Lectures on quantum gases

Lecture 5

Feshbach resonances

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Where do we stand?

- 1. We introduced Breit-Wigner resonances
- 2. We analyzed s-wave resonances near threshold
- 3. We noticed that negative effective ranges did not appear
- 4. We introduced a tunnel barrier
- 5. We introduced the width parameter R^* to discriminate between broad and narrow s-wave resonances
- 6. We found that for a weak tunnel coupling $r_e < 0$
- 7. We introduced atoms with spin
- 8. We found triplet and singlet potentials
- 9. We introduced the Feshbach problem

Feshbach resonance **<u>Closed</u>** channel \mathcal{E}_{qb} $\varepsilon_v = -\kappa^2 - \kappa^2$ k^2 k^2 Open channel threshold energy Closed channel: below threshold Closed channel: above threshold ${\mathcal E}$ ${\mathcal E}$ a > 0a < 0 \mathcal{E}_{qb} ()0 $\varepsilon_v = -\kappa^2$ k^2 k^2

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Feshbach resonance



With cold alkali atoms we can tune to a Feshbach resonance at arbitrary, <u>fixed</u> (low) collisional energy by varying the magnetic field: Zeeman tuning

$$\mathcal{H} = \frac{1}{2\mu} \left(p_r^2 + \frac{\mathbf{L}^2}{r^2} \right) + \mathcal{V}(r) + \gamma_e B S_z$$
$$\mathcal{H}_0 \qquad |\psi\rangle = |R_{v,l}^S\rangle |S, M_S\rangle |l, m_l\rangle$$

Solve radial wave equation for given l, S and M_s : $R_{S,l}'' + \frac{2}{r}R_{S,l}' + [\varepsilon - U_{S,l}(r)]R_{S,l} = 0$ $U_{S,l}(r) = U_S(r) + \frac{l(l+1)}{r^2}$

$$U_{S,l}(r) = U_S(r) + \frac{l(l+1)}{r^2} + \frac{2\mu}{\hbar^2}\gamma_e B M_S$$

Solutions for given l, S and M_s :

$$\begin{array}{ll} \text{Continuum states } \varepsilon > 0: \quad \varepsilon_k = k^2 + \frac{2\mu}{\hbar^2} \gamma_e B \, M_S \\ \text{Bound states } \varepsilon < 0: \quad \varepsilon_{v,l}^S = -\kappa_{v,S}^2 + l \, (l+1) \, \mathcal{R}_{v,l}^S + \frac{2\mu}{\hbar^2} \gamma_e B \, M_S \\ & \uparrow \\ \langle R_{v,l}^S | \mathcal{H}_0 | R_{v,l}^S \rangle & \qquad \uparrow \\ \mathcal{R}_{v,l}^S = \langle R_{v,l}^S | r^{-2} | R_{v,l}^S \rangle \end{array}$$

 $V_1(r)$

Hamiltonian including spin Zeeman term:

$$\mathcal{H} = \frac{1}{2\mu} \left(p_r^2 + \frac{\mathbf{L}^2}{r^2} \right) + \mathcal{V}(r) + \gamma_e B S_z - (\gamma_1 i_{z1} + \gamma_2 i_{z2}) B$$

Add nuclear Zeeman terms (unlike atoms):

$$\mathcal{H}_Z = -\gamma_1 \mathbf{i}_1 \cdot \mathbf{B} - \gamma_2 \mathbf{i}_2 \cdot \mathbf{B}$$

$$\Delta E_Z = -\left(\gamma_1 m_1 + \gamma_2 m_2\right) B$$

Good basis states: $|\psi\rangle = |R_l^S\rangle |l, m_l\rangle |S, M_S\rangle |i_1, m_1\rangle |i_2, m_2\rangle$

Effective potential (including rotational and magnetic shifts):

$$U_{S,l}(r) = U_S(r) + \frac{l(l+1)}{r^2} + \frac{2\mu}{\hbar^2} \left[\gamma_e B M_S - (\gamma_1 m_1 + \gamma_2 m_2) B\right]$$

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Hamiltonian including spin Zeeman term:

$$\mathcal{H} = \frac{1}{2\mu} \left(p_r^2 + \frac{\mathbf{L}^2}{r^2} \right) + \mathcal{V}(r) + \gamma_e B S_z - \gamma_n B I_z$$

Add nuclear Zeeman terms (identical atoms):

$$\mathbf{I} = \mathbf{i}_1 + \mathbf{i}_2 \qquad M_I = m_1 + m_2$$
$$\mathcal{H}_Z = -\gamma_n \mathbf{i}_1 \cdot \mathbf{B} - \gamma_n \mathbf{i}_2 \cdot \mathbf{B} = -\gamma_n \mathbf{I} \cdot \mathbf{B}$$
Good basis states: $|\psi\rangle = |R_l^{S,I}\rangle |l, m_l\rangle |S, M_S\rangle |I, M_I\rangle$

Effective potential (including rotational and magnetic shifts):

$$U_{S,l}(r) = U_S(r) + \frac{l(l+1)}{r^2} + \frac{2\mu}{\hbar^2} \left[\gamma_e B M_S - \gamma_n B M_I\right]$$

Add hyperfine interactions (unlike atoms):

$$\mathcal{H}_{\rm hf} = rac{a_1}{\hbar^2} \mathbf{i}_1 \cdot \mathbf{s}_1 + rac{a_2}{\hbar^2} \mathbf{i}_2 \cdot \mathbf{s}_2 \qquad \qquad \mathbf{f} = \mathbf{s} + \mathbf{i}$$

Is $M_F = M_S + M_I$ a good quantum number? $M_I = m_1 + m_2$ $M_S = m_{s_1} + m_{s_2}$

Answer: yes!

$$\mathbf{i} \cdot \mathbf{s} = i_z s_z + \frac{1}{2}(i_+ s_- + i_- s_+)$$

Is S still a good quantum number?

$$\mathcal{H}_{\rm hf} = \mathcal{H}_{\rm hf}^{+} + \mathcal{H}_{\rm hf}^{-}$$
$$\mathcal{H}_{\rm hf}^{\pm} = \frac{a_1}{2\hbar^2} \mathbf{i}_1 \cdot (\mathbf{s}_1 \pm \mathbf{s}_2) \pm \frac{a_2}{2\hbar^2} \mathbf{i}_2 \cdot (\mathbf{s}_1 \pm \mathbf{s}_2)$$

Is S still a good quantum number?

$$\mathcal{H}_{\rm hf} = \mathcal{H}_{\rm hf}^{+} + \mathcal{H}_{\rm hf}^{-}$$
$$\mathcal{H}_{\rm hf}^{\pm} = \frac{a_1}{2\hbar^2} \mathbf{i}_1 \cdot (\mathbf{s}_1 \pm \mathbf{s}_2) \pm \frac{a_2}{2\hbar^2} \mathbf{i}_2 \cdot (\mathbf{s}_1 \pm \mathbf{s}_2)$$

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$$\mathcal{H}_{\rm hf} = \mathcal{H}_{\rm hf}^{+} + \mathcal{H}_{\rm hf}^{-}$$

$$\mathcal{H}_{\rm hf}^{\pm} = \frac{a_1}{2\hbar^2} \mathbf{i}_1 \cdot (\mathbf{s}_1 \pm \mathbf{s}_2) \pm \frac{a_2}{2\hbar^2} \mathbf{i}_2 \cdot (\mathbf{s}_1 \pm \mathbf{s}_2)$$

$$\mathcal{H}_{\rm hf}^{+} = \frac{a_1}{2\hbar^2} \mathbf{i}_1 \cdot \mathbf{S} + \frac{a_2}{2\hbar^2} \mathbf{i}_2 \cdot \mathbf{S}$$

$$\mathcal{H}_{\rm hf}^{+} = \frac{a_1}{2\hbar^2} \mathbf{I} \cdot \mathbf{S} \qquad \mathcal{H}_{\rm hf}^{+} \text{ can change } M_S \text{ but not } S \text{ and } M_F$$

$$\mathbf{I} \cdot \mathbf{S} = I_z S_z + \frac{1}{2} (I_+ S_- + I_- S_+)$$

With \mathcal{H}_{hf}^+ in hamiltonian *S* remains a good quantum number! Analysis shows that \mathcal{H}_{hf}^- converts singlet in triplet and vice versa

Hamiltonian including spin Zeeman term:

$$\mathcal{H} = \frac{1}{2\mu} \left(p_r^2 + \frac{\mathbf{L}^2}{r^2} \right) + \mathcal{V}(r) + \gamma_e B S_z - (\gamma_1 i_{z1} + \gamma_2 i_{z2}) B + \mathcal{H}_{hf}^+ + \mathcal{H}_{hf}^-$$

$$(-\gamma_n B I_z)$$

all terms conserve M_F

only term

not singlet/triplet conserving

Good basis states:
$$|\psi\rangle = |R_l^S\rangle |l, m_l\rangle |S, M_S\rangle |i_1, m_1\rangle |i_2, m_2\rangle$$

 $|\psi\rangle = |R_l^{S,I}\rangle |l, m_l\rangle |S, M_S\rangle |I, M_I\rangle$

Effective potential:

$$U_{S,l}(r) = U_S(r) + \frac{l(l+1)}{r^2} + \frac{2\mu}{\hbar^2} \left[\gamma_e B M_S - (\gamma_1 m_1 + \gamma_2 m_2) B \right]$$
(- $\gamma_n B M_I$)

Example: two ⁶Li atoms



Example: two ⁶Li atoms



Example: two ⁶Li atoms



Diagonalization of Hamiltonian

$$\mathcal{H} = \frac{1}{2\mu} \left(p_r^2 + \frac{\mathbf{L}^2}{r^2} \right) + \mathcal{V}(r) + \gamma_e B S_z - \gamma_n B I_z + \mathcal{H}_{hf}^+ + \mathcal{H}_{hf}^-$$
$$|\sigma\rangle$$
$$|\alpha\rangle = |R_{v,l}^S\rangle |l, m_l\rangle |S, M_S; I, M_I\rangle$$

 $\langle \beta | \mathcal{H} | \alpha \rangle = \left(\varepsilon_v^{S,l} + \gamma_e B M_S - \gamma_n B M_I \right) \delta_{\alpha\beta} + \langle \beta | \mathcal{H}_{\rm hf}^+ + \mathcal{H}_{\rm hf}^- | \alpha \rangle$ $= [\varepsilon_{v}^{S,l} + E_{\sigma}(B)]\delta_{\sigma,\sigma'} + \langle \sigma' | \mathcal{H}_{hf}^{+} | \sigma \rangle \delta_{v,v'} + \langle \sigma' | \mathcal{H}_{hf}^{-} | \sigma \rangle \langle \mathcal{R}_{v'}^{S',l} | \mathcal{R}_{v}^{S,l} \rangle$ = 0 = 0 = 0 $for S \neq S'$ for S = S' \mathcal{H}_{hf}^{+} S = 0Franck-Condon factor $\mathcal{H}_{\mathrm{hf}}^{-}$ (only part that depends on radial wavefunction) S = 1 \mathcal{H}_{hf}^+ S = 0v'3 fit parameters: ε^0 , ε^1 , $\langle R_{v'}^{S',l} | R_v^{S,l} \rangle$

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Franck-Condon factor



b. Asymptotic bound states ($r_{cl} > r_x$)

can be calculated numerically starting from Van der Waals tail

singlet/triplet overlap



So we can fit the resonances without knowing the radial wavefunctions!

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⁶Li-⁴⁰K pairs

 $\mathcal{H}_{hf+Z} = \frac{a_1}{\hbar^2} \mathbf{i}_1 \cdot \mathbf{s}_1 + \frac{a_2}{\hbar^2} \mathbf{i}_2 \cdot \mathbf{s}_2 + \gamma_s \left(\mathbf{s}_1 + \mathbf{s}_2\right) \cdot \mathbf{B} - \left(\gamma_1 \mathbf{i}_1 + \gamma_2 \mathbf{i}_2\right) \cdot \mathbf{B}$







Include l = 1 molecular levels



Conclusion: l = 0 and l = 1 levels can be related through C₆ coefficient



Relation to scattering length



Review paper C. Chin et al., R.M.P., 82 1225 (2010)

Relation to scattering length



Relation to scattering length

strength parameter:

$$s \equiv \frac{r_0}{R^*} = \frac{a_{bg}}{r_0} \frac{\mu_{\text{eff}}}{E_0} \Delta$$



Fano lineshape



width from: elastic cross section



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width from: elastic cross section



for $a_{bq} \neq 0$ asymmetric line shape: Fano profile

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Narrow versus broad resonances

Importantly:

The meaning of broad and narrow depends on the context

- Broad as measured in magnetic field
- Broad as compared to the collisional band of energies
- Broad due to strong Feshbach coupling (large R*)

experimental

adiabatic expansion of Gaussian trap induces evaporation:





characterization of a Feshbach resonance



End of Les Houches Lectures

Thank you