse with slowly varying envelope

cold atoms on resonance : $v_g < 0$



S. Jennewein et al., Phys. Rev. A 94, 053828 (2016)

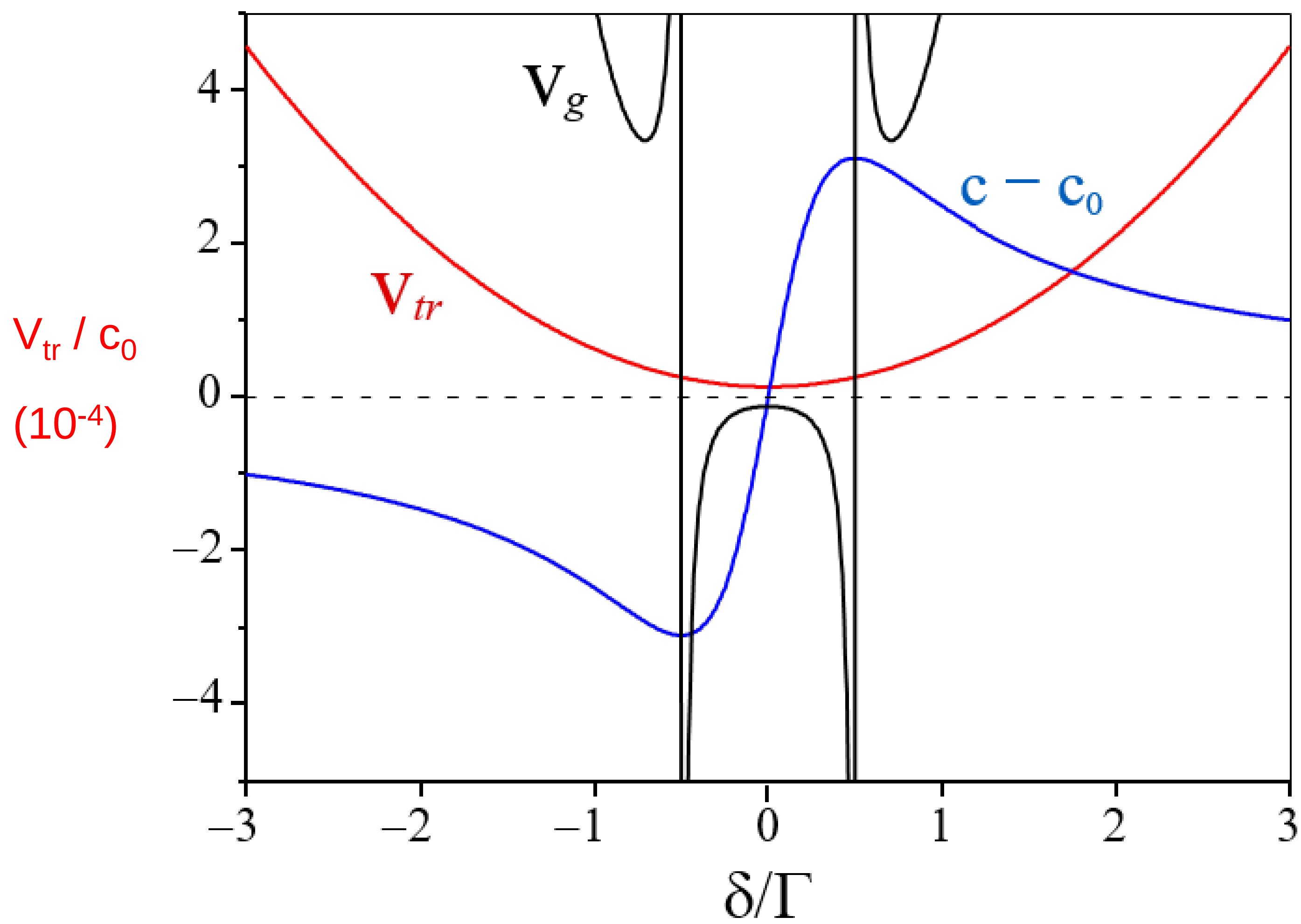
- Transport velocity : propagation of scattered wave energy



$$0 < v_{tr} < c_0$$

A. Lagendijk, B.v.Tiggelen, Phys. Rep. **270**, 143 (1996).

Velocities in resonant scattering



$$v_g = \frac{c}{n + \omega \frac{dn}{d\omega}}$$

can be > c
(for $\frac{dn}{d\omega} < 0$)

and even <0

Time dependent diffuse transmission through a slab

$$T_{diff}(t) \propto \sum_n (-1)^{n+1} n^2 e^{-n^2\pi^2 Dt/L^2} \xrightarrow[t\to\infty]{} e^{-\pi^2 Dt/L^2}$$

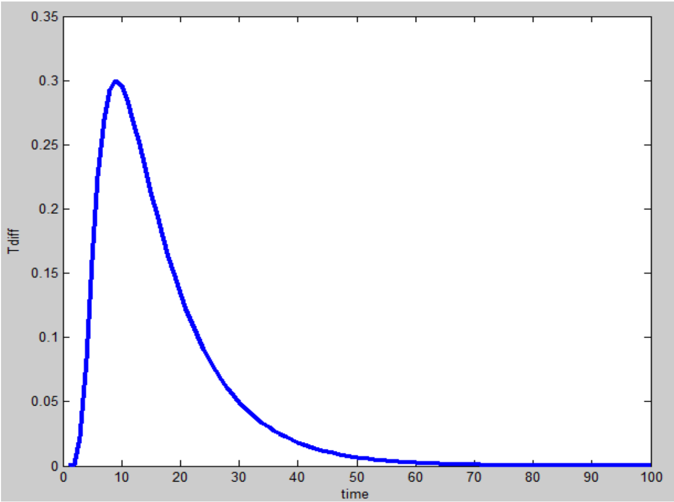
$$\tau_0 = \frac{L^2}{\pi^2 D}$$

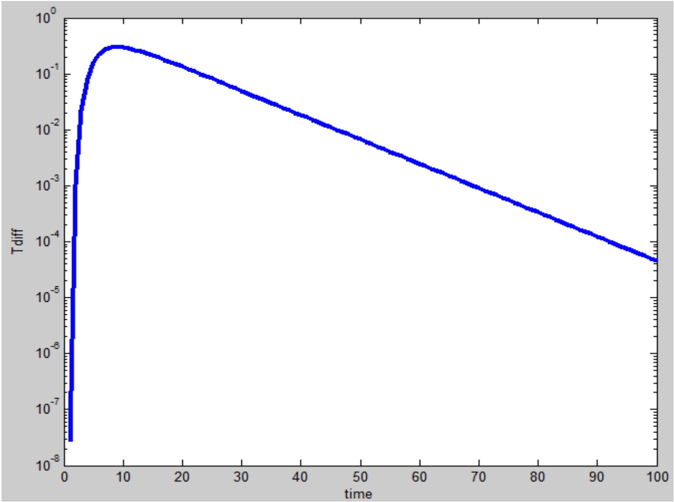
$$D = \frac{1}{3}\frac{l^2}{\tau_{tr}} = \frac{1}{3} l v_{tr}$$

l : mean free path

τ_{tr} : transport time

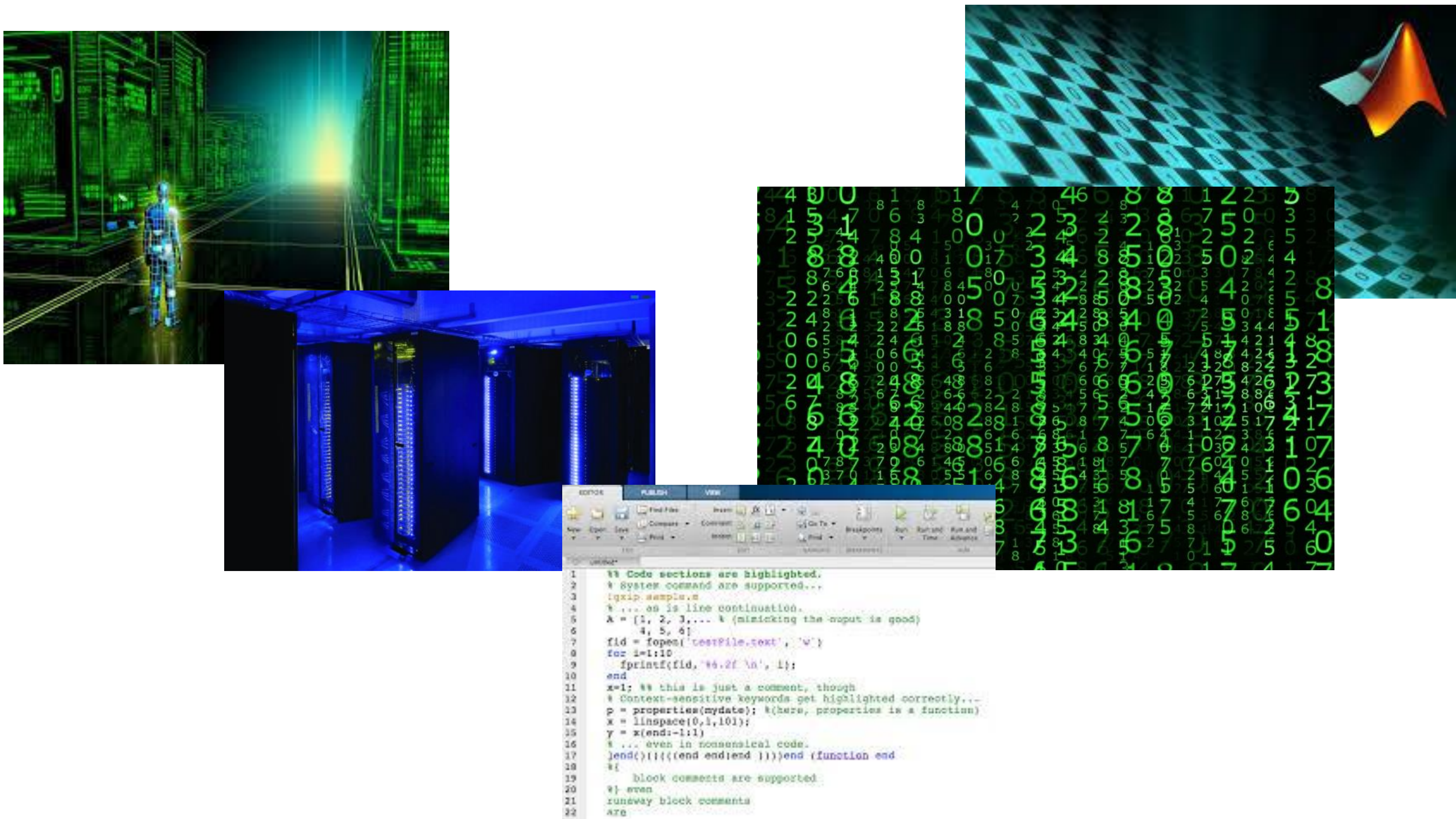
v_{tr} : transport velocity





R. Berkovitz, M. Kaveh, J. Phys. Condens. Matter 2, 307 (1990)
Martin Störzer et al., PRL 96, 063904 (2006)

Numerical Simulations



www.kaiserlux.de/coldatoms/LesHouches2019Kaiser.html

Rechercher

Link to Mathlab codes of Robin Kaiser's lecture in Les Houches 2019

Random Walk based codes :

- RandomWalk_LH2019.m
- Ohm_LH2019.m
- TransmissionProfile_N100000.m
- RT_LH2019.m
- RandomWalk_CBS.m

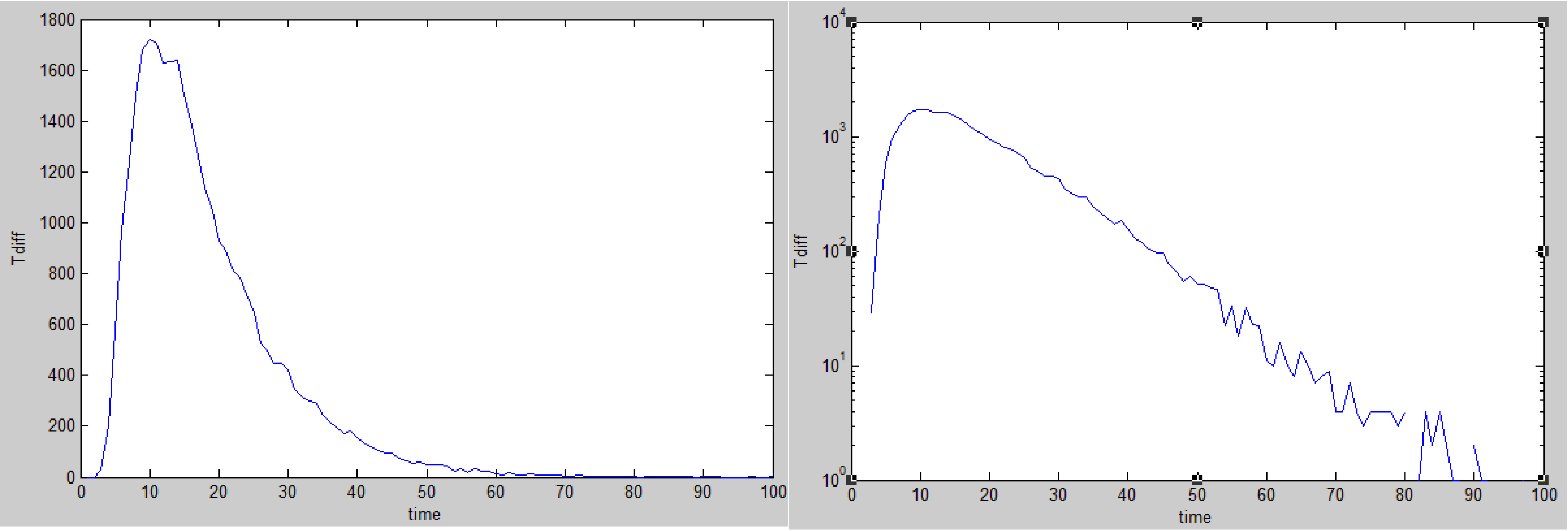
Coupled Dipole based codes :

- CoupledDipoles_LH2019.m

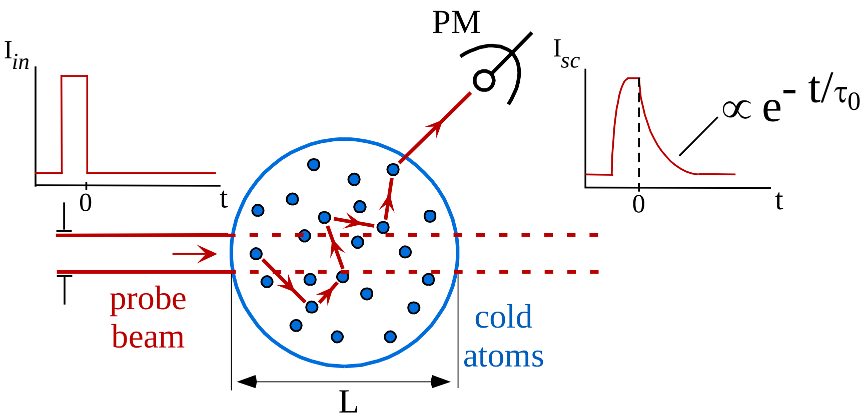
Contact : Robin Kaiser
e-mail: Robin.Kaiser@inphyni.cnrs.fr
http://www.kaiserlux.eu/coldatoms/

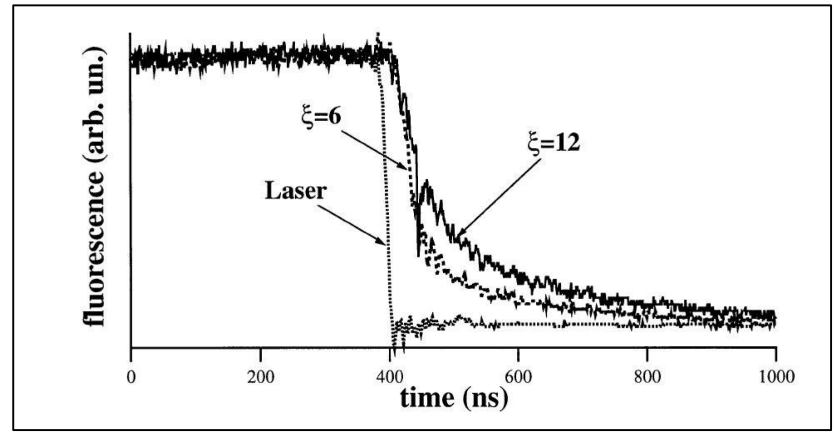
```
% Random walk (RW) in a finite slab
%  All lengths (Lz, R, ell, x0, y0) have to be given in the same unit: e.g in 1/k.
% The outputs are: the Trajectory, the number of scattering events Nsc (0 for transmitted photons), the output angles theta

clear all;
% Parameters of the slab
LzMax=1000; % length of the slab
Lz=LzMax;
R=2000; %transverse size
ell=20; %scattering mean free path
Tdiff=0;


Nph=10000; % number of photons (use Nph=10000 - 6 secondes computing - or more for a nice curve)
for iph=1:1:Nph
clear X Y Z RR S; %clear the parameters of the loop
% Initial Conditions
Nsc=0;  % number of scattering events already done
cos_phi = 1; theta=0; % direction of inicident propagation
x=0; %initial position along x
y=0; %initial position along y
z=0; % Initial impact with the vapor (NOT the 1st scattering event!)
r=sqrt(x^2+y^2);

% length of the 1st step using an exponential distribution (Beer-Lambert)
step = -ell*log(rand);  % use the inverse function f^(-1) to draw the random number following a distribution f: exp -> log

% The first step is ballistic in forward direction
z = z+step;

while all(z<=Lz  && z>0  && r<R) % Next scattering event inside the slab

    Nsc = Nsc+1;  % add 1 to number of scattering events

    % draw a new random direction
    theta = 2*pi*rand;
    cos_phi = -1+2*rand;  % uniform between -1 and 1 (always  check that the distributi
    sin_phi = sqrt(1-cos_phi^2);  % always >0 since phi is between 0 and pi

    % length of the next step in an exponential ditribution (Beer-Lambert)
    step = -ell*log(rand);

    % new position
    x = x + step*sin_phi*cos(theta);
    y = y + step*sin_phi*sin(theta);
    z = z + step*cos_phi;
    r = sqrt(x^2+y^2);

end

if (z>Lz)
    Tdiff=Tdiff+1; %just to count diffuse transmission
    RTtime(iph)=Nsc; % register number of events, which we put to time
end

end;

Tdiff
ps=hist(RTtime, 100); %histrgram of transit times
ps(1)=0; %remove points which have not been scattered
plot(ps(1,1:100),'DisplayName','ps(1,1:100)','YDataSource','ps(1,1:100)');figure(gcf)
```

Time dependent diffuse transmission through a slab



Time Resolved Experiments

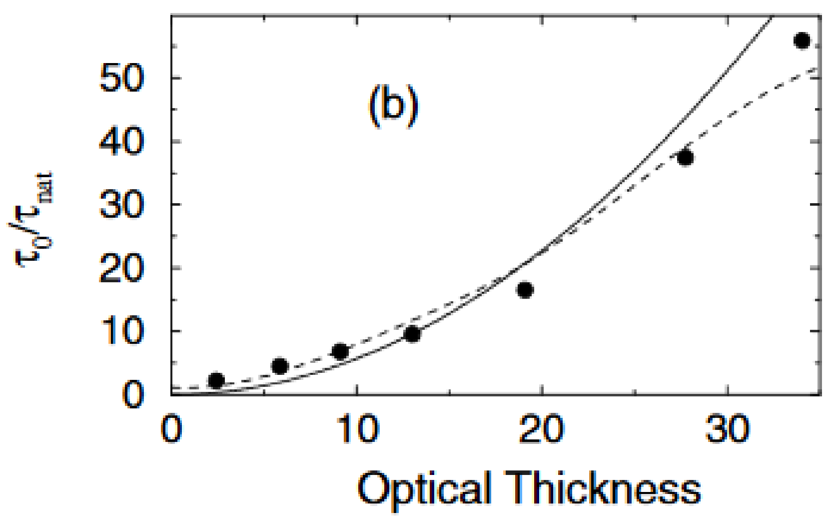


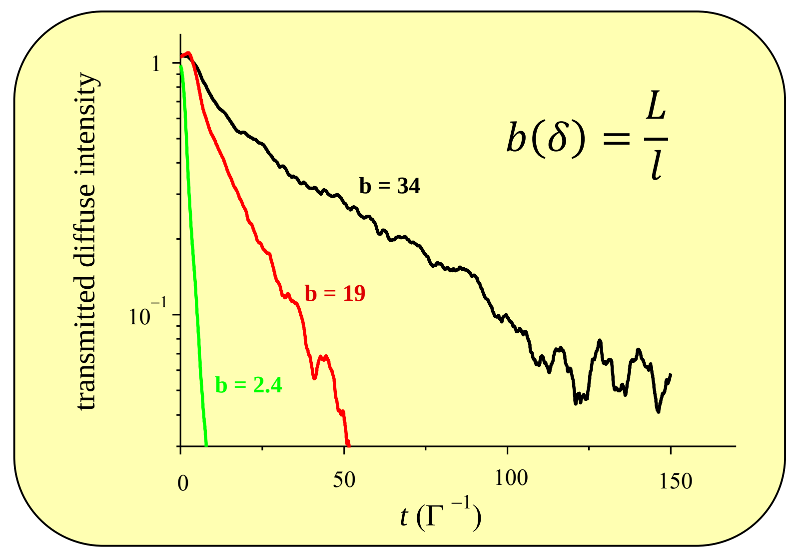


A. Fioretti et al., Opt. Comm. **149**, 415 (1998)

$$\tau_0 = \frac{L^2}{\pi^2 D} = \frac{b(\delta)^2\, l^2}{\pi^2 D} = \frac{3}{5.35\pi^2}\, b(\delta)^2\, t_{tr}$$

$$D = \frac{1}{3}\frac{l^2}{\tau_{tr}} = \frac{1}{3}\, l v_{tr}$$

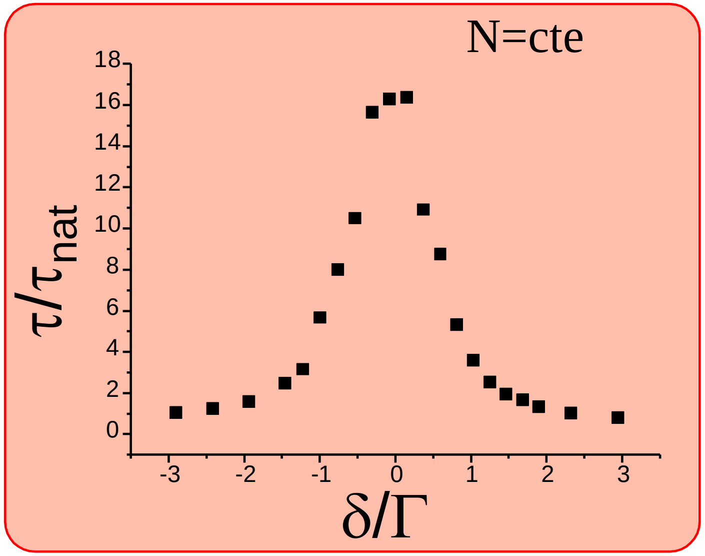


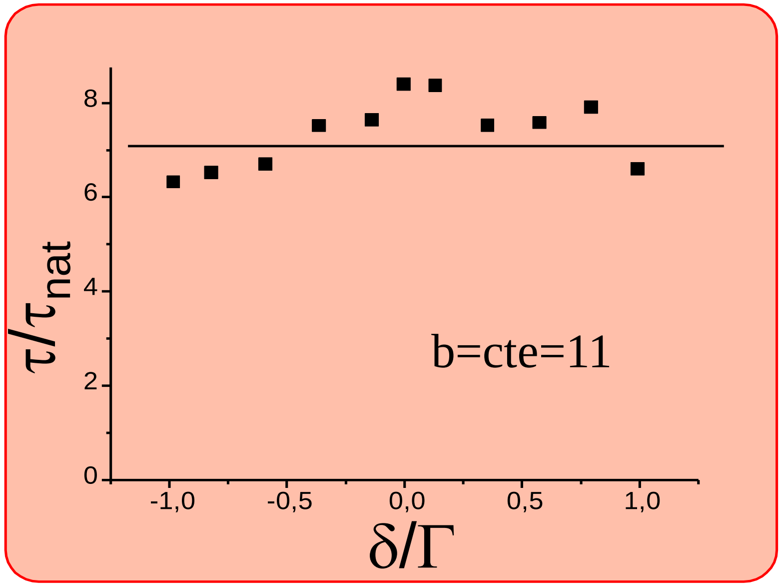


G. Labeyrie et al., Phys. Rev. Lett. **91**, 223904 (2003)

Transport time for light in cold atoms

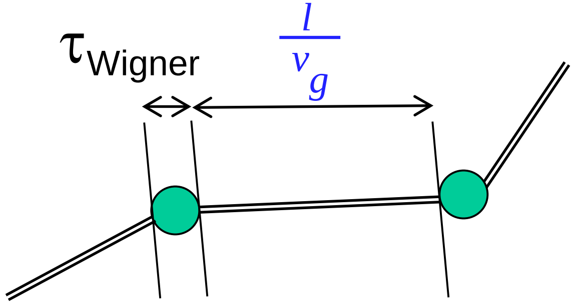
$$\tau = \frac{3}{5.35\pi^2} b(\delta)^2 t_{tr}$$





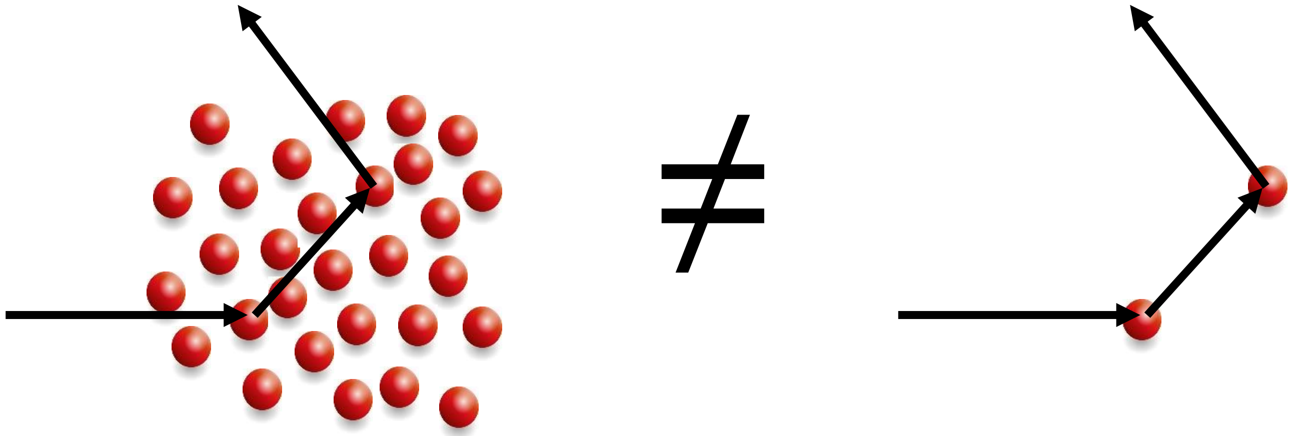
Transport Time :
Independent of δ

$$\tau_{\mathrm{tr}} = \tau_{\mathrm{W}} + \frac{\ell_{\mathrm{sc}}}{v_{\mathrm{g}}}$$



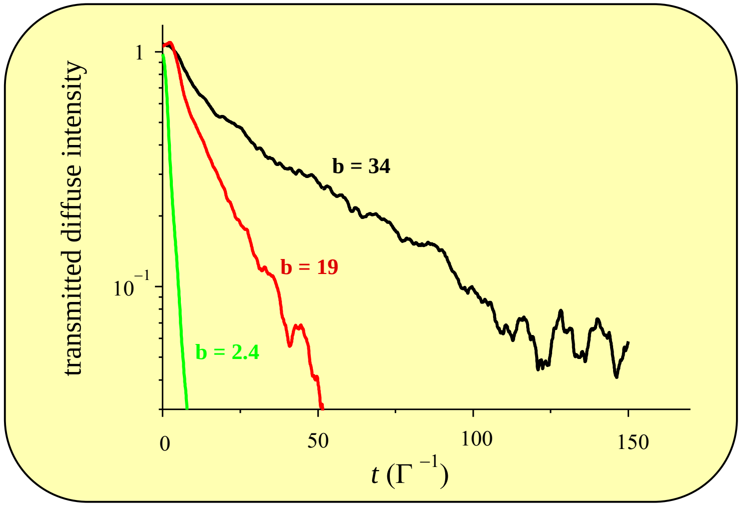
Double scattering in cold atoms :

1) Diffusion (scattering : phase + amplitude)
2) Propagation in effective medium (index effect)



Enhanced effects with magnetic field (Hanle + Faraday effects)

Time Resolved Experiments



$$D = \frac{1}{3}\,\frac{l_{tr}^{2}}{\tau_{tr}}$$

transport mean-free path

transport time

$$= \frac{1}{3}\, l_{tr}\, v_{tr}$$

transport velocity

$$\frac{v_{tr}}{c_0} = \frac{\ell_{tr}}{c_0 \tau_{tr}} \approx 3 \cdot 10^{-5}$$

Slow diffusion of light ≠ diffusion of slow light

Lecture 1 : Multiple Scattering of Light in Cold Atoms

1.1 Steady State Results : Ohm's Law for Photons

Single Atom response: optical Bloch equations
Random Walk : Beer-Lambert's Law, diffusion, Ohm's law, Transmission profile
Numerical Simulations

1.2 Time dependant Scattering : radiation trapping

Phase, group and transport velocity
Slow diffusion
Numerical Simulations

1.3 Multiple Scattering + gain : random lasers

Multiple scattering beyond linear optics

‘quantum’ at the single atom level

Pump-probe spectroscopy

G. Grynberg, C. Robilliard, Phys. Rep. **355**, 335 (2001)

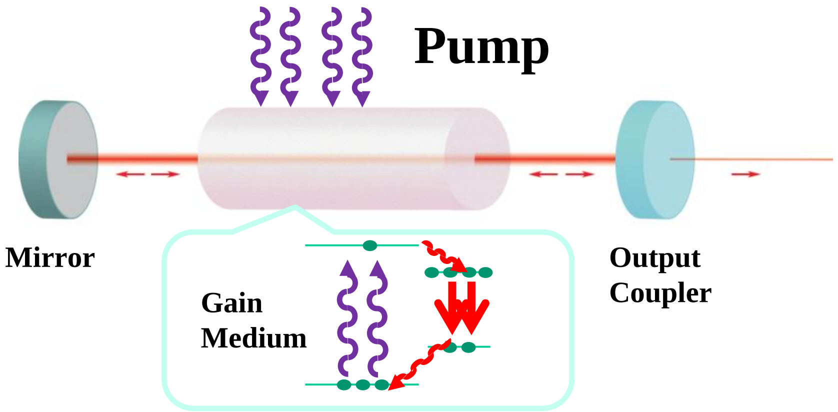
Random lasers

H. Cao, Waves Random Media **13**, 1 (2003).
H. Cao, J. Phys. A **38**, 10497 (2005).
D. Wiersma, Nature Physics **4**, 359 (2008).

Cold atom random lasers

W. Guerin et al., Phys. Rev. Lett. 101 ,093002 (2008)
L.S. Froufe-Perez & al, PRL. 102, 173903 (2009)
W. Guerin et al., J. Opt. 12, 024002 (2010)

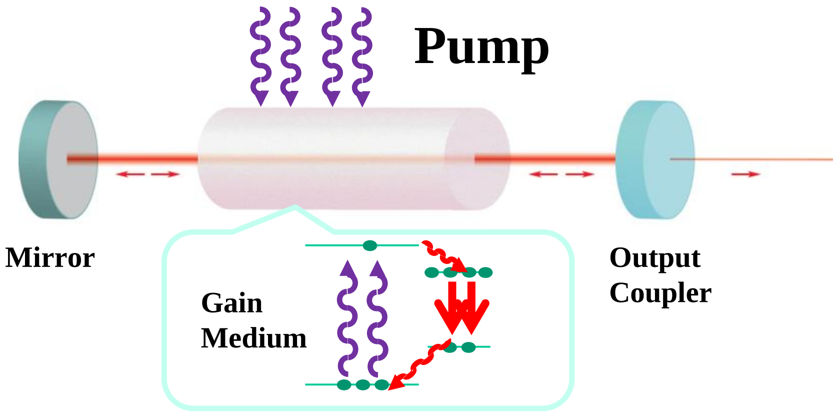
- Cavity Laser



Ingredients:

- Gain Medium
- Cavity
  → Feedback & Mode Selection

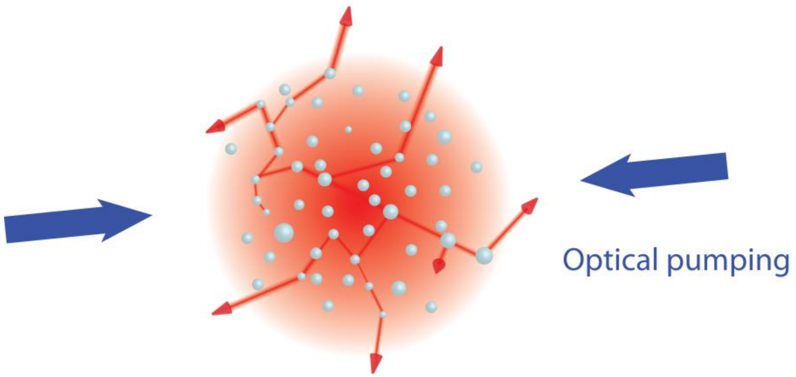
- Cavity Laser



Ingredients:

- Gain Medium
- Cavity
  → Feedback & Mode Selection

- Random Laser



D. Wiersma, Nat. Phys. 4 359 (2008)

- Gain Medium
- **Multiple scattering**

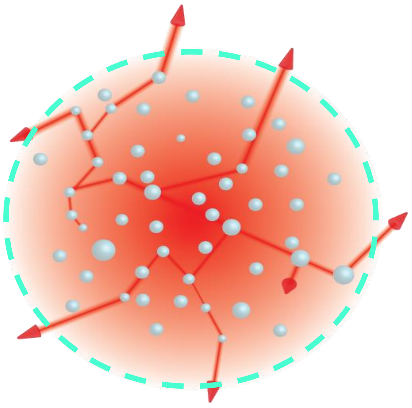
  *V.S. Letokhov, Sov. Phys. JETP **26**, 835 (1968)*

1939–2009

Main ingredient for a random laser :

Gain in **volume** (ℓ_g) vs **Losses** at **surface** (via diffusion : ℓ_{sc})

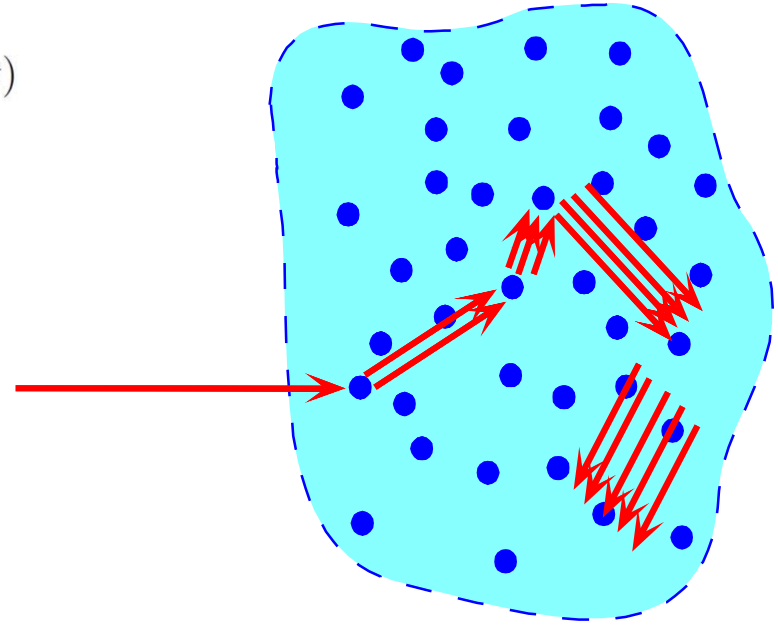
$\Rightarrow$ **Critical volume / mass : photonic bomb**



1) Diffusion equation with gain + boundary conditions

$$\frac{1}{c}\frac{\partial \Phi_\omega(\mathbf{r},t)}{\partial t} = D\Delta\Phi_\omega(\mathbf{r},t) + Q_\omega(\mathbf{r},t)N_0\Phi_\omega(\mathbf{r},t)$$

2) Unfolded path longer than gain length



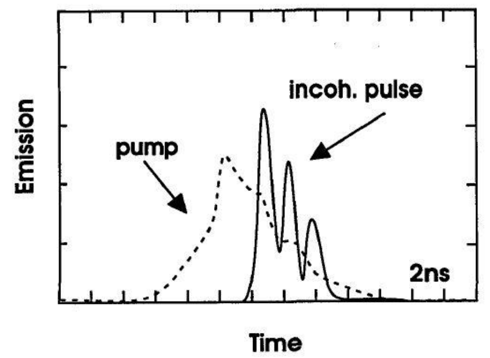
$N\,\ell_{sc} > \ell_g$

$N \propto b^2 = (L/\,\ell_{sc})^2$

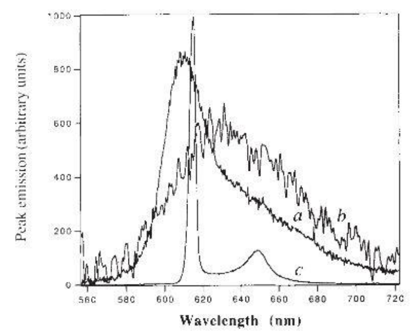
$$\boxed{L > 2\pi\sqrt{\frac{l_g l_{sc}}{3}}}$$

Pionneering experiments by

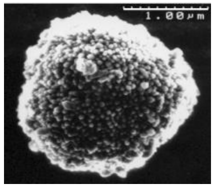
A. Migus et al. : laser crystal into a powder
J. Opt. Soc. Am. **B10,** 2358 (1993)



N. Lawandy et al. : suspended TiO_2 laser dye
Nature **368,** 436 (1994)



H. Cao et al. : ZnO powder
PRL, **82**, 2278 (1999)

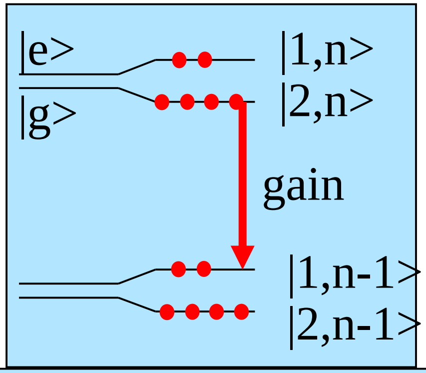


<u>Some reviews papers:</u>
H. Cao, Waves Random Media **13**, 1 (2003).
H. Cao, J. Phys. A **38**, 10497 (2005).
D. Wiersma, Nature Physics **4**, 359 (2008).

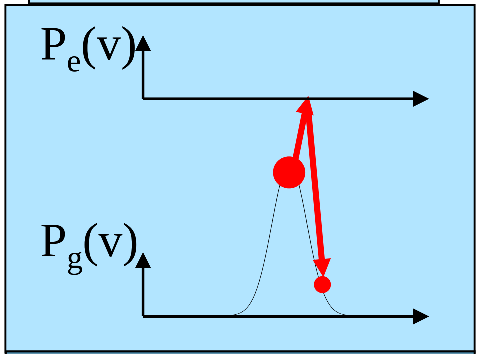
Lasing with Cold Atoms

Gain Mechanisms with Cold Atoms

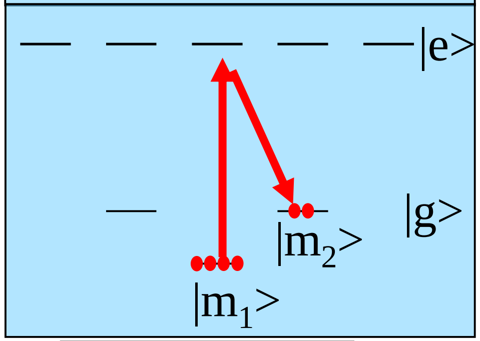
- **Mollow gain** (used in ‘lasing without inversion’)
  $\Rightarrow$ internal degree of freedom of 2 level system
  $\Rightarrow$ excited/ground state population inversion : $\pi_e > \pi_g$
  $\Rightarrow$ requires high pump intensities



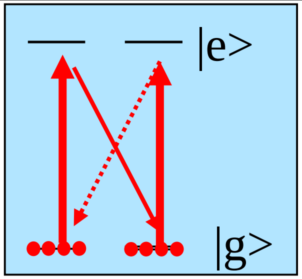
- **Recoil induced resonances / Rayleigh gain**
  (used in ‘Collective Atomic Recoil Laser’)
  $\Rightarrow$ external degree of freedom (motion)
  $\Rightarrow$ population inversion : $\pi(v+dv)>\pi(v)$ (free moving atoms)
  $\Rightarrow$ vibrational levels of ground population inversion : $\pi_n > \pi_{n'}$
  (atoms bound in lattice)



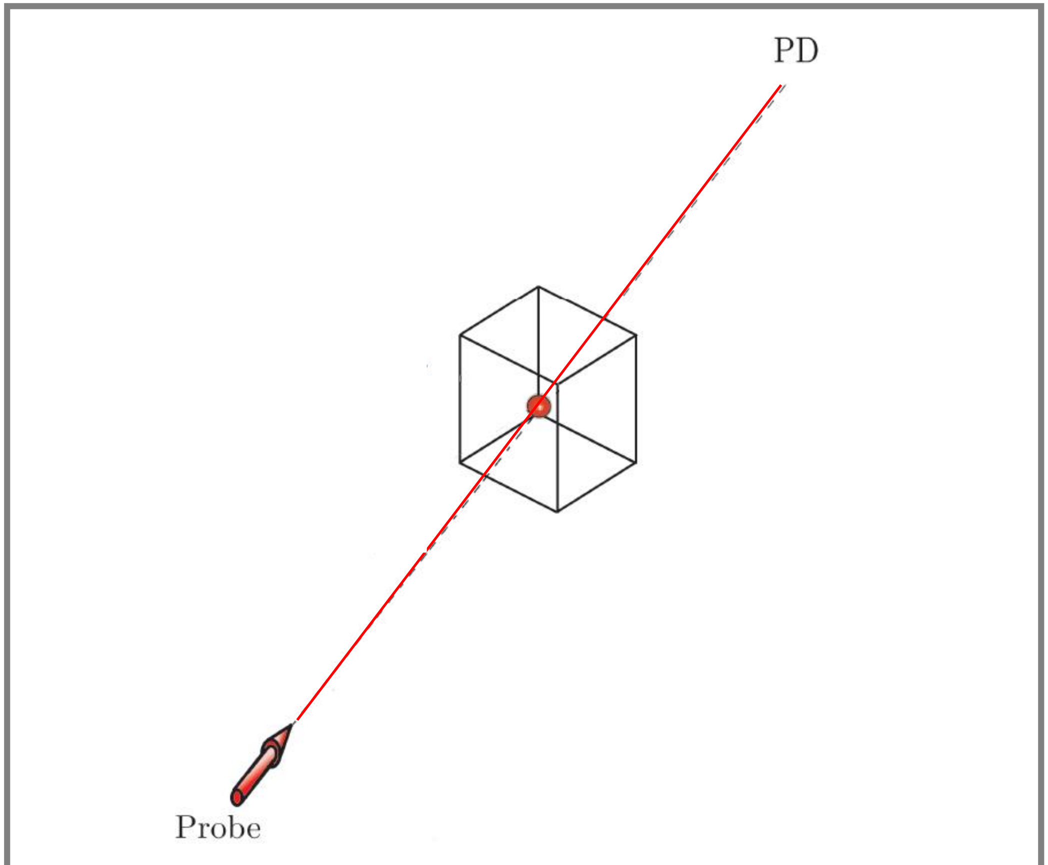
- **Raman gain**
  $\Rightarrow$ internal degree of freedom of multilevel system
  $\Rightarrow$ population inversion : $\pi_{g1} > \pi_{g2}$
  $\Rightarrow$ large in cold atoms

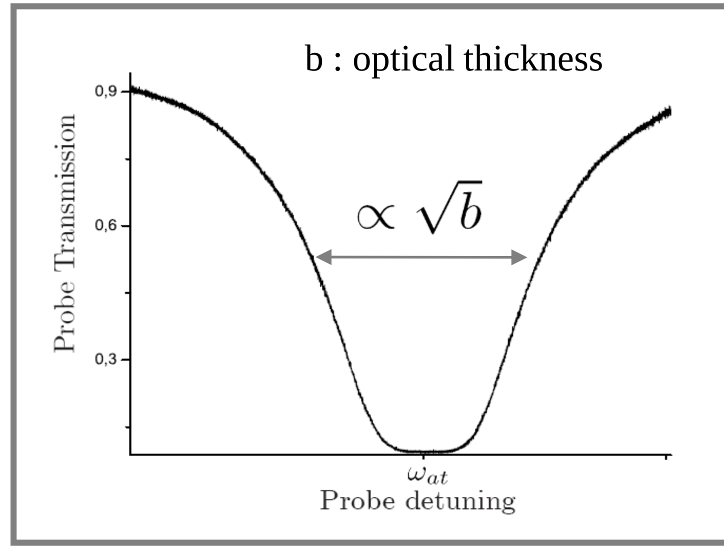


- **Four Wave Mixing**
  $\Rightarrow$ phase conjugation
  $\Rightarrow$ self-alignement / multimode

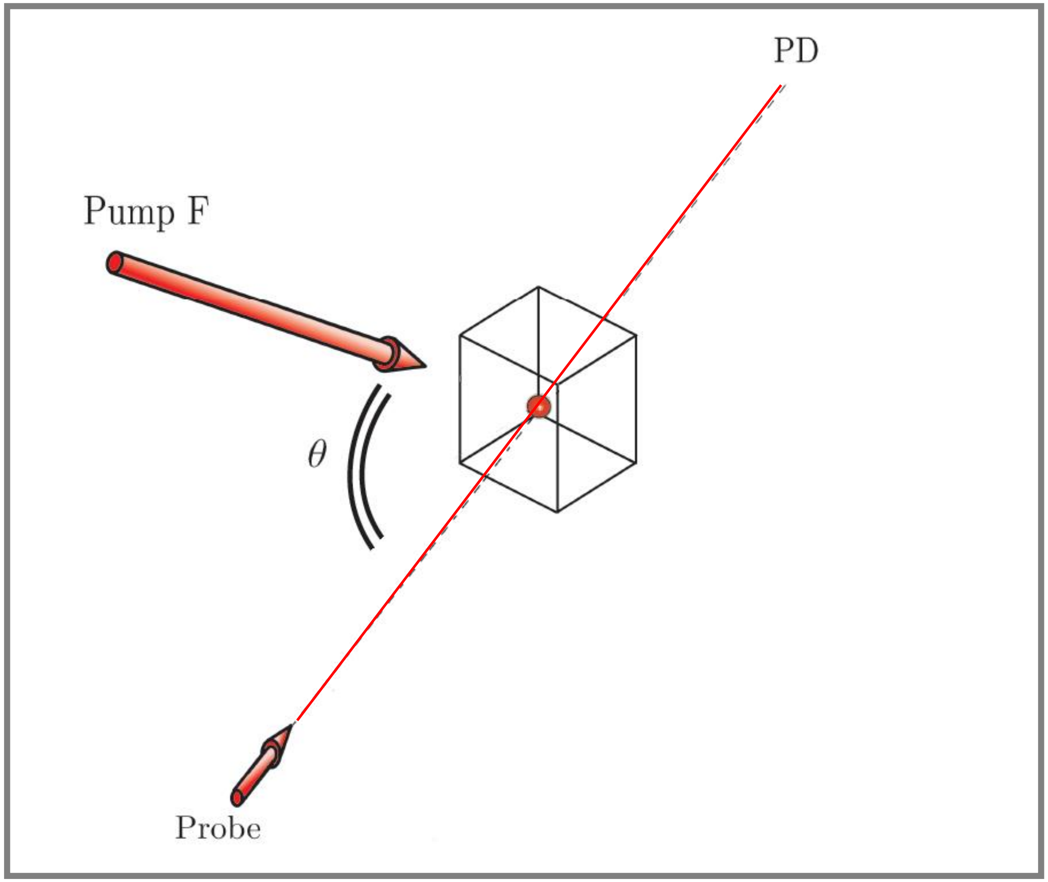


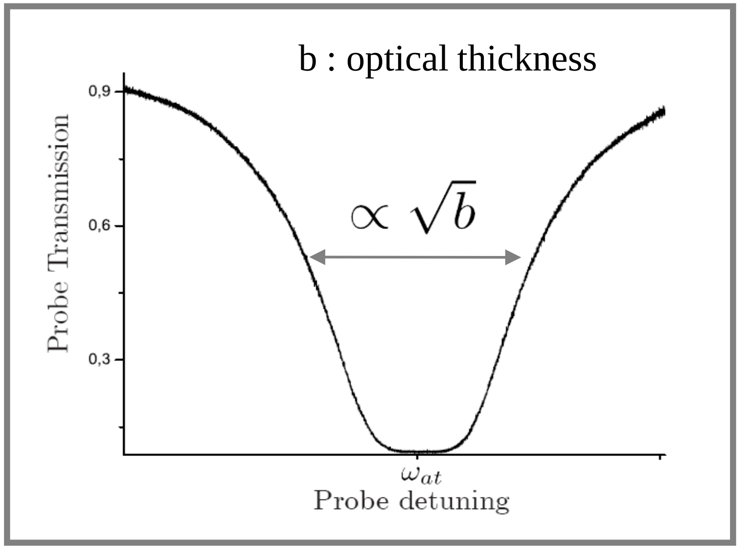
Gain Mechanisms with Cold Atoms

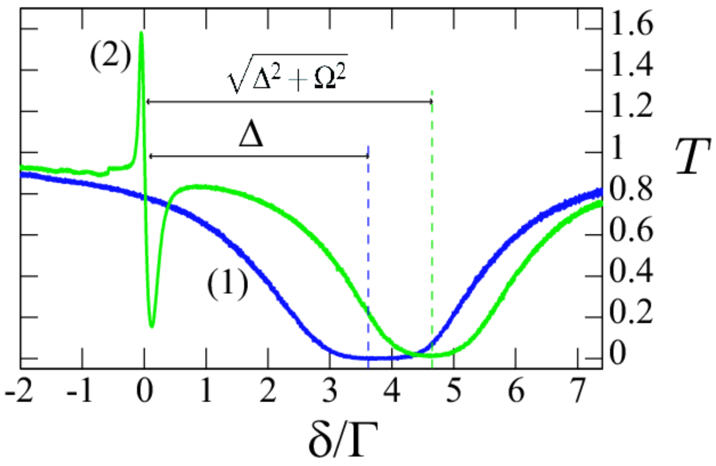




Gain Mechanisms with Cold Atoms

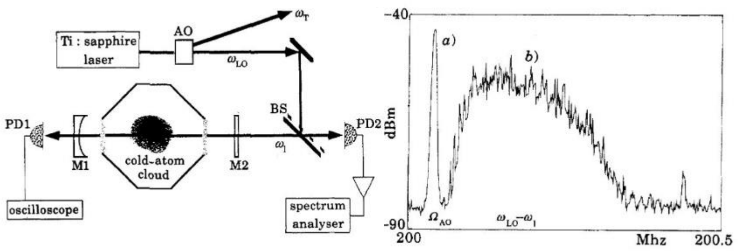






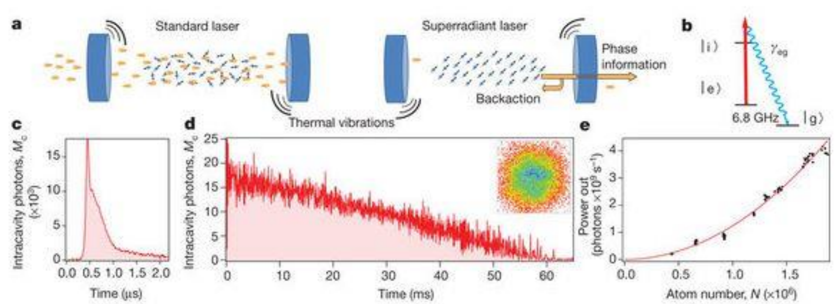
Lasing with Cold Atoms in a Cavity :

- Raman Lasers with cold atoms



L. Hilico, C. Fabre, E. Giacobino, Europhys. Lett. **18**, 685 (1992)

Figure 1: **A steady-state superradiant laser.**

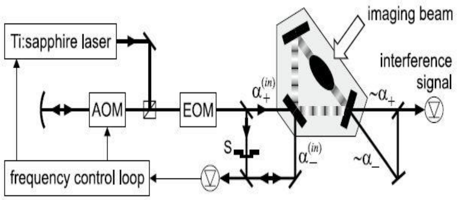


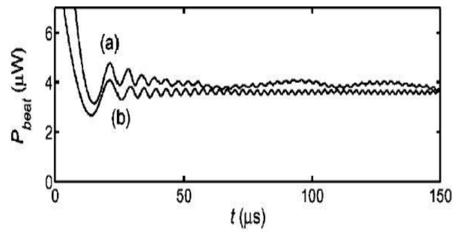
a, Left: in a standard, good-cavity laser far above threshold, many photons (yellow) circulate inside the cavity, extracting energy from the largely incoherent atomic gain medium (blue). Thermal vibrations of the mirror surfaces modulat...

J. Bohnet, *et al*., Nature **484**, 78 (2012).

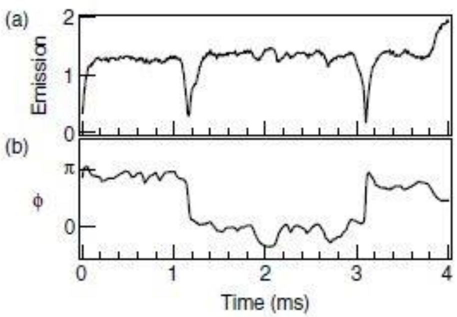
Lasing with Cold Atoms in a Cavity :

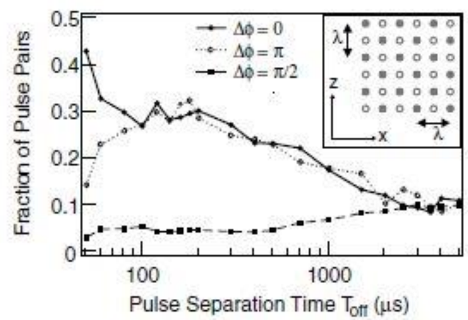
- « Recoil Induced Resonance » Lasers with cold atoms



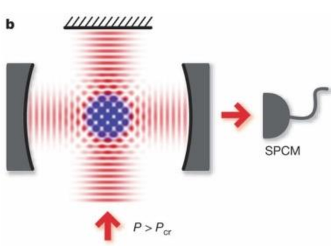


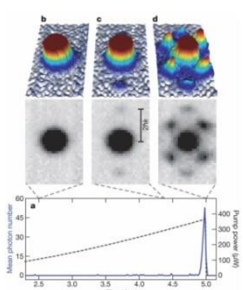
S. Slama, et al., PRL, **91**, 183601 (2003)





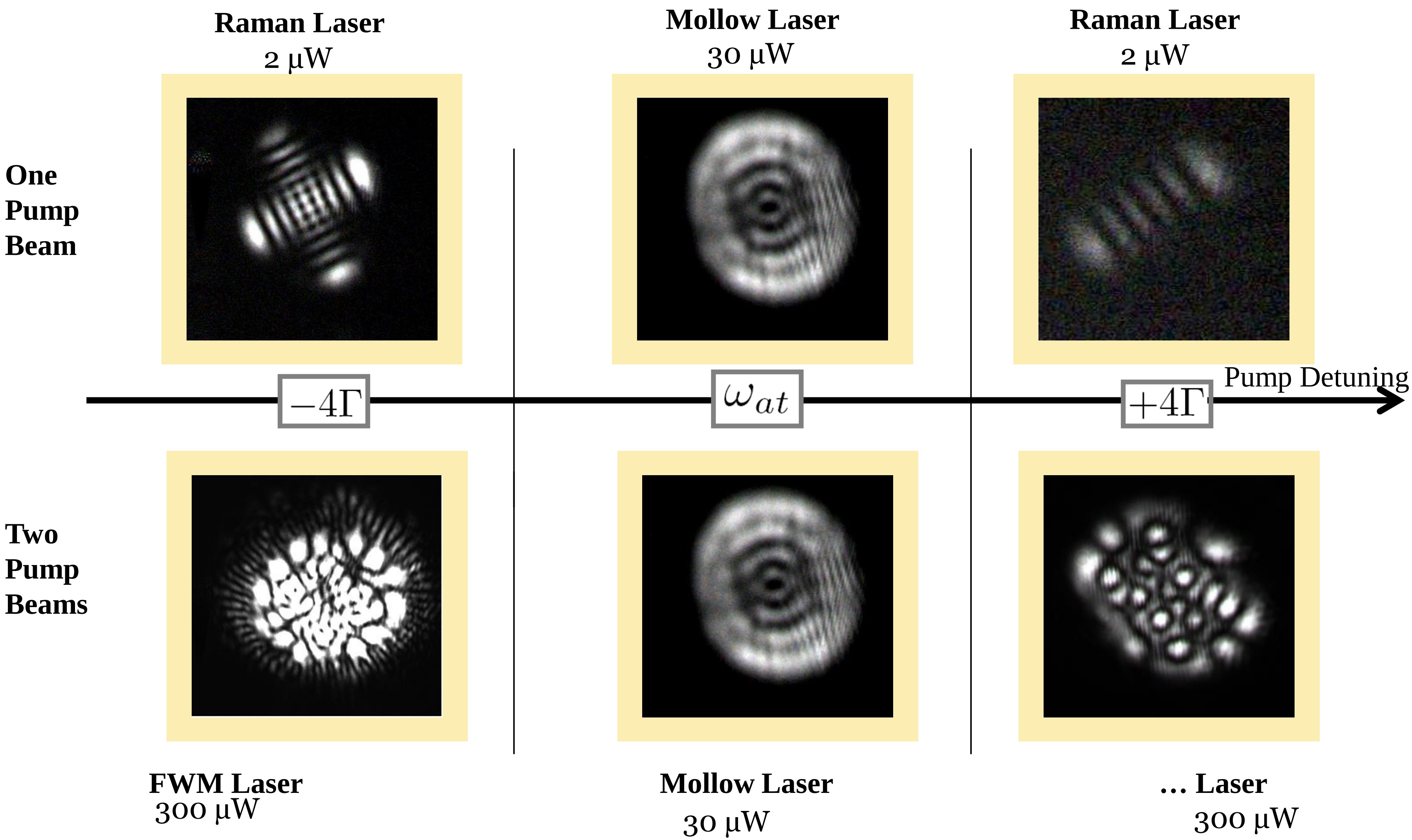
A. Black, et al., PRL, **91**, 203001 (2003)





K.Baumann, et al. Nature **464**, 1301 (2010)

Lasing with Cold Atoms in a Cavity :

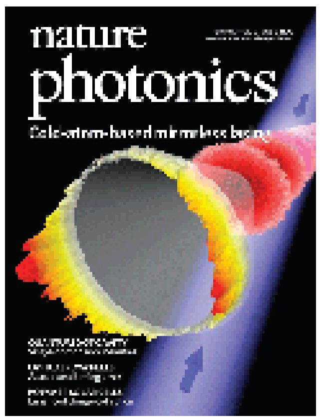


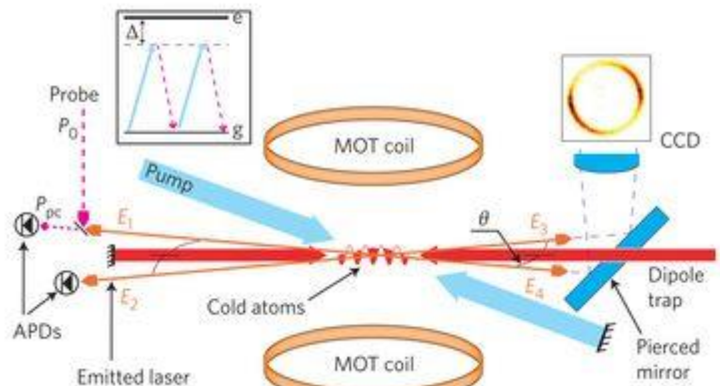
W. Guerin, et al. PRL,**101**,093002 (2008)

Lasing without external mirrors

Laser DFB (distributed feedback laser)

Optical parametric oscillation with distributed feedback in cold atoms





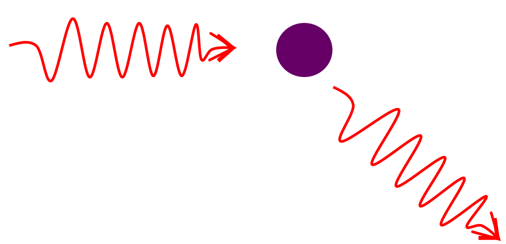
A. Schilke et al., Nature Photon. **6**, 101 (2011).

- **Multiple Scattering of Light by Cold Atoms**
- **Lasing with Cold Atoms:**

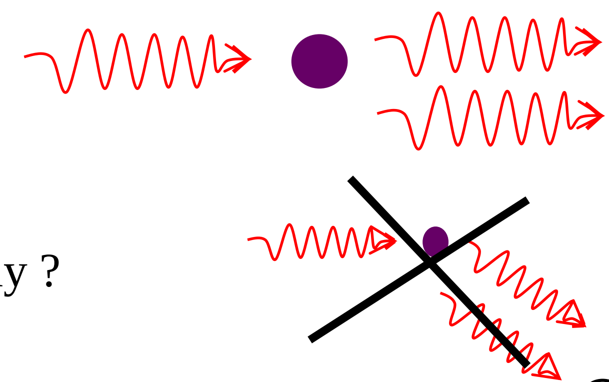
How to combine these ingredients ?

Problem : the scatterers and amplifiers are **the same atoms** !

Scattering



Gain



Is it possible to get both simultaneously ?

Pumping, necessary to get gain, reduces drastically the scattering cross-section ☹

Letokhov **diffusive** model

$$\boxed{L > 2\pi\sqrt{\frac{\ell_{sc}\ell_g}{3}}}$$

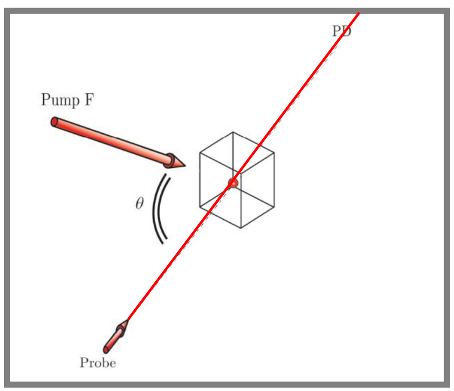
(sphere geometry)

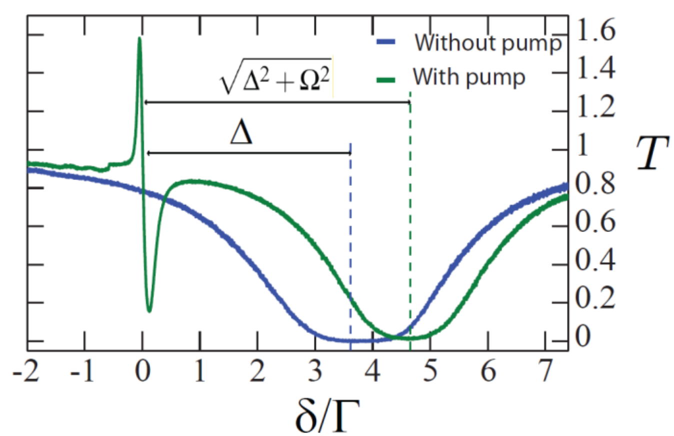
ℓ_g = linear gain length

ℓ_{sc} = mean free path $\quad \ell_{sc} = \dfrac{1}{n\,\sigma_{sc}}$

Transmission experiments: $\quad T = e^{-L/\ell_{ex}}$

with the extinction length $\quad \dfrac{1}{\ell_{ex}} = \dfrac{1}{\ell_{sc}} - \dfrac{1}{\ell_g}$





The atomic polarizability

$$\sigma_{ex} = k\,\mathrm{Im}(\alpha) = \sigma_0\,\mathrm{Im}(\tilde{\alpha})$$

$$\sigma_{sc} = \frac{k^4}{6\pi}|\alpha|^2 = \sigma_0|\tilde{\alpha}|^2$$

$$\sigma_0 = \frac{3\lambda^2}{2\pi} = \frac{6\pi}{k^2}$$ = on-resonance atomic cross-section

$$\alpha = \frac{\sigma_0}{k}|\tilde{\alpha}|$$ = polarizability (with ~ : dimensionless)

$$L > 2\pi\sqrt{\frac{\ell_{sc}\ell_g}{3}}$$

$$\frac{1}{\ell_{ex}} = \frac{1}{\ell_{sc}} - \frac{1}{\ell_g}$$

On-resonance optical depth : $b_0 = n\,\sigma_0 L$

$$b_0 > \frac{2\pi}{\sqrt{3|\tilde{\alpha}|^2\left(|\tilde{\alpha}|^2 - \mathrm{Im}(\tilde{\alpha})\right)}}$$

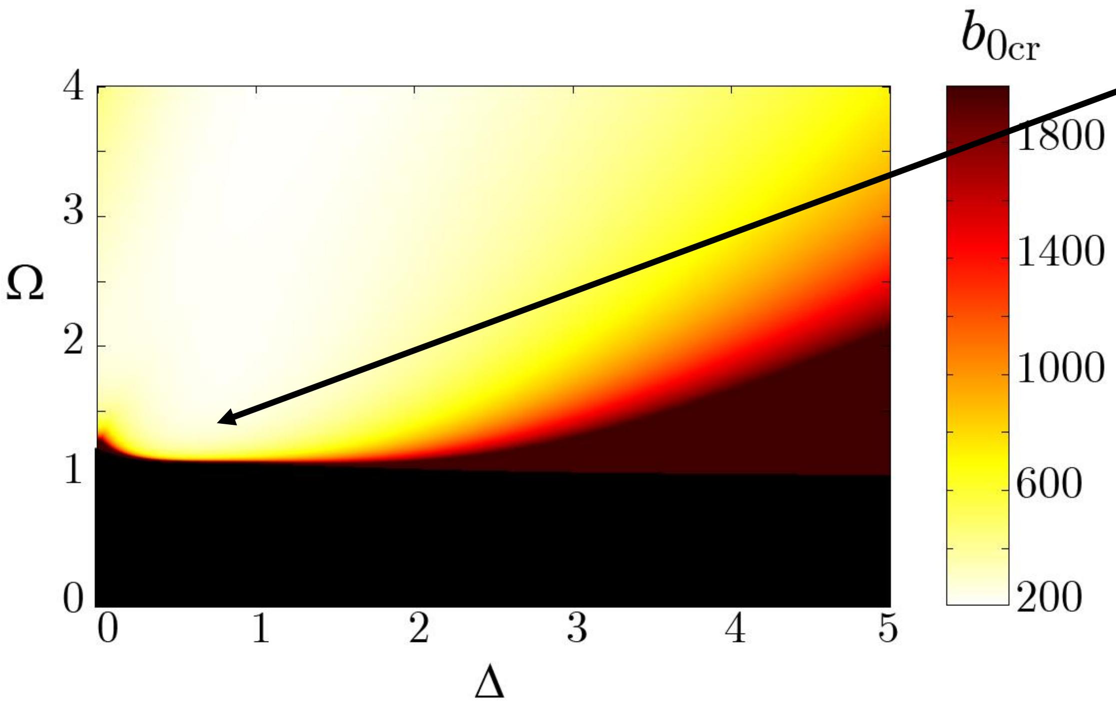
α depends on :
- Gain scheme
- Rabi frequency Ω of the pump
- atom-pump detuning Δ
- detuning δ from the pump

Example : Mollow gain

$$\tilde{\alpha}(\Omega,\Delta,\delta) = -\frac{1}{2}\,\frac{1+4\Delta^2}{1+4\Delta^2+2\Omega^2} \times \frac{(\delta+i)(\delta-\Delta+i/2)-\Omega^2\delta/(2\Delta-i)}{(\delta+i)(\delta-\Delta+i/2)(\delta+\Delta+i/2)-\Omega^2(\delta+i/2)}$$

with Ω, Δ, δ in unit of Γ.

minimum : $b_{0,cr} \approx 200.$



1) A threshold exists !

2) ± within reach

3) signature ?

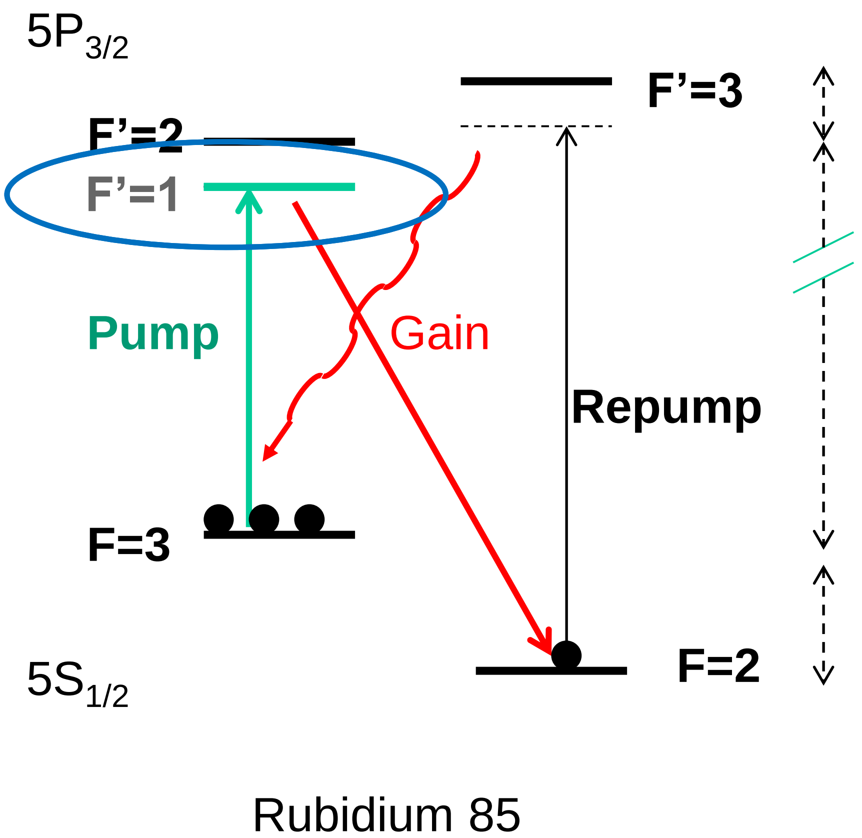
L.S. Froufe-Perez & al, PRL. **102**, 173903 (2009)

Best choice of gain scheme ?

Gain mechanism	Evaluation method	b_{0cr}	Validity of the diffusion approx.	Other problem	Ref.
Mollow gain	Analytical α	~ 200	✗	Pump penetration ☹	[1]
NDFWM	Exp. & Num.	∞	✓	Inelastic scattering ☹	
Raman gain (Zeeman)	Exp.	~ 200	✗	Detection ☹	[2]
Raman gain (Hyperfine)	Num.	~ 90	✗		
Raman gain (Hyperfine) + additional scattering	Num.	~ 30	✓		

[1] Phys. Rev. Lett. **102**, 173903 (2009).
[2] Opt. Express **17**, 11236 (2009).

Our choice for gain : Hyperfin Raman Gain



gain ☺ : $b_{0,cr} = 90$

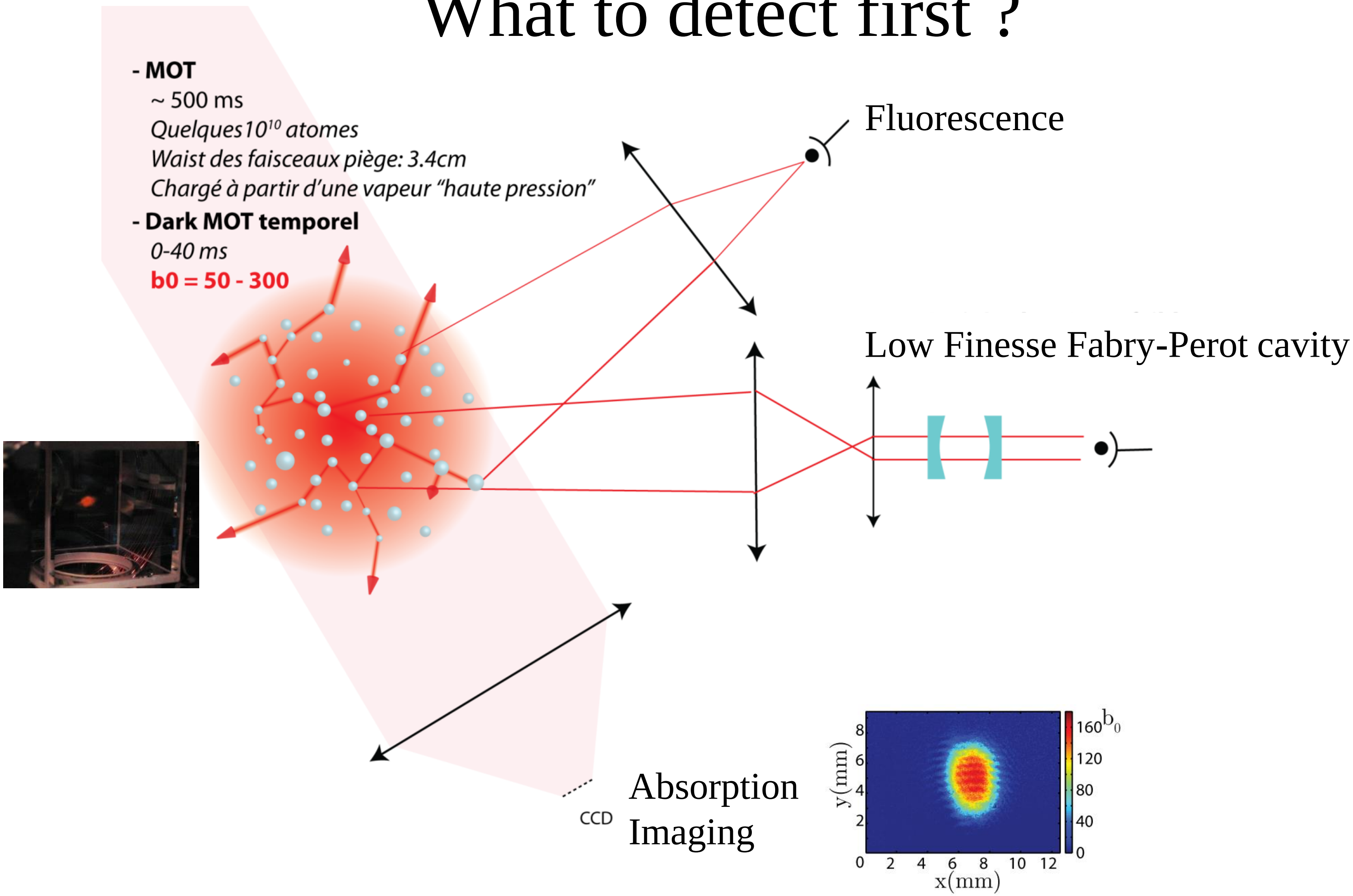
scattering ☹

Added scattering

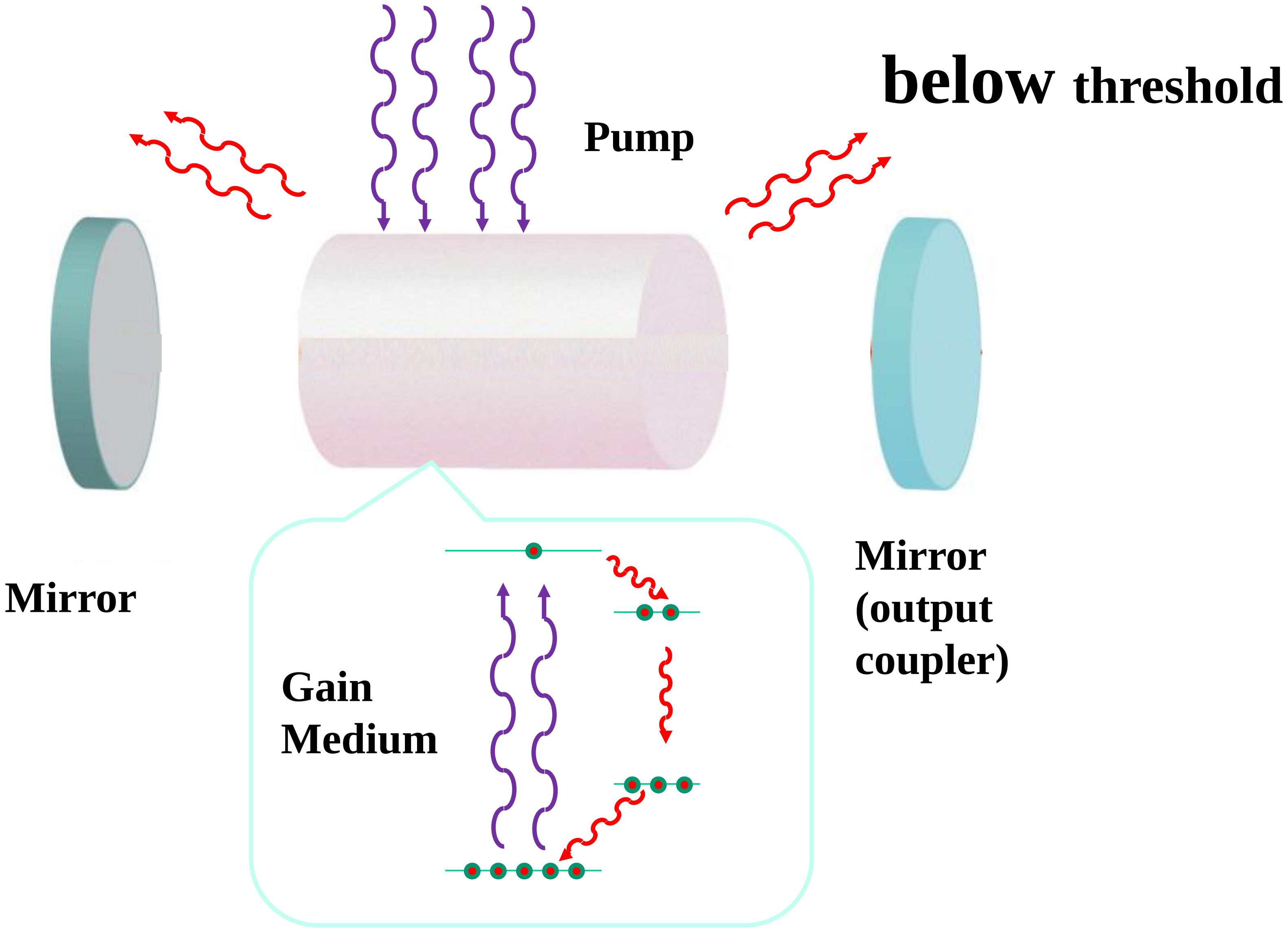
F=2 – F'=1 : scattering ☺

$\mathbf{b_{0,cr}=30}$

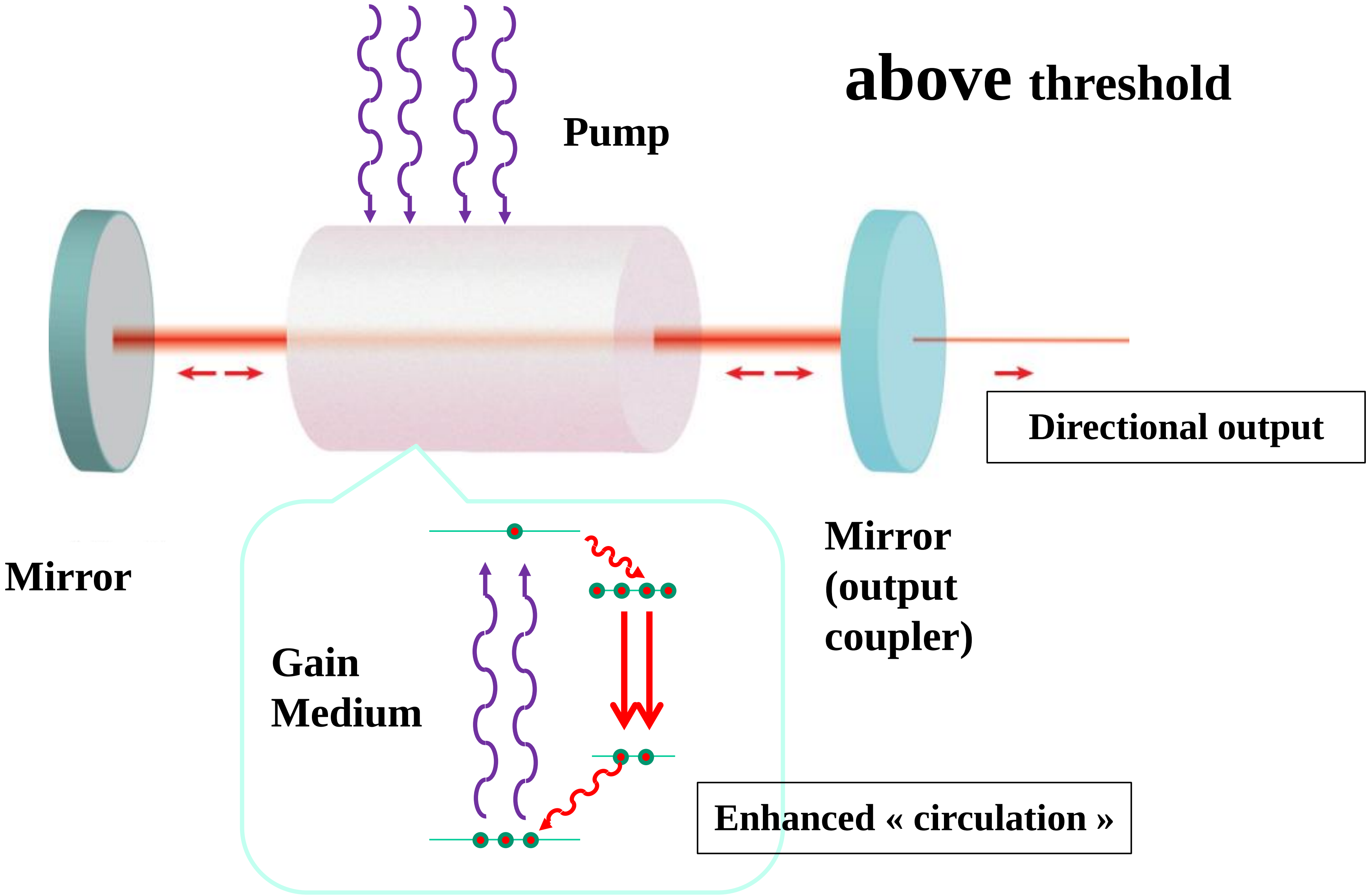
The Experimental Setup : What to detect first ?



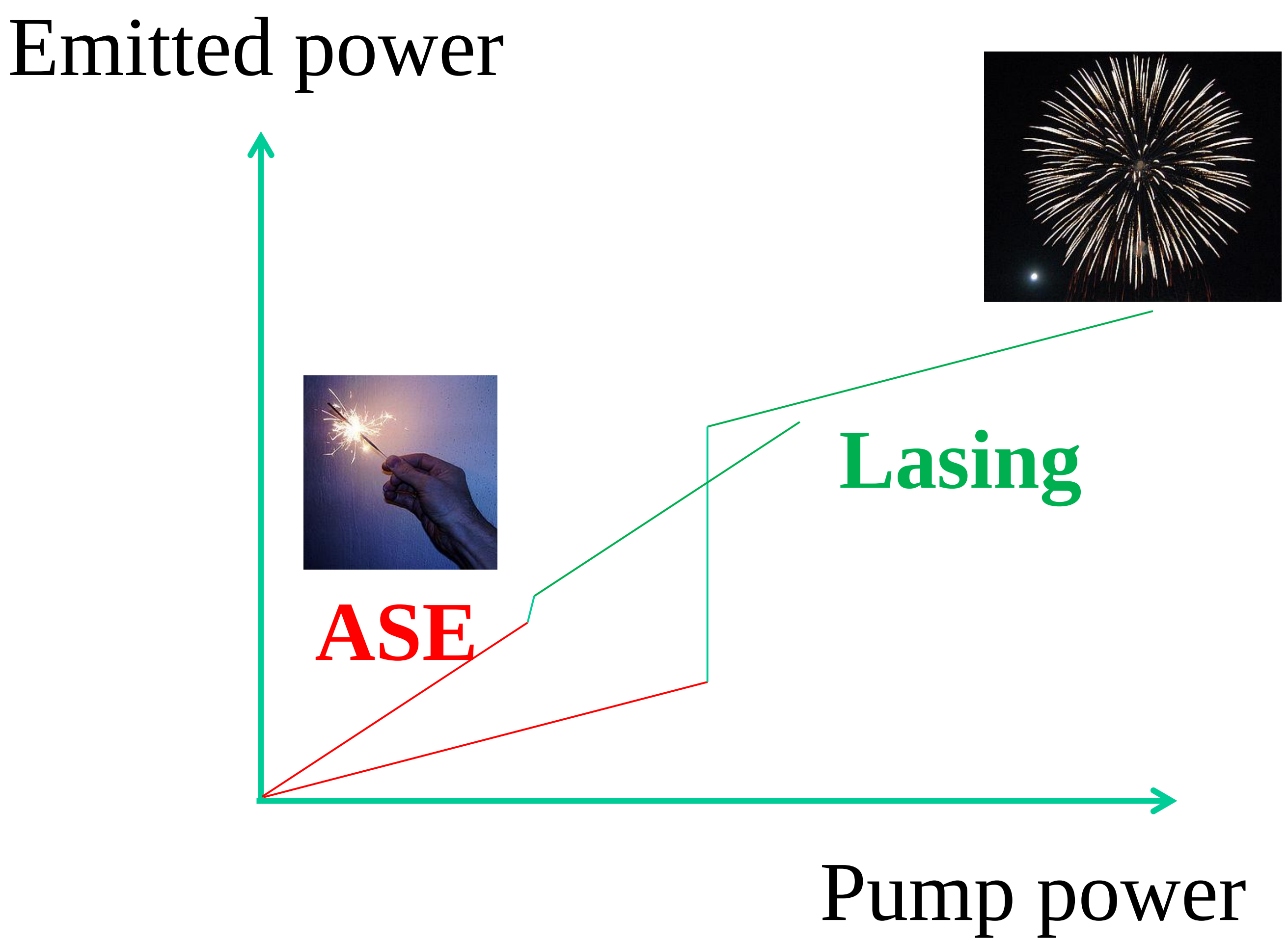
A simple laser model



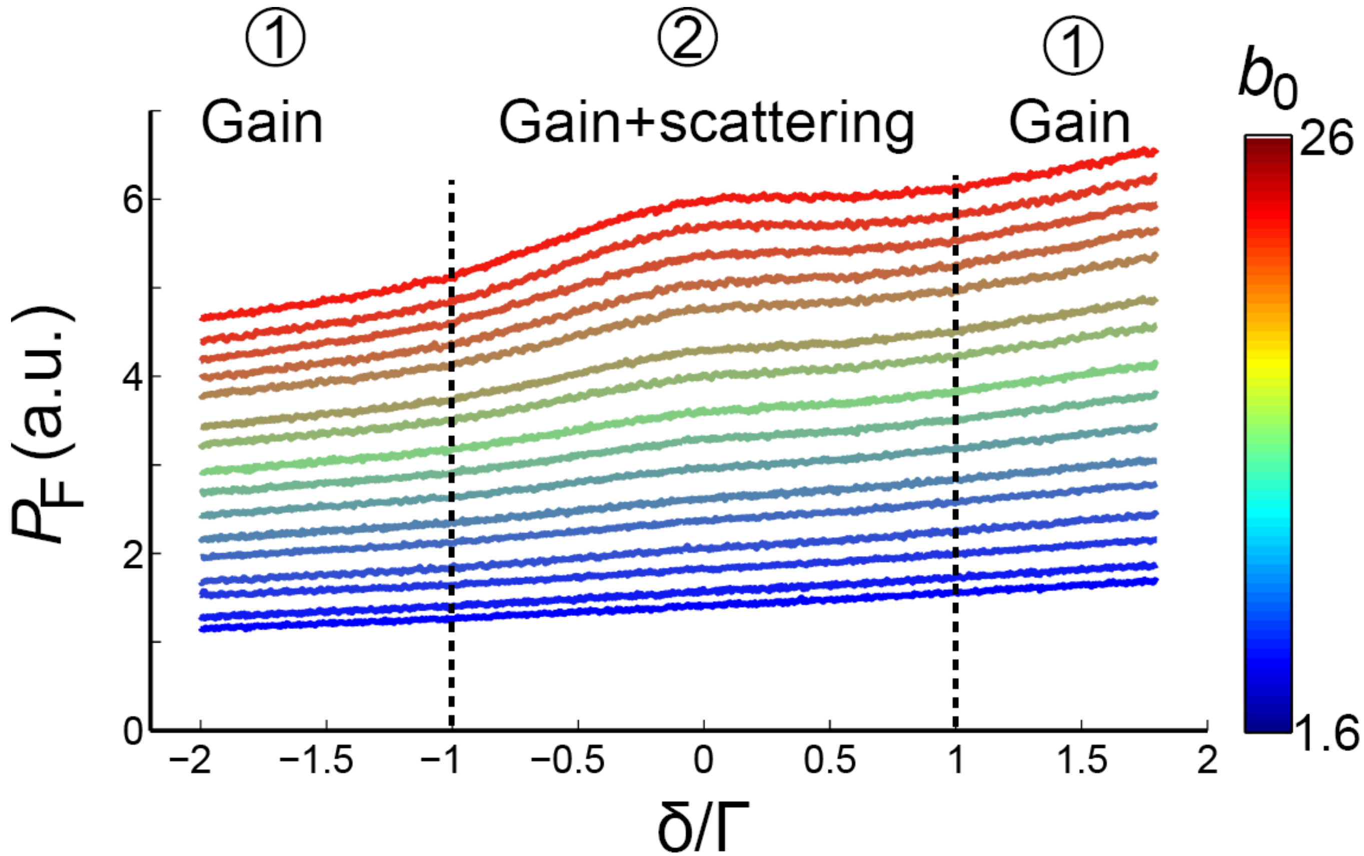
A simple laser model



ASE and gain saturation

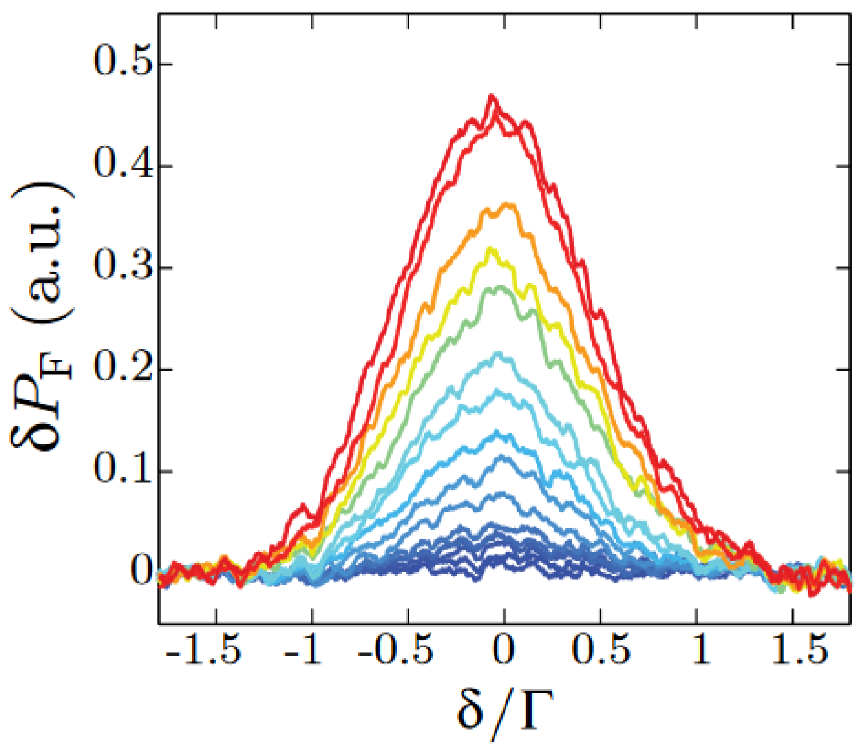


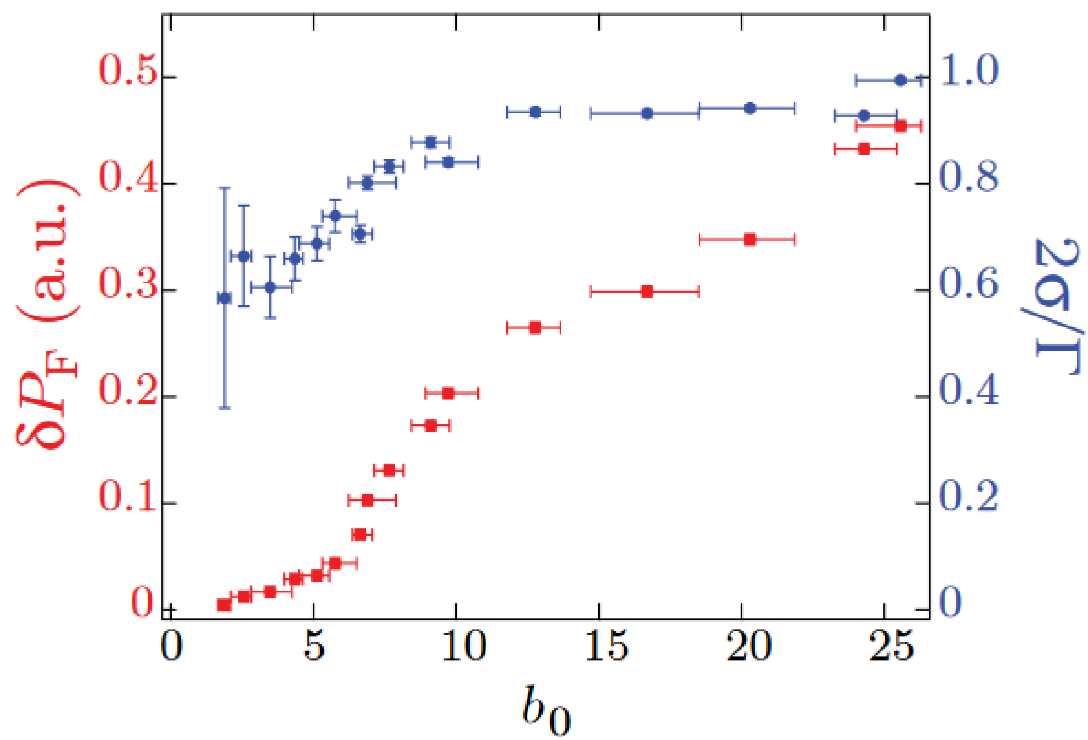
- No increase due to redirection of emitted photons (as in cavity laser)
- N=c^{te} ! vary compression = change b_0 @ c^{te} N



Q. Baudouin et al., Nat. Phys. 9, 357 (2013)

Fit of the wings → subtract the "ASE" background





Microscopic modelling for cold atom random lasing

1) Input parameters : pump and repump lasers $\{\Omega_{Raman}, \Delta_{Raman}, \Omega_{pump}, \Delta_{pump}\}$
system « size » b_0

2) Ω_{RL} , Δ_{RL} => optical Bloch equations (4 level, 3 lasers : 2 external + selfgenerated RL)
=> density matrix solution
=> σ_{sc} , σ_{ext}, σ_g
=> 4+1 model : σ_{sc}+ scattering $\sigma_{2\text{-}1}$,

3) Compute $2\pi \sqrt{(\sigma_0^2 / 3\sigma_g\sigma_{sc})} = b_{0,Letokhov}$
4) Start with small Ω_{RL} & find Δ_{RL} to get smallest $b_{0,Letokhov}$
5) If $b_{0,exp} < b_{0,Letokhov}$: even without gain saturation, sample too small : no RL
If $b_{0,exp} > b_{0,Letokhov}$: our experiment must have more scattering :
increase Ω_{RL} leading to gain saturation and larger $b_{0,Letokhov}$

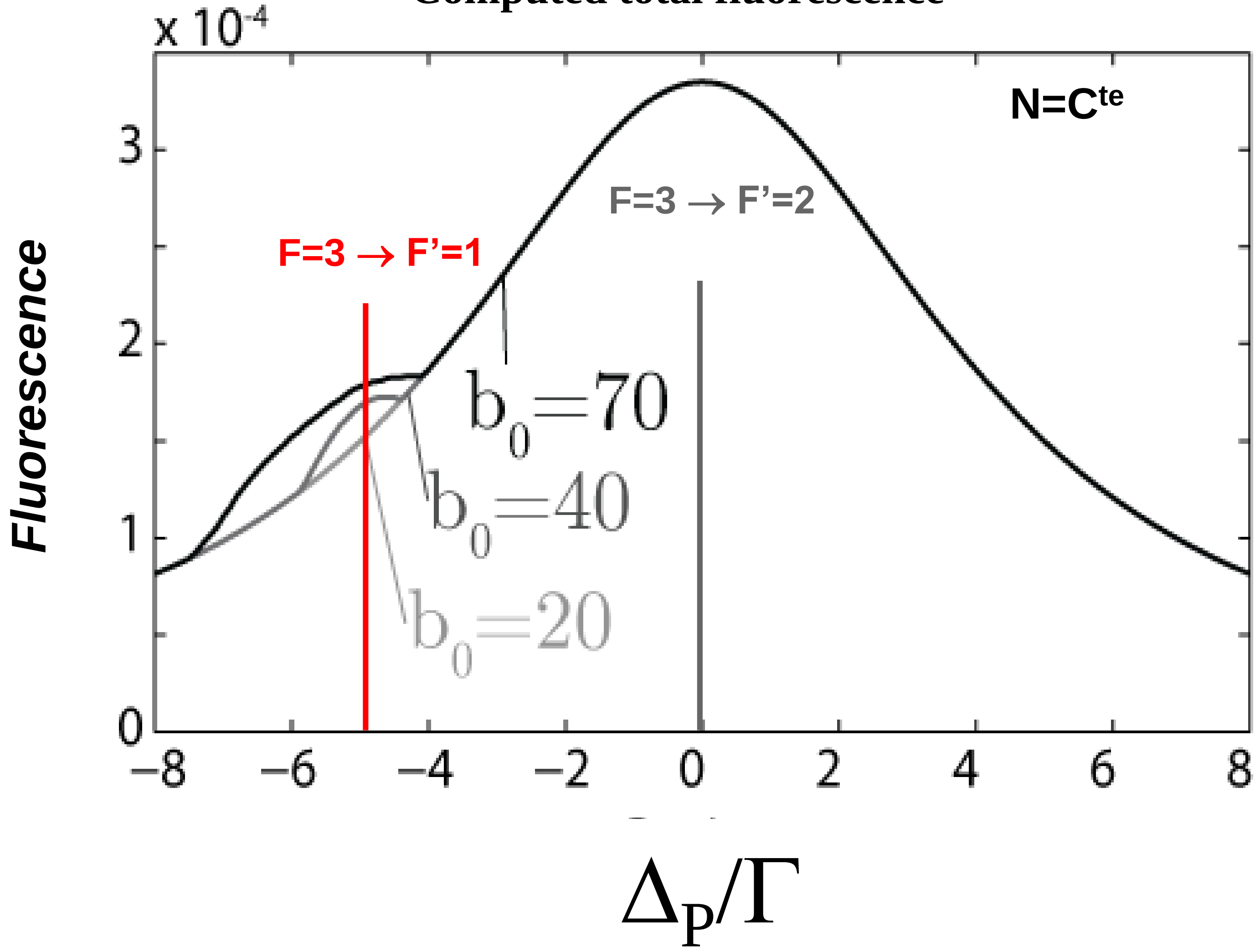
6) $b_{0,exp} = b_{0,Letokhov}$: we know Ω_{RL} (including saturation, which sets losses = gain)
7) Compute pump and repump extinction = total fluorescence ☺

☹ misses Raman hyperfin gain below threshold

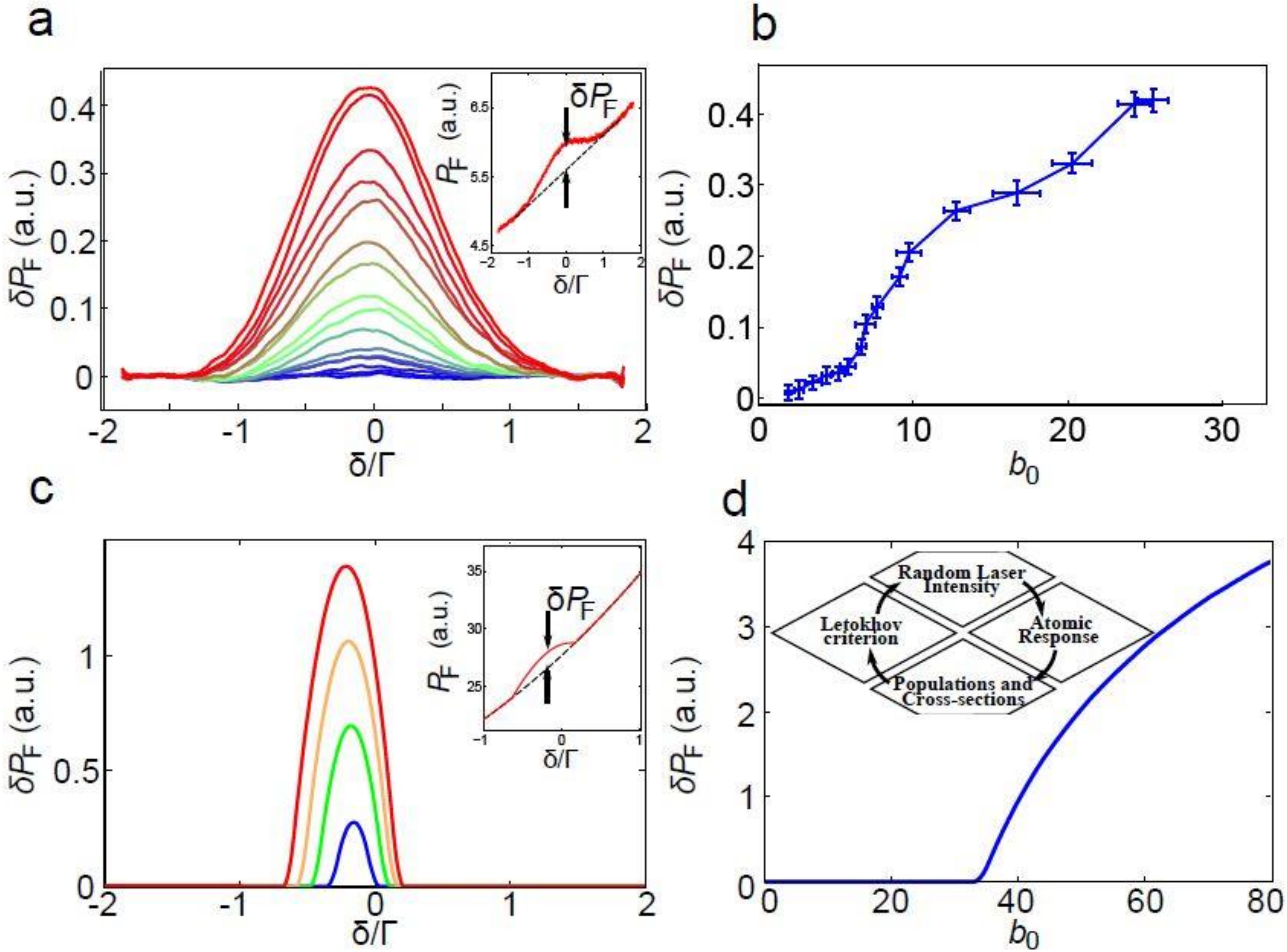
Microscopic modelling for cold atom random lasing

Input parameters : pump and repump lasers {Ω_P, Δ_P, Ω_{rep}, Δ_{rep}}, system « size » b_0

Computed total fluorescence

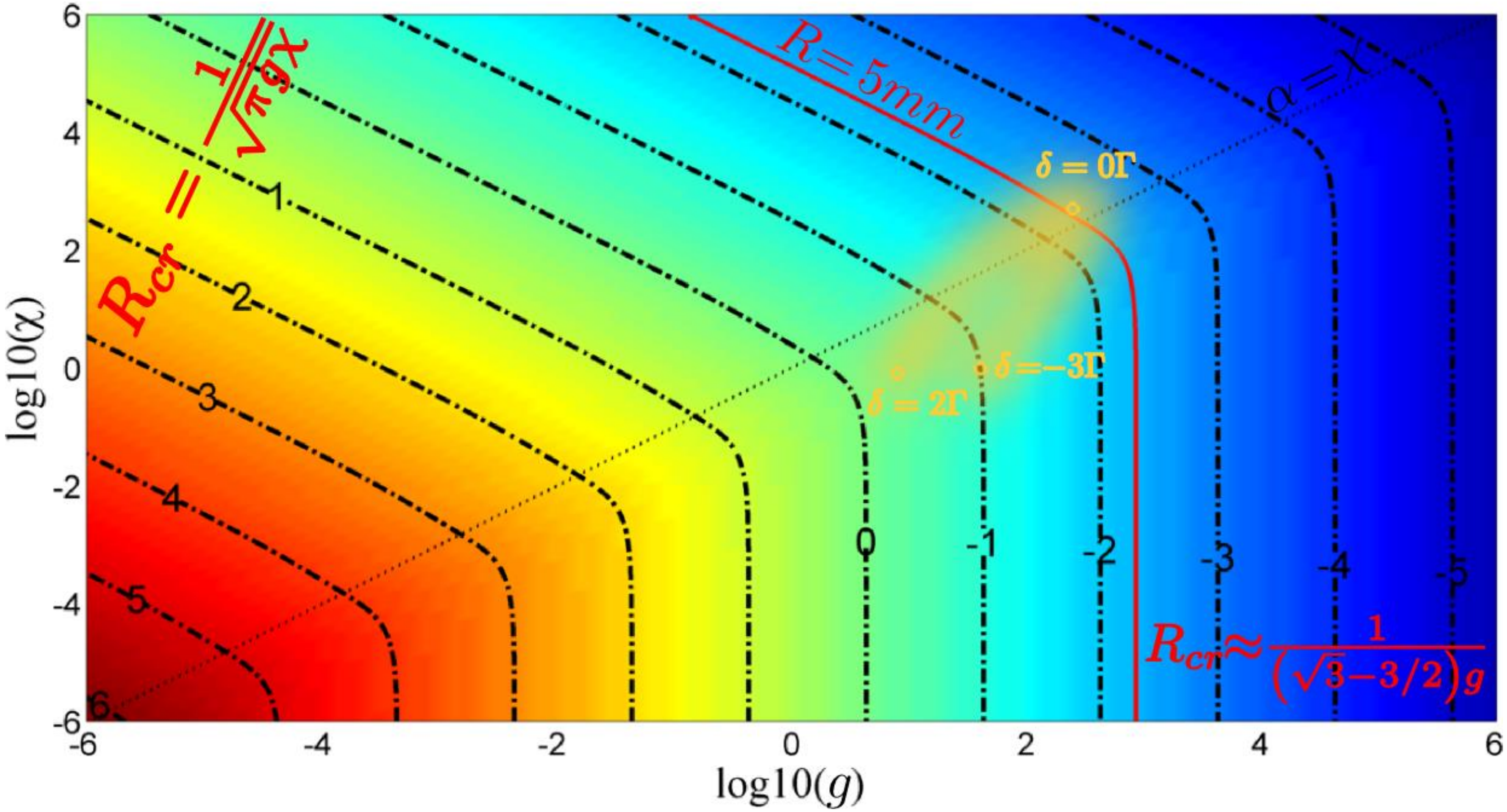


Microscopic modelling for cold atom random lasing



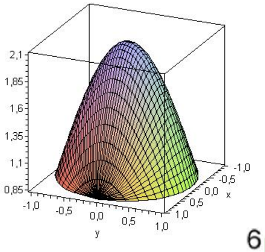
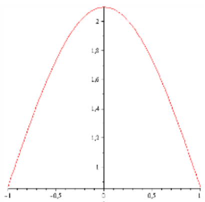
Generalised Letokhov threshold

Model decomposition of radiative transfer equation



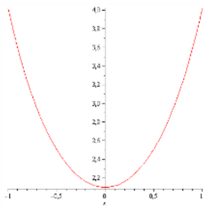
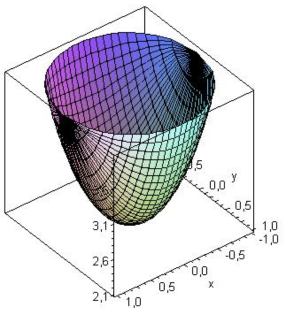
W. Guerin et al., J. Opt. Soc. Am. B 33, 1888 (2016)

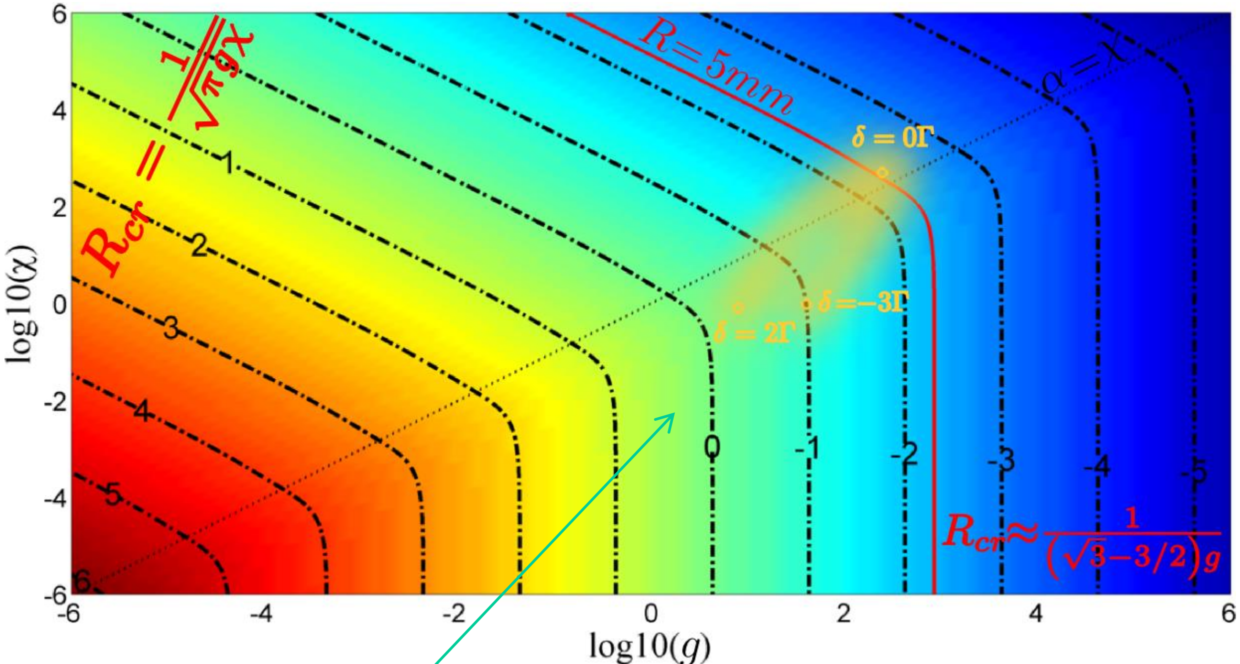
Shape of Lasing modes

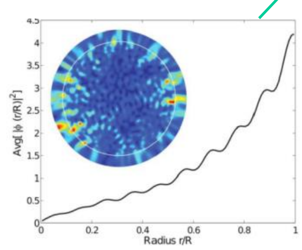



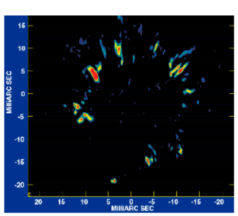
Diffuse Random Laser

ASE





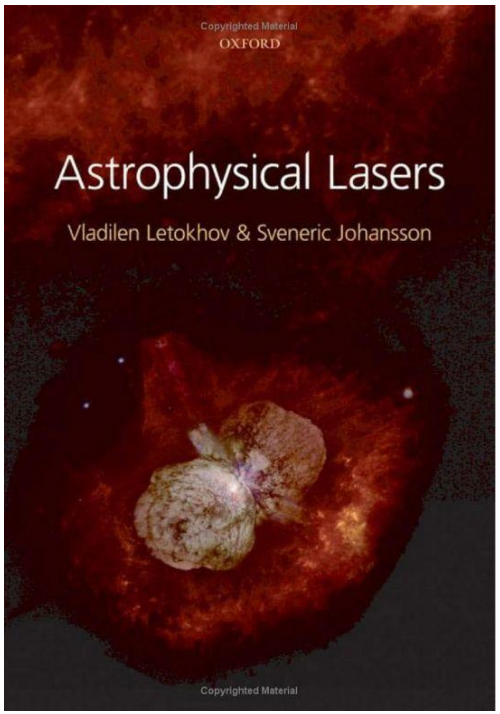


43 GHz (SiO) maser in TX-cam star

A&A, **432**, 531, 2005

Random lasing in Space ?

V. Letokhov, Stimulated Radio Emission of the interstellar Medium, JETP 4, 321 (1966)



Letokhov, V. S. & Johansson S. *Astrophysical Lasers* (Oxford University Press, 2009).

Table 1
Maser/laser action in space

Microwaves	Molecules	OH, 18.5 cm	Weaver et al. (1965)
	>100 molecular species	H_2O, 1.35 cm	Cheung et al. (1969)
			Townes (1997)
Submillimeters	Atom	H^{**}	Strelnitskii et al. (1996)
IR	Molecule (Mars, Venus)	CO_2, 10 μm	Johnson et al. (1976)
			Mumma et al. (1981)
Optical waves	Ion, atom	$Fe^{+} \simeq 1$ μm	Johansson and Letokhov (2002)
	η Carinae, gas condensations	OI, 8446 Å	Johansson and Letokhov (2005a–c)

S. Johansson ,V. Letokhov, New Astronomy Reviews, **51**, 443 (2007)

That's all for now

Next lecture : wave / interference effects