

Dipole-dipole interactions between atoms for many-body physics and quantum information

Lecture 1: Dipole-dipole interaction between atoms

Lecture 2: Basics of Rydberg physics. Arrays of cold atoms. Rydberg blockade & QIP

Lecture 3: Many-body physics with Rydberg atoms.

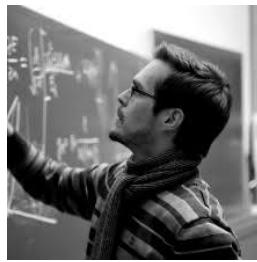
The team (atom-tweezers-io.org)



Theory



H.-P. Büchler
(Stuttgart)



N. Lang
(Stuttgart)



S. Weber
(Stuttgart)

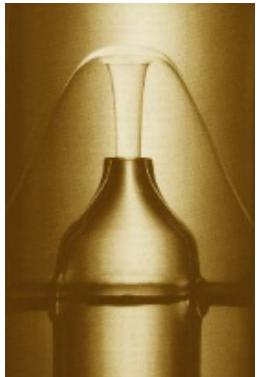


Outline

1. Quantum simulation and spin models
2. Simulation of Ising model using van der Waals interactions
3. Topological systems using resonant dipole interactions

Many-body physics and quantum simulation

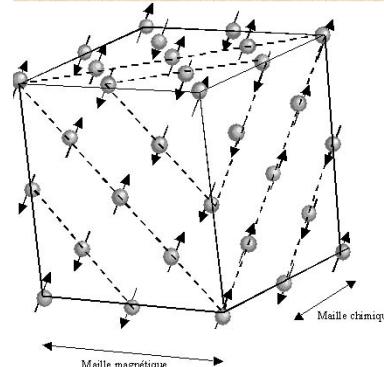
Goal: Understand ensembles of **interacting quantum particles**



superfluidity



superconductivity

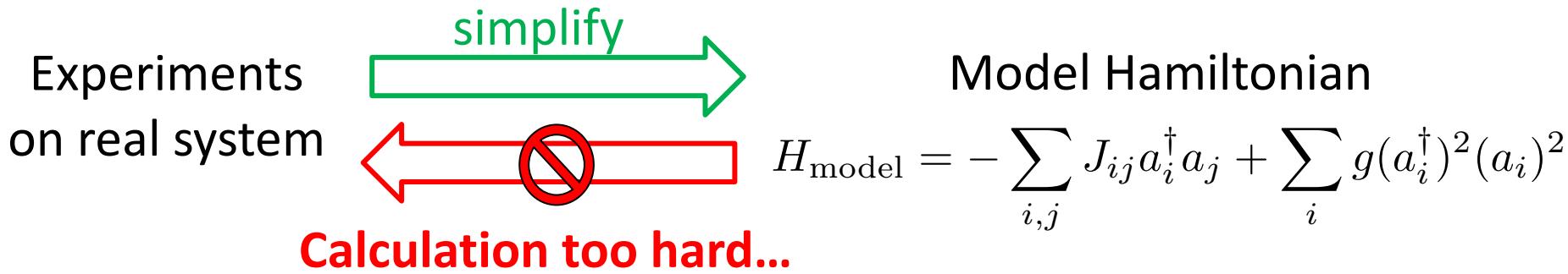


magnetism



neutron star

Questions: phase diagram, excitation spectrum, dynamics, transport...



Exponential scaling of Hilbert space

Ex: N -spin $\frac{1}{2}$ $\Rightarrow \dim \mathcal{H} = 2^N$

Record ab-initio: $N \sim 42$

Many-body physics and quantum simulation

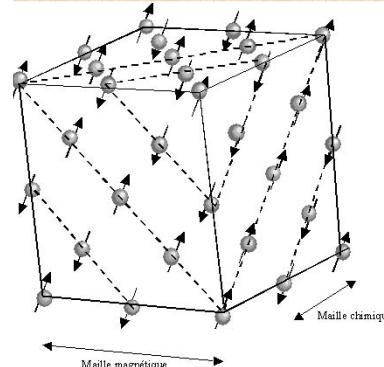
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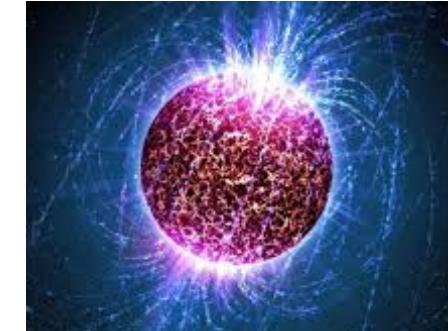
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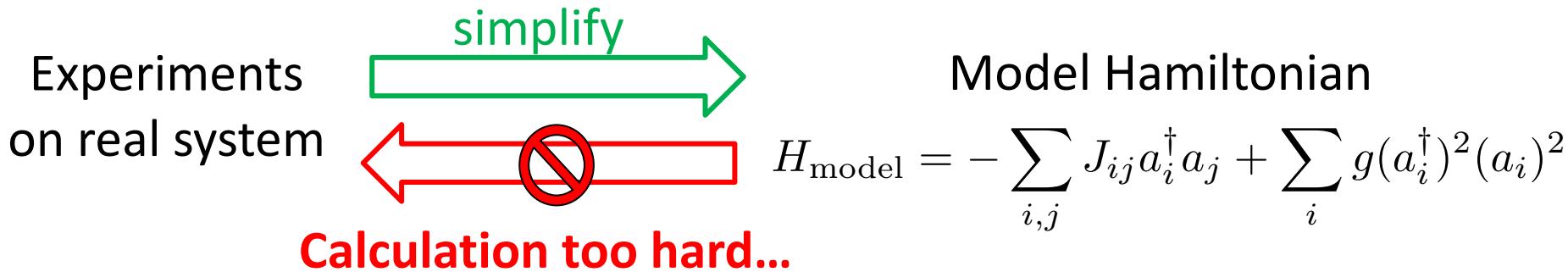


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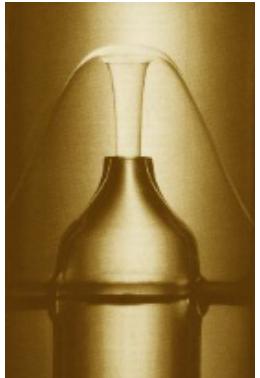
Questions: phase diagram, excitation spectrum, dynamics, transport...



Careful: approximate methods ($10^2 < N < 10^5$) exist!!!!
DMRG, Monte Carlo, density functionnal, mean field...

Many-body physics and quantum simulation

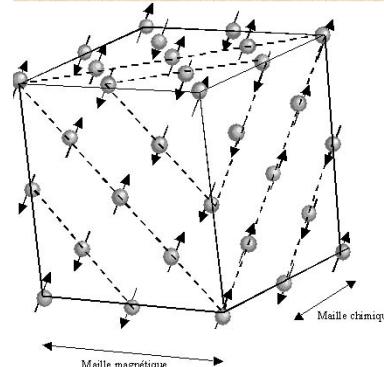
Goal: Understand ensembles of **interacting quantum particles**



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Questions: phase diagram, excitation spectrum, dynamics, transport...

Experiments
on real system

simplify

Model Hamiltonian

“Preparable”

$$H_{\text{model}} = - \sum_{i,j} J_{ij} a_i^\dagger a_j + \sum_i g(a_i^\dagger)^2 (a_i)^2$$

Lab...

$$|\psi(t)\rangle = e^{-\frac{i}{\hbar} H_{\text{model}} t} |\psi(0)\rangle$$

Measure outcome

of simulator:
ground state =
supercond.?



Quantum simulator =
engineer ensemble of
“atoms” ruled by H_{model}

The original idea...



Simulating Physics with Computers, Int. J. Theo. Phys. **21** (1982)

International Journal of Theoretical Physics, Vol. 21, Nos. 6/7, 1982

Simulating Physics with Computers

R.P. Feynman

Richard P. Feynman

4. QUANTUM COMPUTERS—UNIVERSAL QUANTUM SIMULATORS

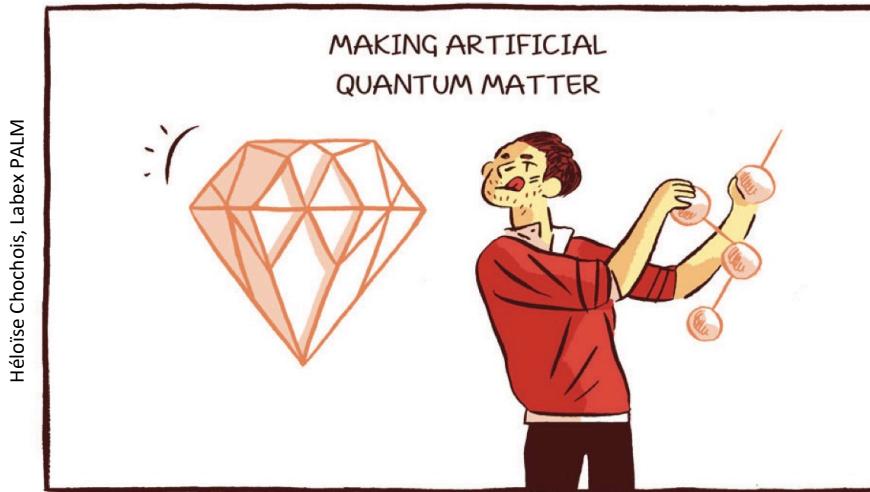
(with it, with quantum-mechanical rules). For example, the spin waves in a spin lattice imitating Bose-particles in the field theory. I therefore believe it's true that with a suitable class of quantum machines you could imitate any quantum system, including the physical world. But I don't know whether the general theory of this intersimulation of quantum systems has ever been worked out, and so I present that as another interesting problem: to work out the classes of different kinds of quantum mechanical systems which are really intersimulatable—which are equivalent—as has been done in the case of classical computers. It has been found that there is a kind of

Many-body physics with synthetic matter



R.P. Feynman

Simulating Physics with Computers, Int. J. Theo. Phys. **21** (1982)



i.e. **well-controlled** quantum systems implementing
many-body Hamiltonians (including “mathematical” ones...)

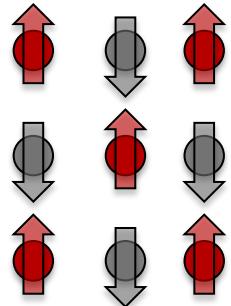
Larger tunability than “real” systems (geometry, parameters...)

+

New types of probe & methods (e.g. out-of-equilibrium)

Spin models: one of the “simplest” many-body problem

Interacting spin $\frac{1}{2}$ particles on a lattice:



$$\hat{H}_{ij} = J_{ij} \mathbf{S}_i \cdot \mathbf{S}_j$$

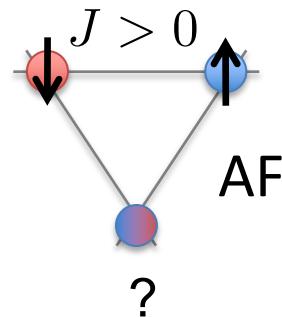
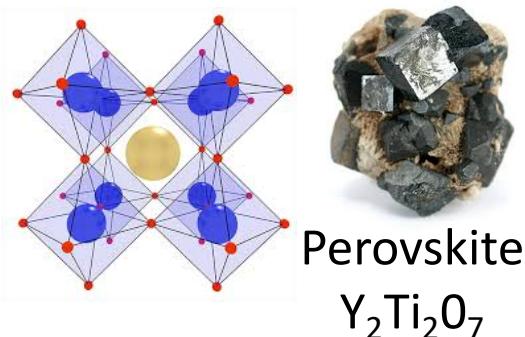
Ising

$$\hat{H} = \sum_{i \neq j} J_{ij} \hat{\sigma}_z^{(i)} \hat{\sigma}_z^{(j)}$$

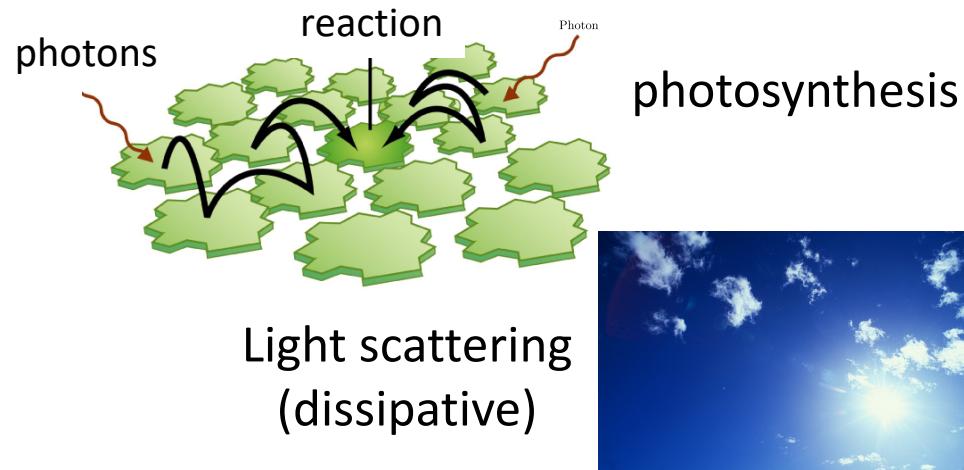
XY model

$$\hat{H} = \sum_{i \neq j} J_{ij} (\hat{\sigma}_i^+ \hat{\sigma}_j^- + \hat{\sigma}_i^- \hat{\sigma}_j^+)$$

Magnetism



Transport of excitations



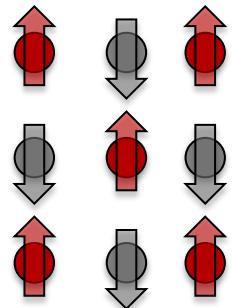
Open questions: **Dynamics** (hard for $N > 30$, long range...)

Entanglement, disorder, **topology**...

Geometry (frustration: spin liquids?) ...

Spin models: one of the “simplest” many-body problem

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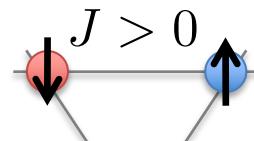
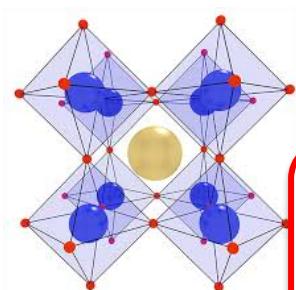
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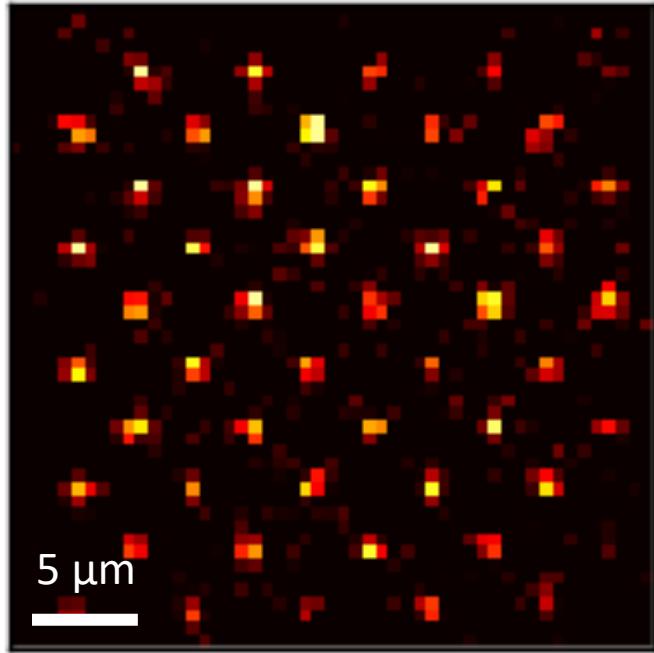
Use control over artificial quantum matter

(dissipative)

Open questions: **Dynamics** (hard for $N > 30$, long range...)
Entanglement, disorder, **topology**...
Geometry (frustration: spin liquids?) ...

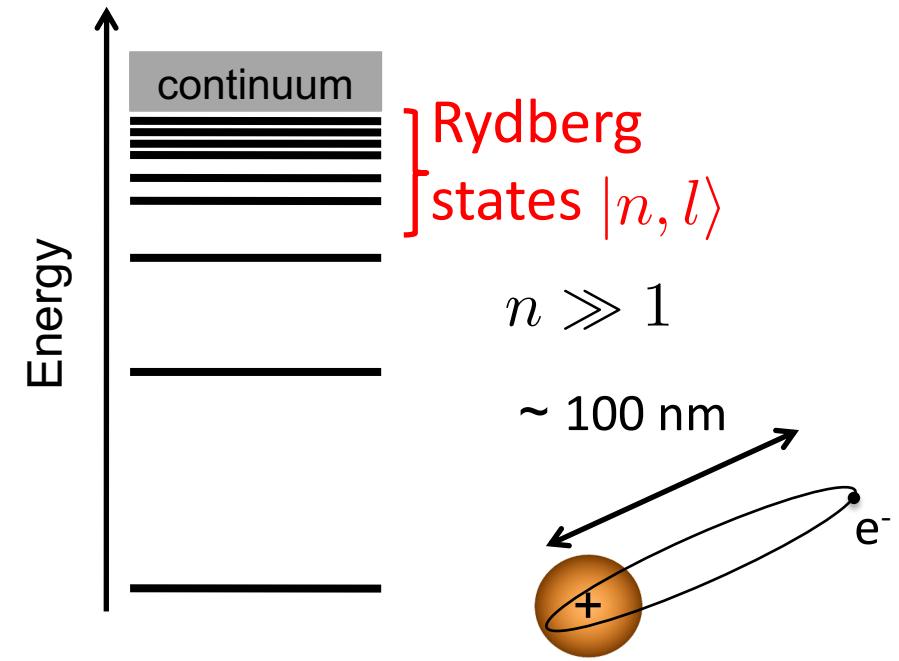
Arrays of interacting Rydberg atoms

Arrays of atoms (~ 70 at.)



Addressable!!

Rydberg atoms



Lifetime $> 100 \mu\text{s}$

Transition dipole: $d \sim n^2 e a_0$

⇒ Large dipole-dipole interactions

$$R = 10 \mu\text{m} \Rightarrow V_{\text{int}}/h \sim 1 - 10 \text{ MHz}$$

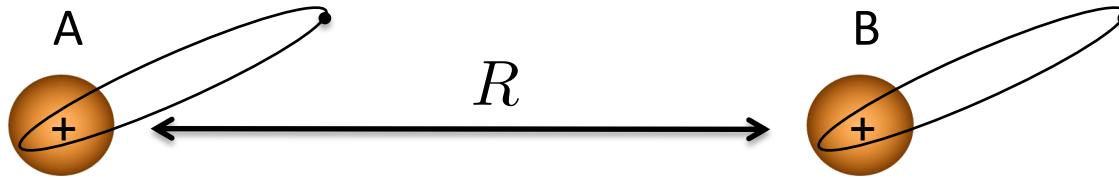
⇒ timescales < μsec

Lukin, Zoller 2000
Saffman, RMP 2010
Browaeys, JPhysB 2016

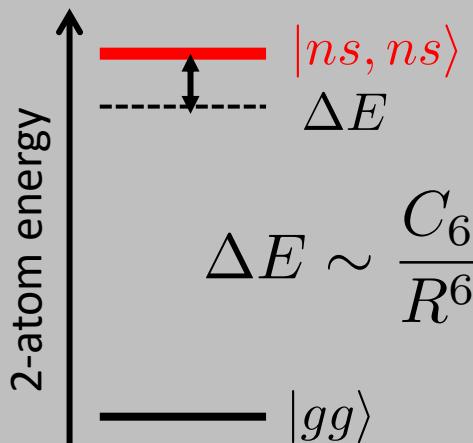
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From van der Waals interactions to Ising model...



van der Waals



$C_6 \propto n^{11} \Rightarrow$ switchable interaction

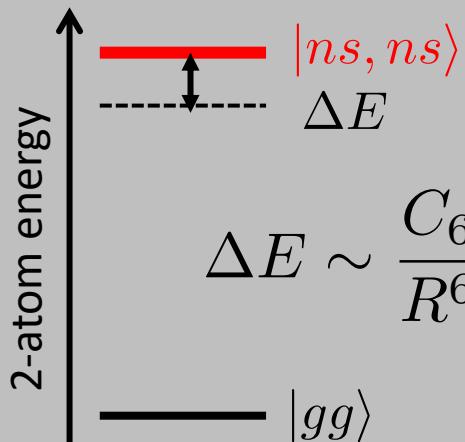
$$\hat{H}_{\text{int}} = \frac{C_6}{R^6} \hat{n}_1 \hat{n}_2$$

Rydberg occupation number

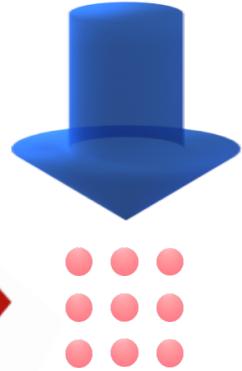
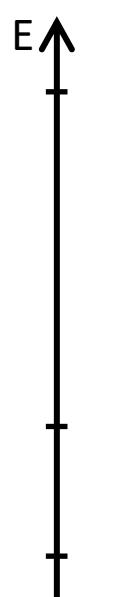
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van der Waals

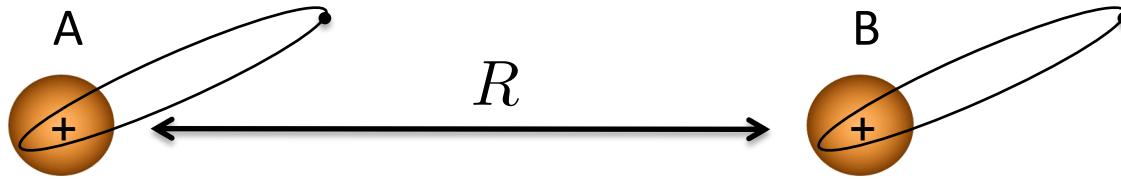


$$\Delta E \sim \frac{C_6}{R^6}$$

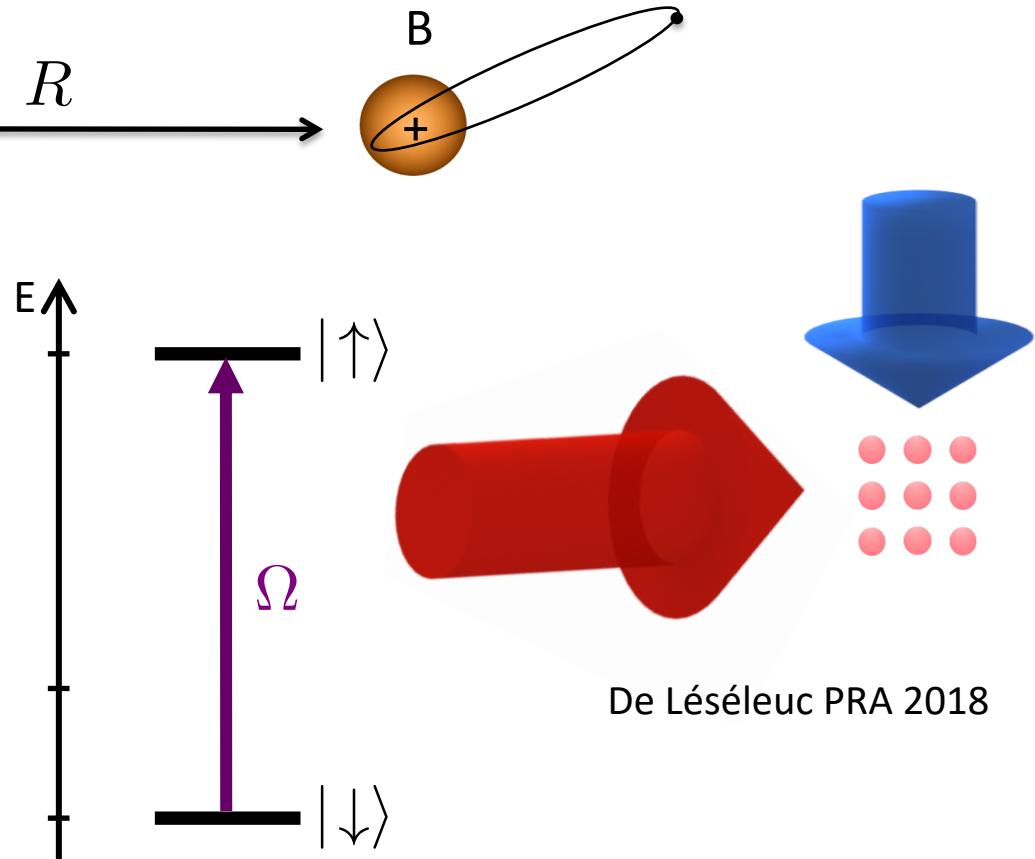
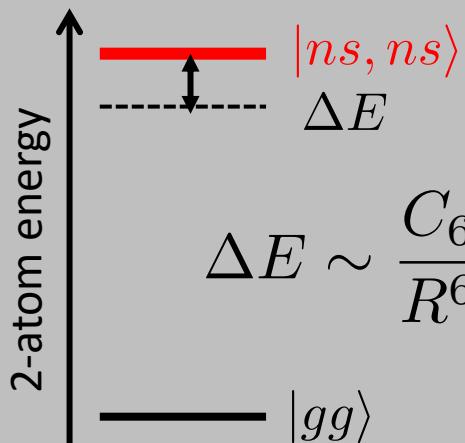


De Léséleuc PRA 2018

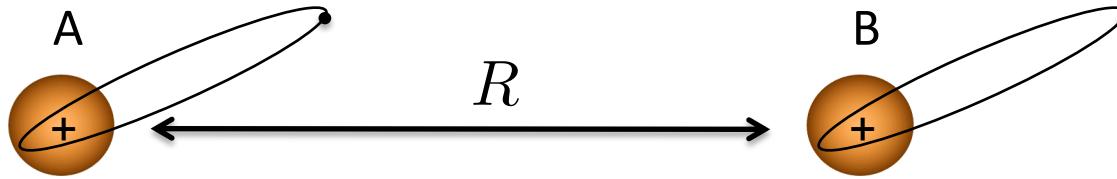
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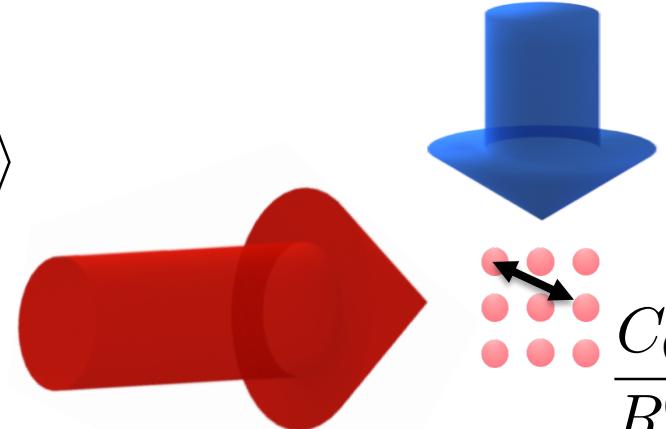
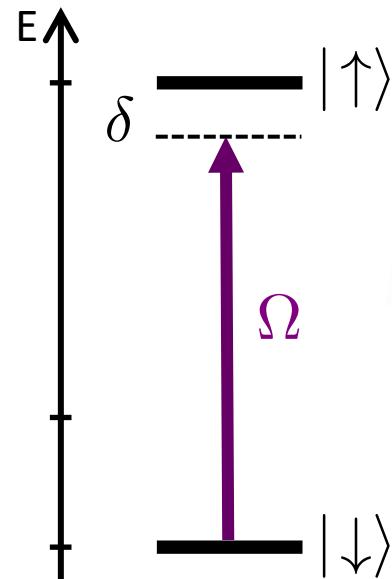
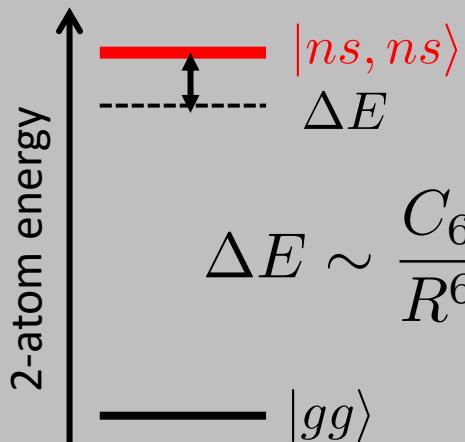
van der Waals



From van der Waals interactions to Ising model...



van der Waals



De Léséleuc PRA 2018

Quantum Ising-like model ($s=\frac{1}{2}$):

$$H = \frac{\hbar\Omega}{2} \sum_i \sigma_x^i - \hbar\delta \sum_i \hat{n}_i + \sum_{i < j} \frac{C_6}{R_{ij}^6} \hat{n}_i \hat{n}_j$$

Experiment.

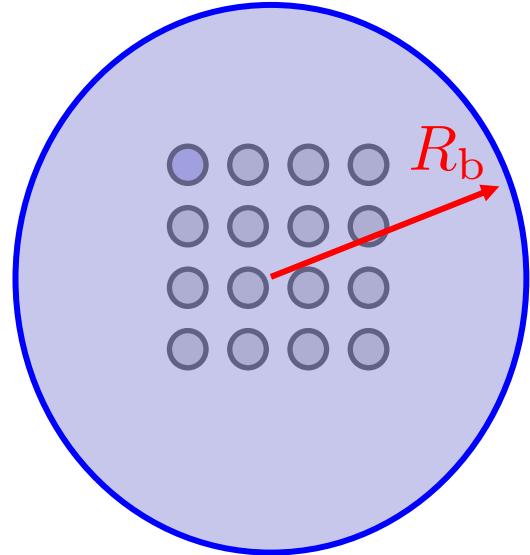
$$\frac{C_6/a^6}{\Omega} = [0 - 20]$$

Transverse B

Longitudinal B

Spin-spin interaction

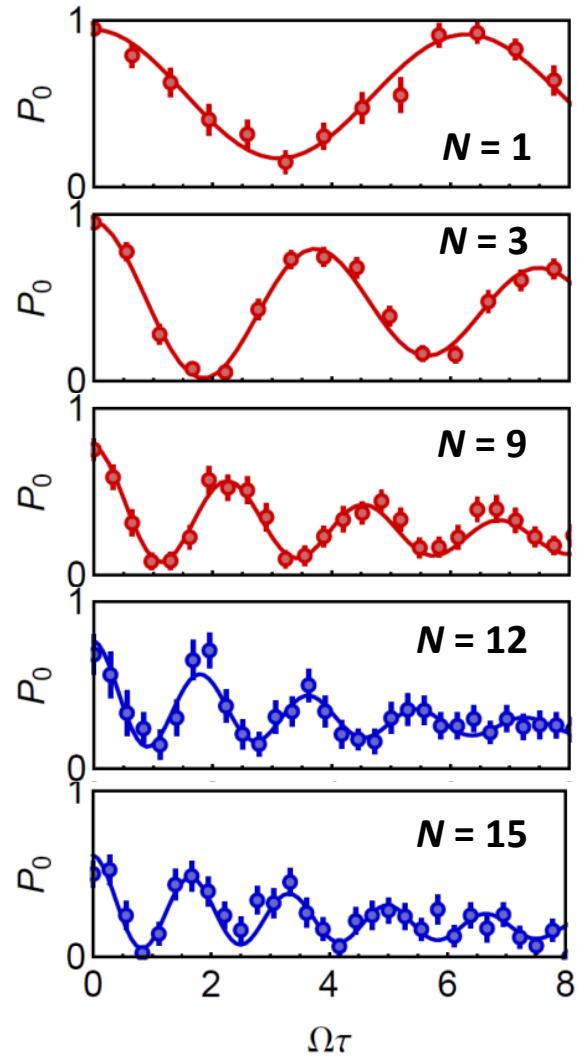
From blockade to many-body physics with “many” atoms



$$\dim H = 2$$

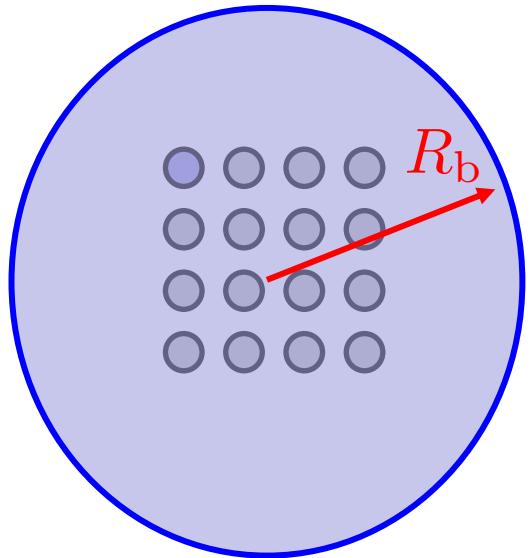
Two (collective) states

$$|ggg\dots\rangle \iff \frac{1}{\sqrt{N}} \sum_i |g\dots r_i g\dots\rangle$$



Labuhn *et al.*, Nature **534**, 667 (2016)
Also: Saffman, Kuzmich, Bloch, Pfau, Ott...

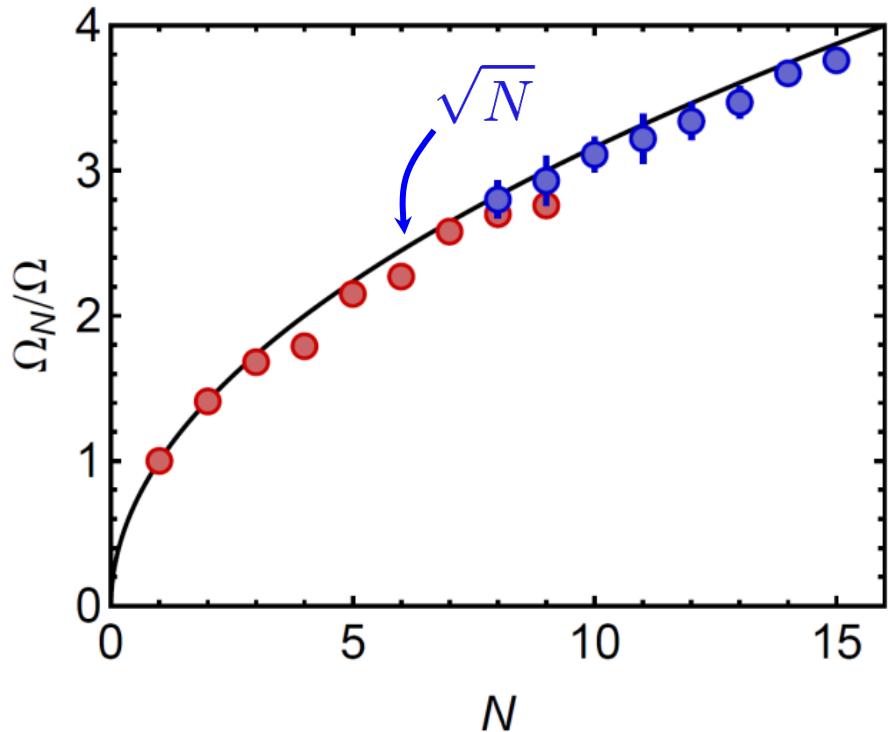
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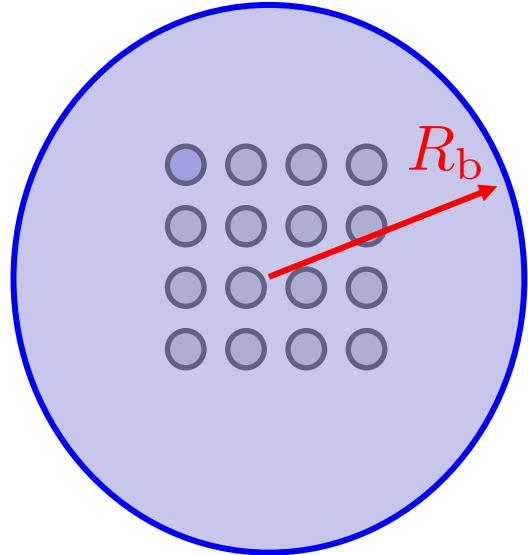
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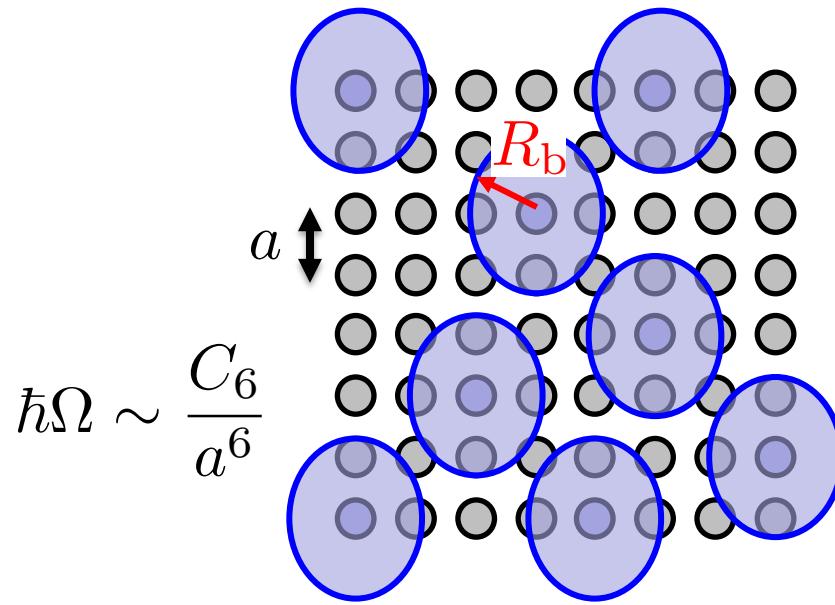


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From blockade to many-body physics with “many” atoms



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$$\dim H \sim 2^N$$

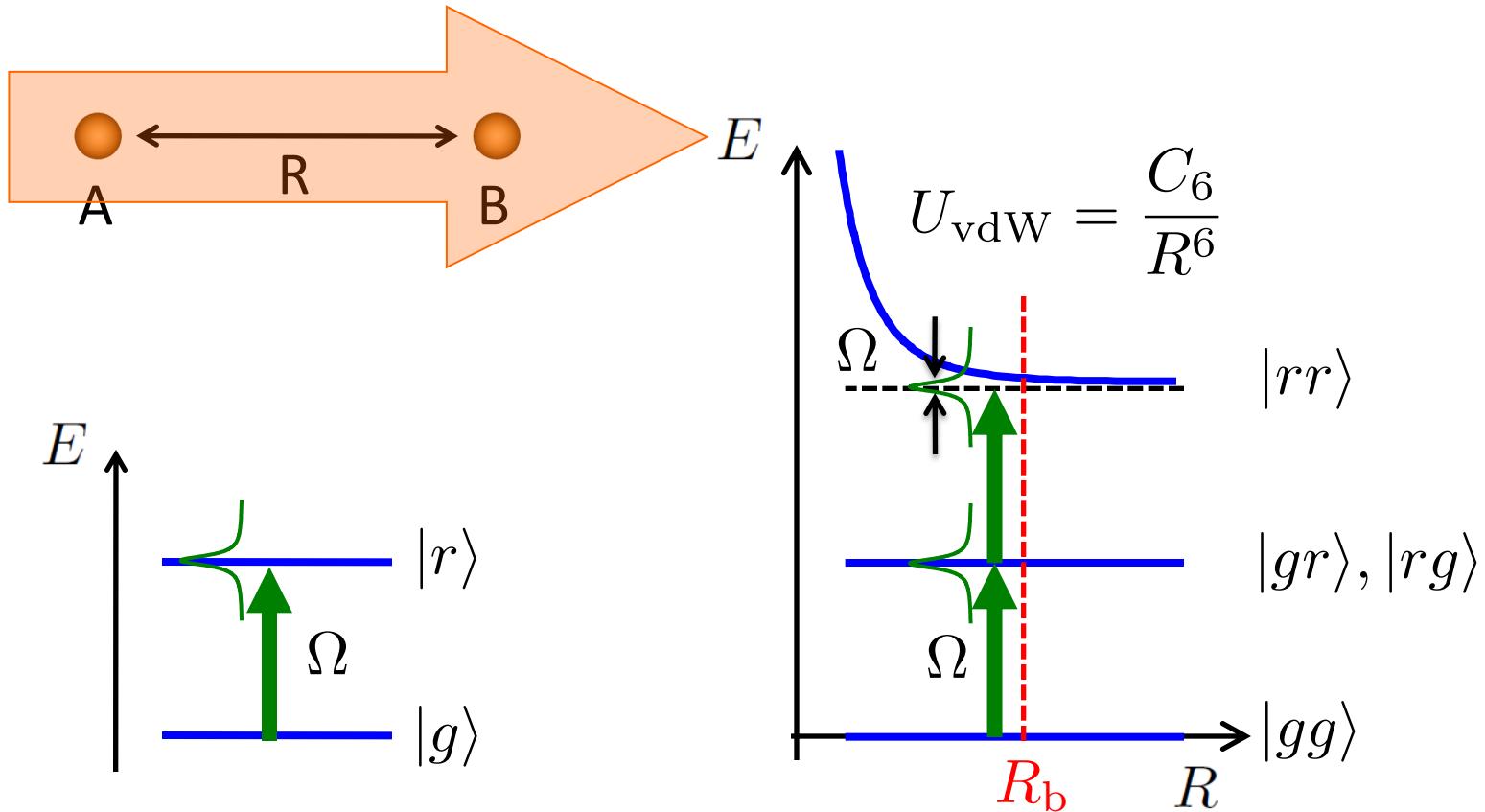
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Strongly correlated many-body system!

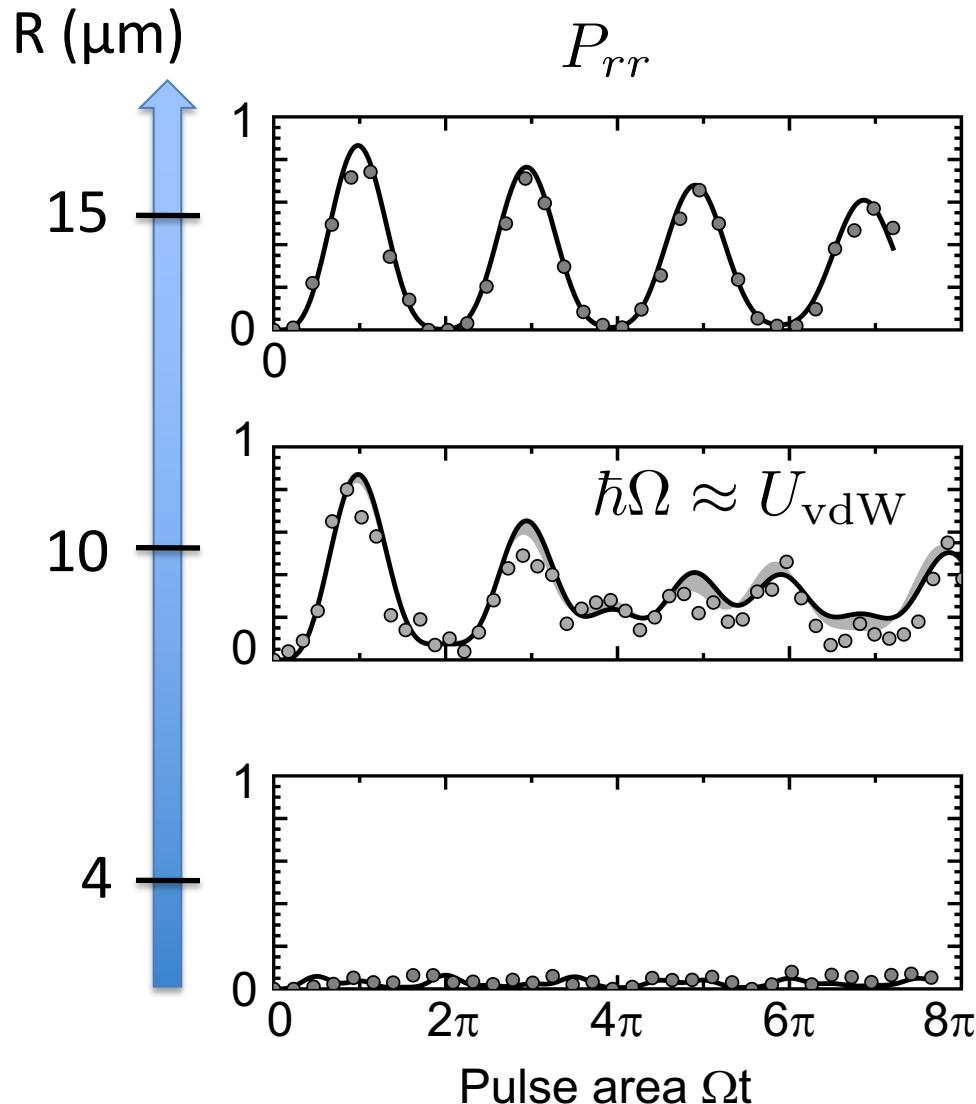
Out-of-equilibrium: sudden variation (“quench”) of parameters
Equilibrium: preparation of ground states

Collective excitation of two interacting Rydberg atoms

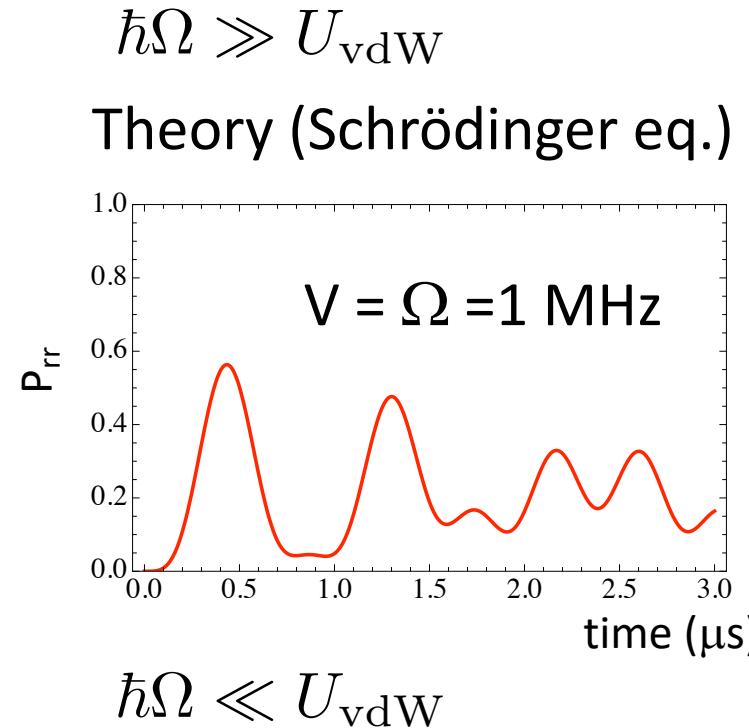


If $\hbar\Omega \approx U_{\text{vdW}}$: dynamics governed by Ω **and** U_{vdW}

From independent atoms to blockade ($62\text{d}_{3/2}$)



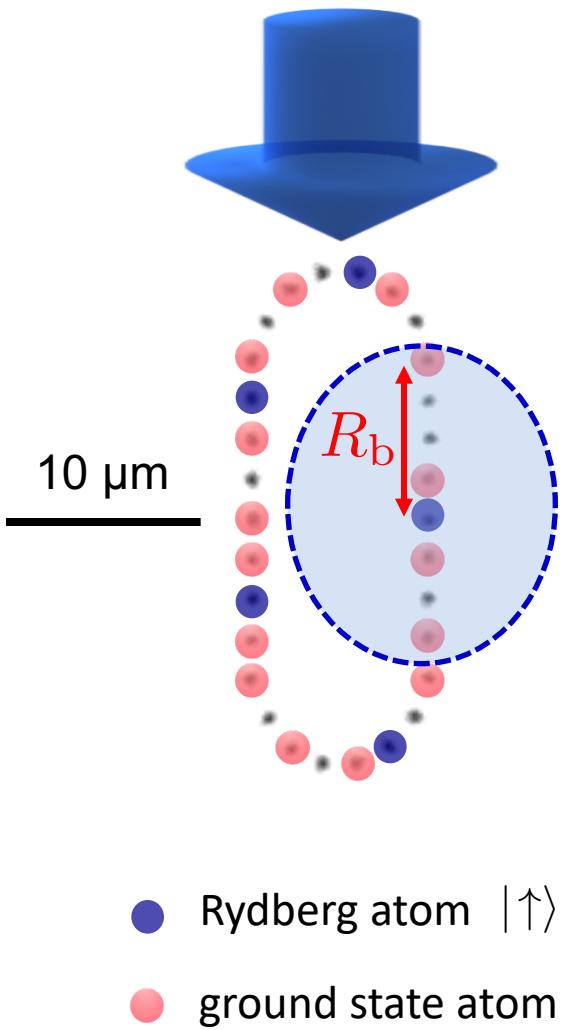
Fit \Rightarrow extract U_{vdW}



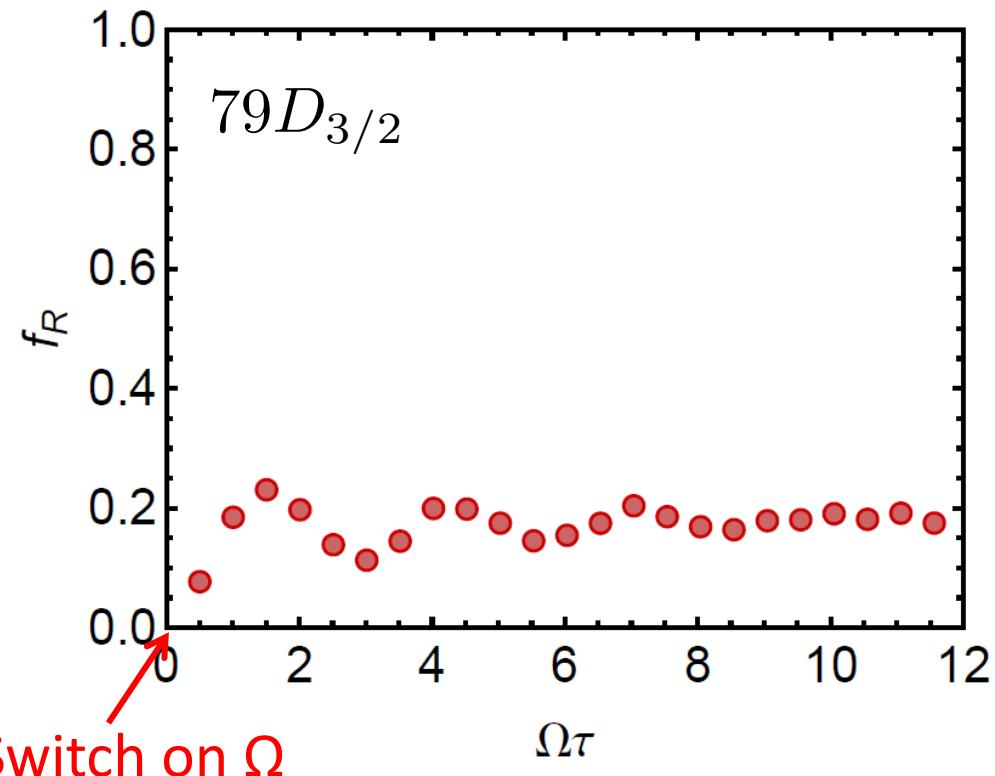
“Quench” in Ising Hamiltonian: 1D with periodic boundary

Labuhn *et al.*, Nature 534, 667 (2016)

Partially loaded 1D ring (30 traps, 20 atoms)



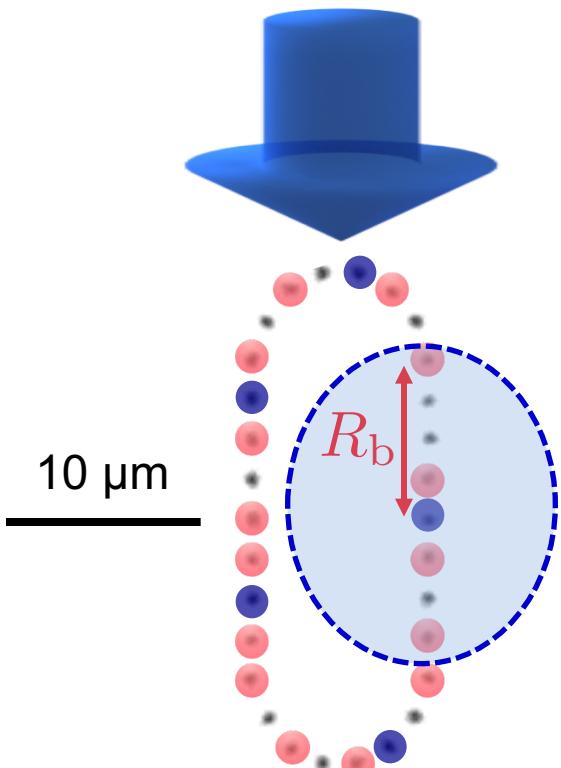
Rydberg fraction: $f_r = \frac{\langle N_r \rangle}{N} \sim \text{magnetization}$



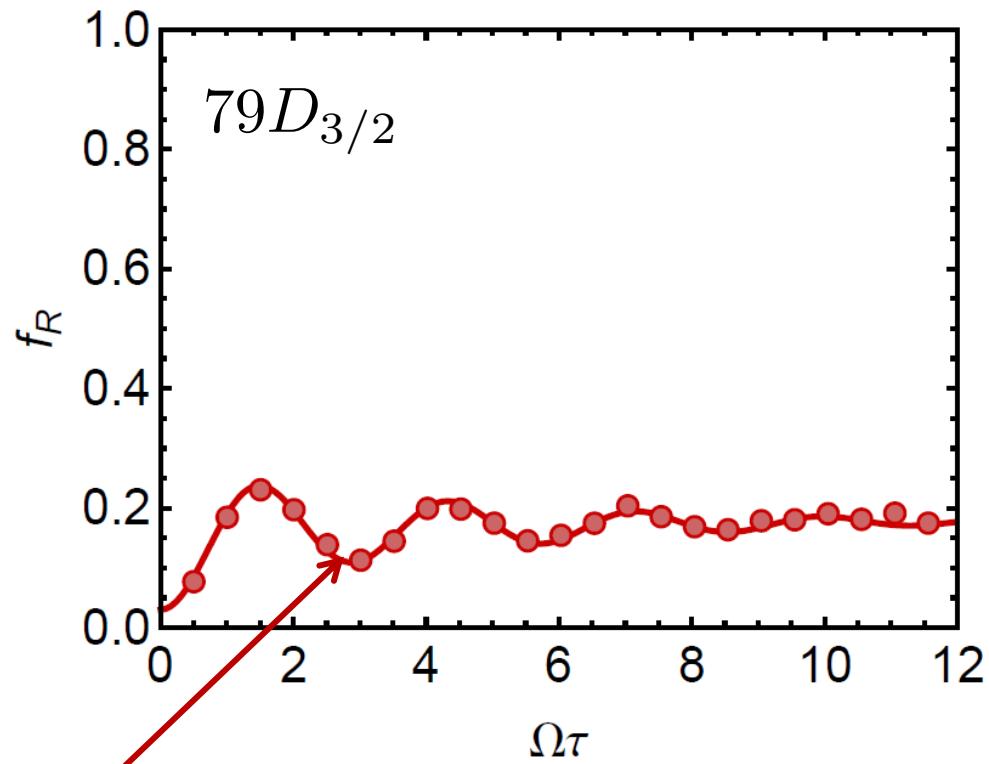
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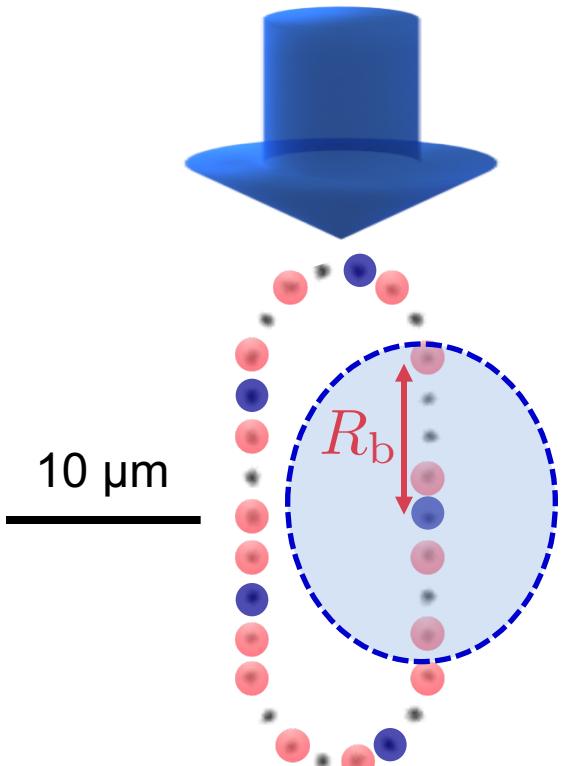


Spin $\frac{1}{2}$ model (Schrödinger), no adjustable parameters...!
Includes detection efficiency (T. Macri)

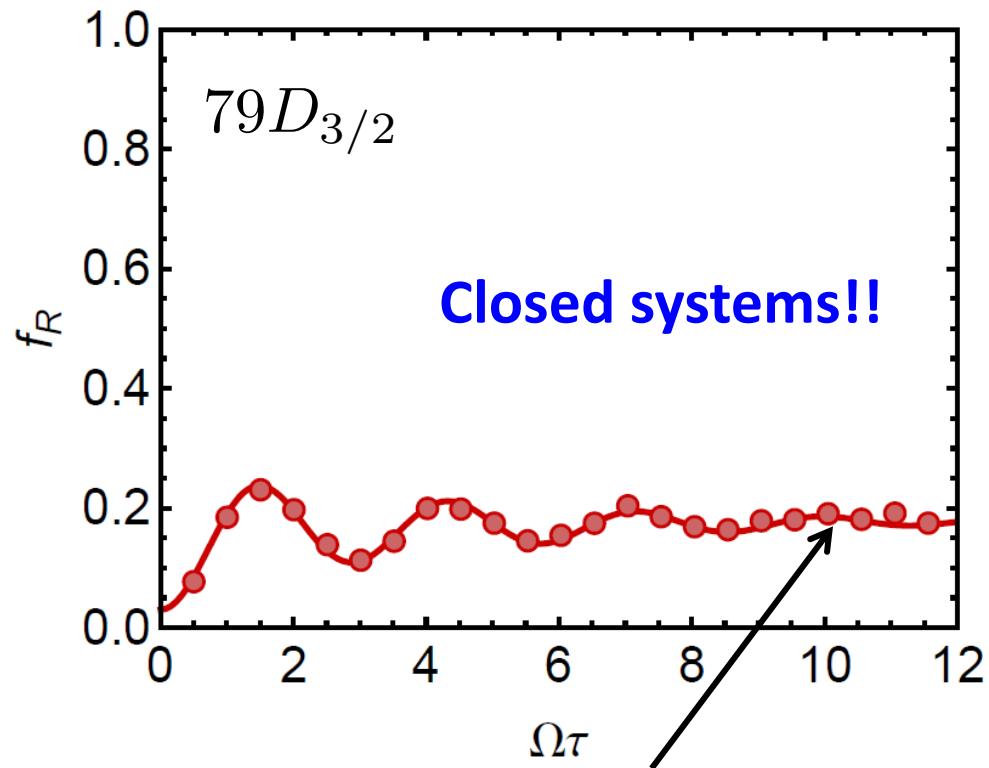
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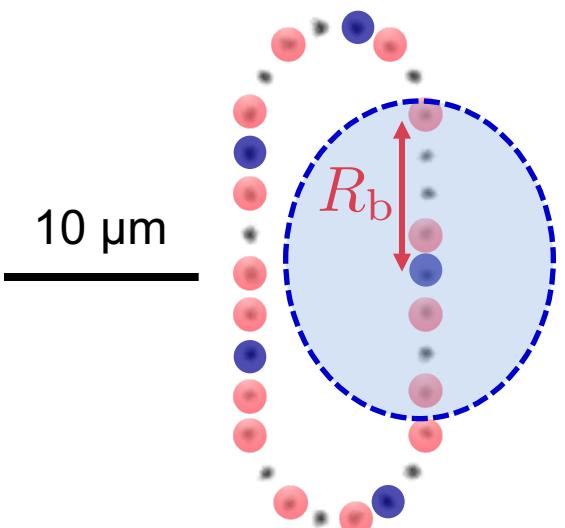


“Damping” = dephasing of 2^{20} eigenenergies

“Quench” in Ising Hamiltonian: 1D with periodic boundary

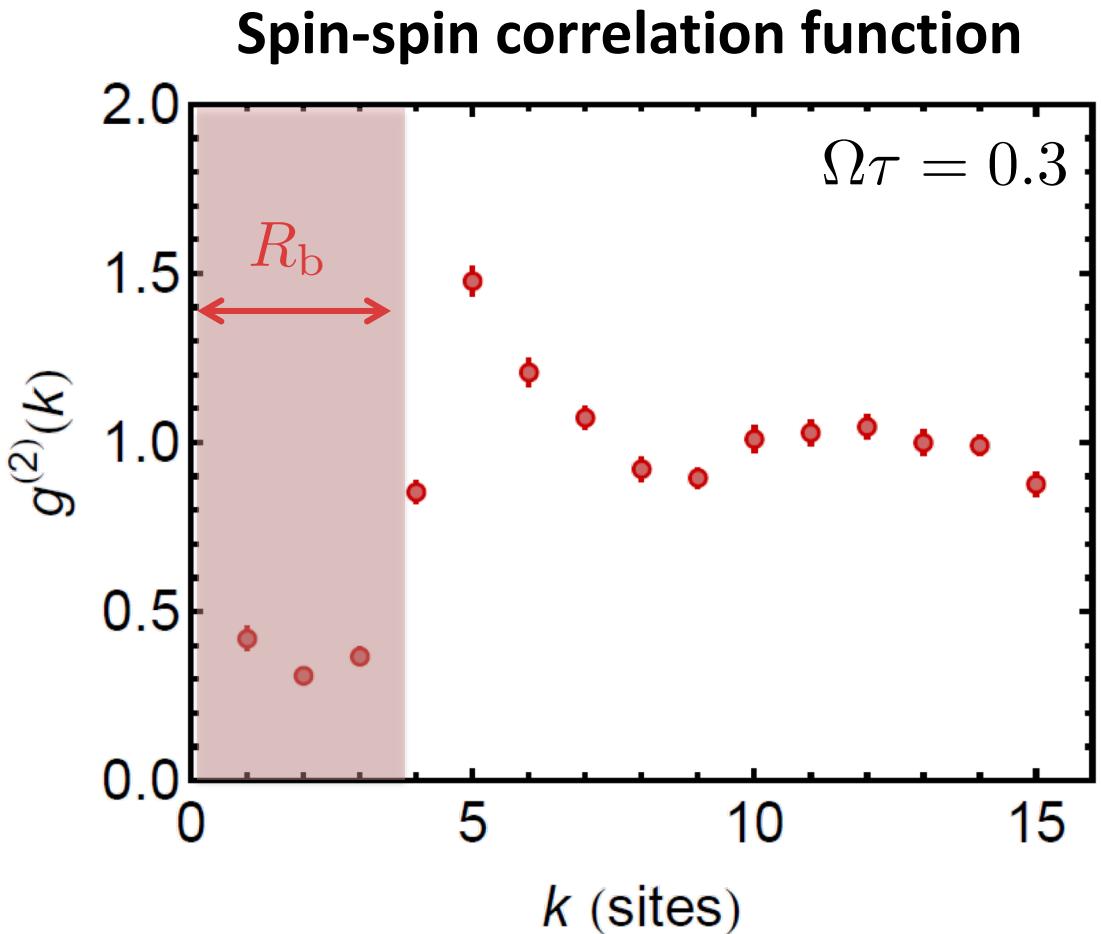
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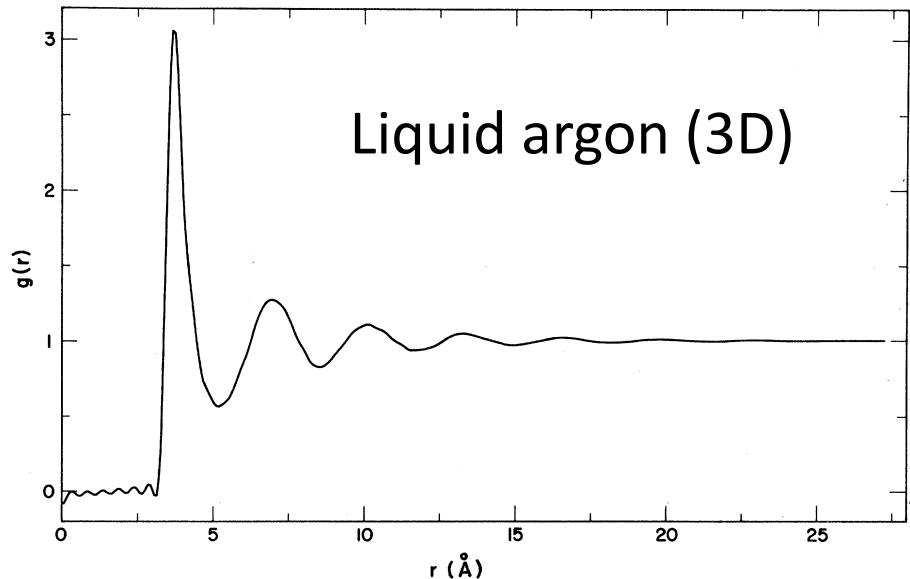


Pair correlation function:

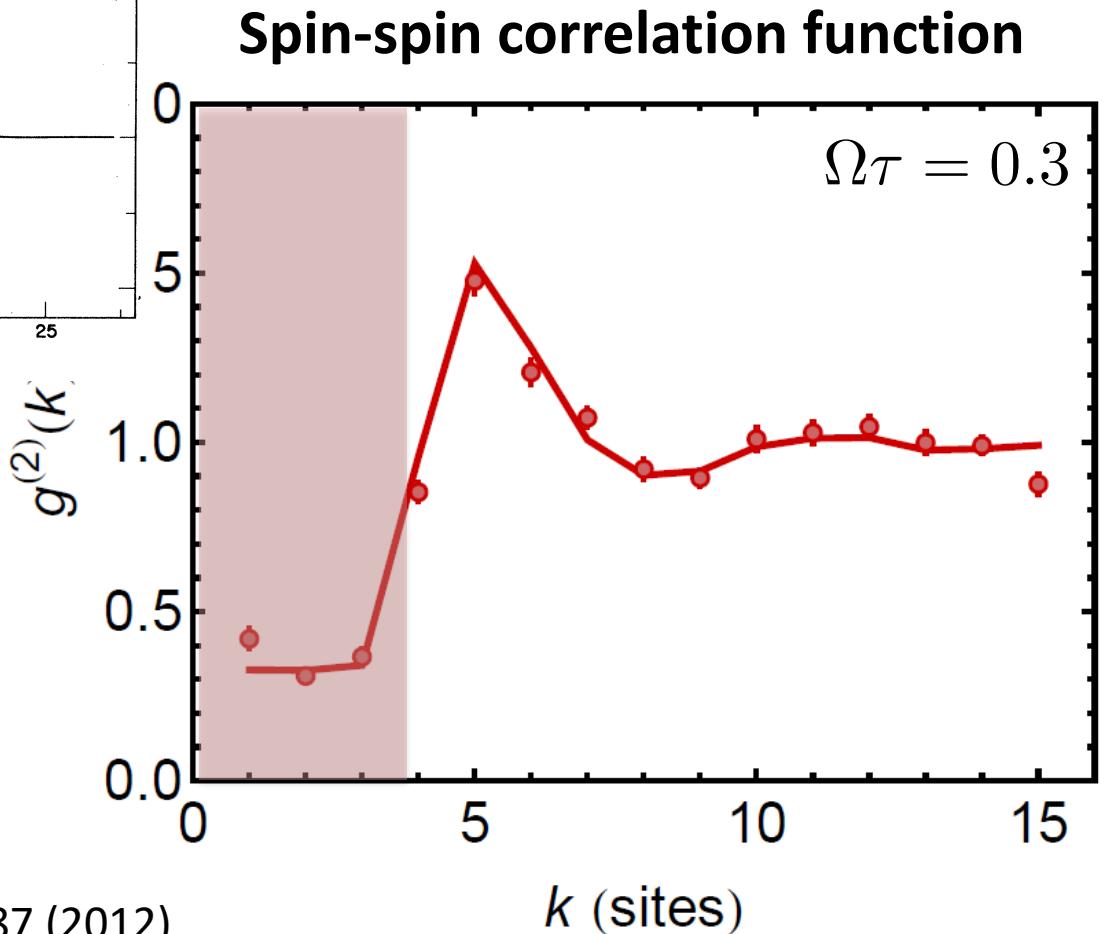
$$g^{(2)}(k) = \frac{1}{N_{\text{tot}}} \sum_i \frac{\langle n_i n_{i+k} \rangle}{\langle n_i \rangle \langle n_{i+k} \rangle}$$



A 1D Ising chain (periodic cond.) = 1D liquid!



Labuhn *et al.*, Nature 534, 667 (2016)

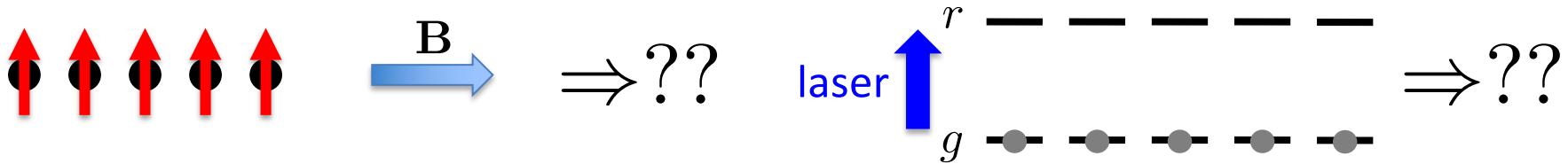


Analogous to 1D liquid:

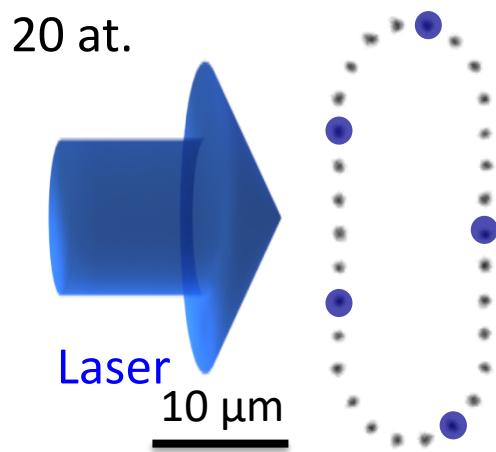
hard sphere R_b

Also in Munich (2D), Nature 491, 87 (2012)
Ates & Lesanovsky, PRA 86, 013408 (2012)
Petrosyan, PRA 87 053414 (2013)

Quench in Ising-like Hamiltonian

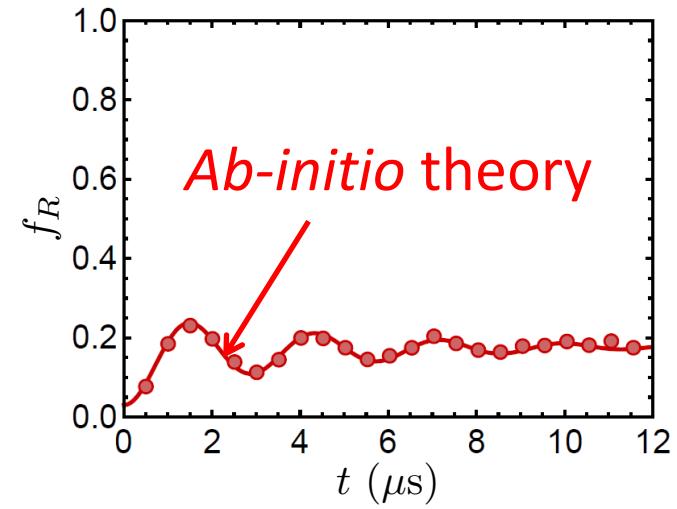


Calculable ($N < 40$) \Rightarrow test

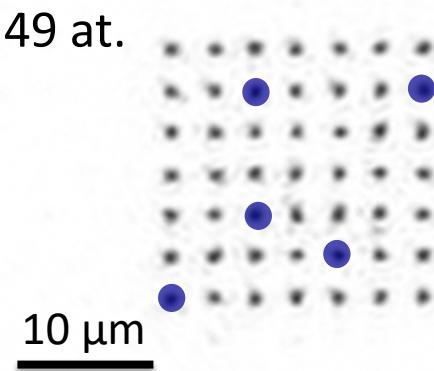


“Magnetization”

$$f_r = \frac{\langle N_r \rangle}{N}$$

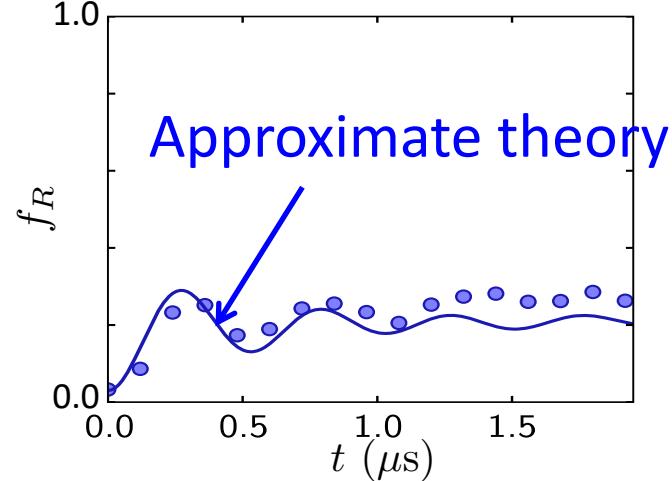


Non-calculable ($N > 40$)



Comparison exp / theo

Validate hypothesis theo
& experiments





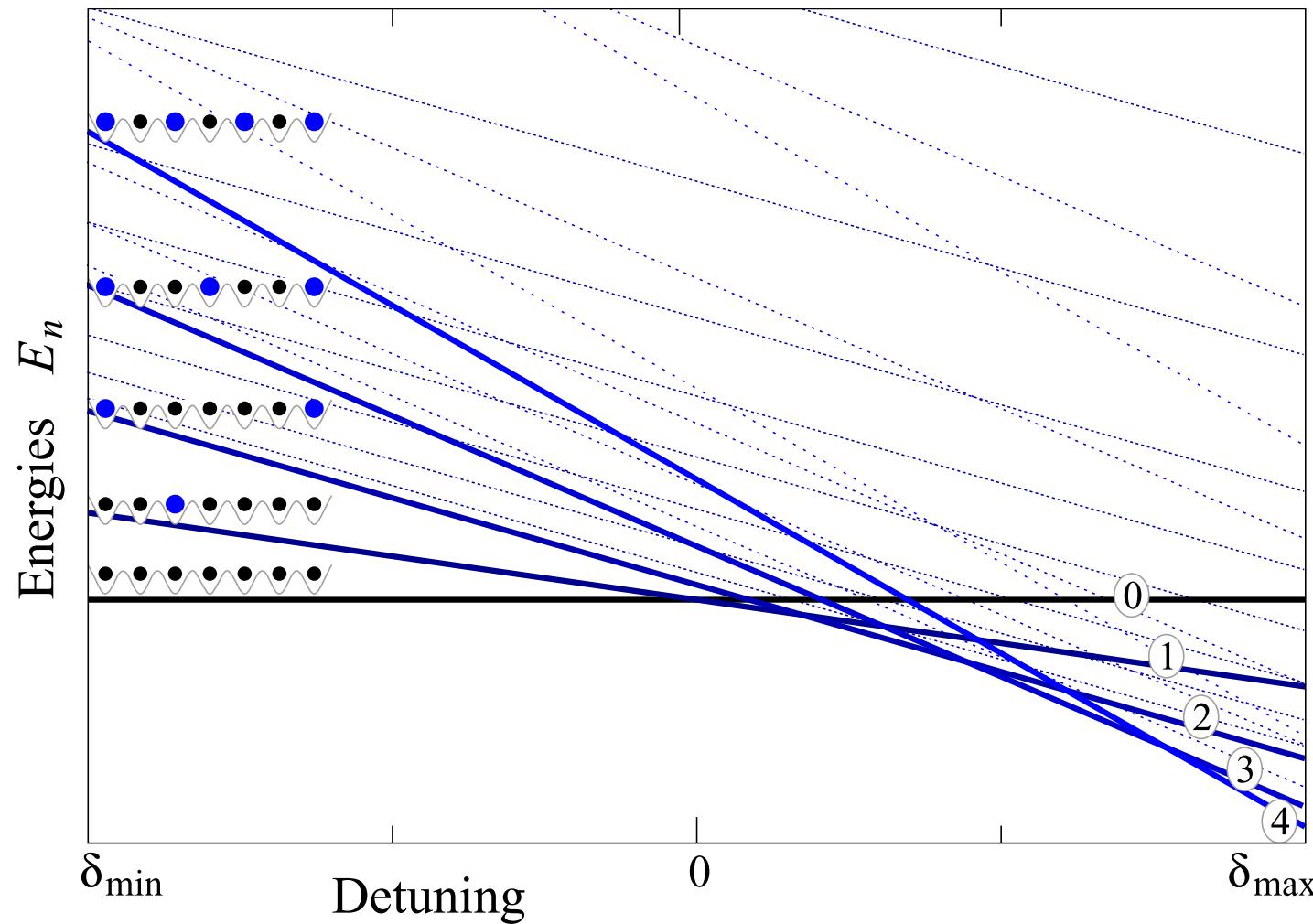
Adiabatic preparation of an Ising antiferromagnet

V. Lienhard, PRX 8, 021070 (2018)

$$H = \sum_i \left(\frac{\hbar\Omega(t)}{2} \sigma_x^i - \hbar\delta(t) \hat{n}_i \right) + \sum_{i < j} \frac{C_6}{R_{ij}^6} \hat{n}_i \hat{n}_j$$

Vary slowly Rabi freq. and detuning

Adiabatic preparation of spatially-ordered 1D Rydberg chains



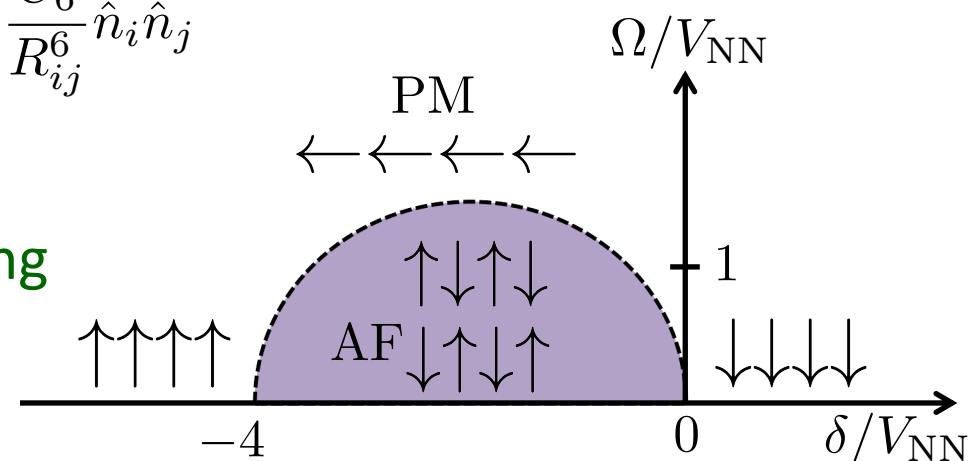


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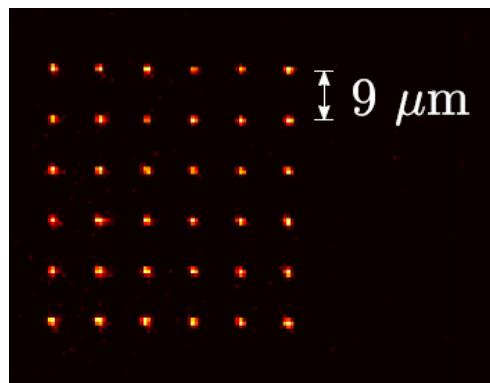
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Vary slowly Rabi freq. and detuning



Nearest nearest neighbor

$N = 36, R_b \sim a$



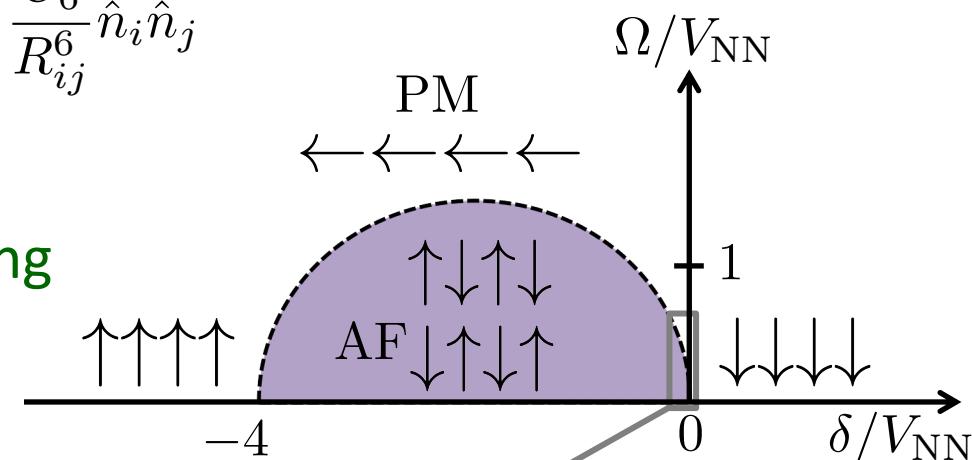


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V. Lienhard, PRX 8, 021070 (2018)

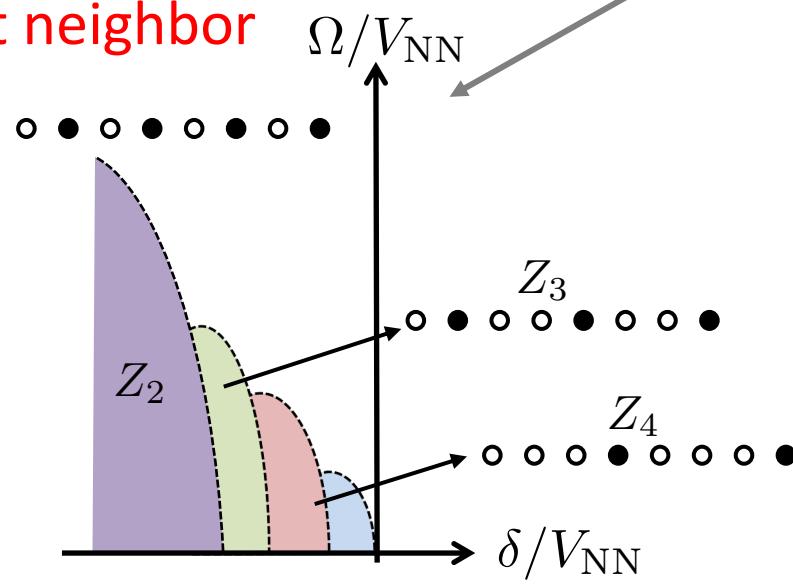
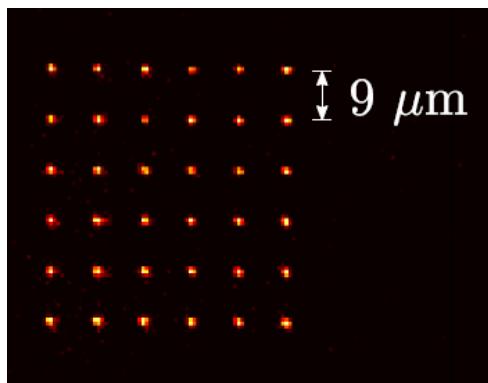
$$H = \sum_i \left(\frac{\hbar\Omega(t)}{2} \sigma_x^i - \hbar\delta(t) \hat{n}_i \right) + \sum_{i < j} \frac{C_6}{R_{ij}^6} \hat{n}_i \hat{n}_j$$

Vary slowly Rabi freq. and detuning



Beyond nearest neighbor

$N = 36, R_b \sim a$



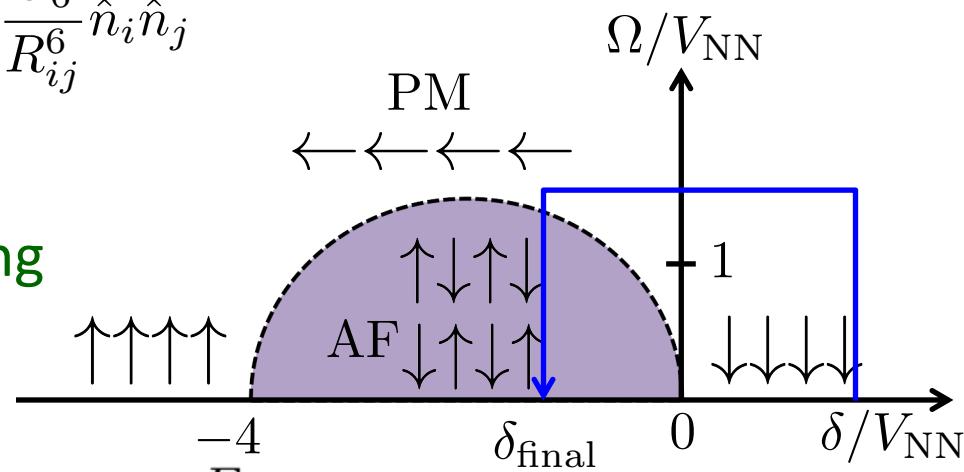


Adiabatic preparation of an Ising antiferromagnet

V. Lienhard, PRX 8, 021070 (2018)

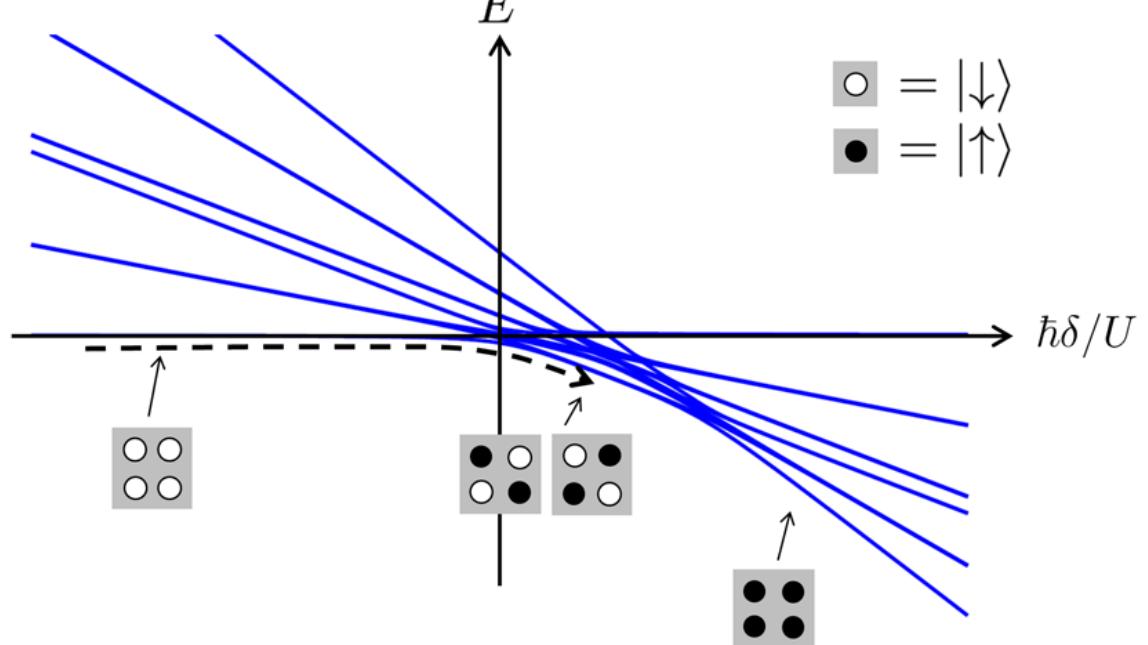
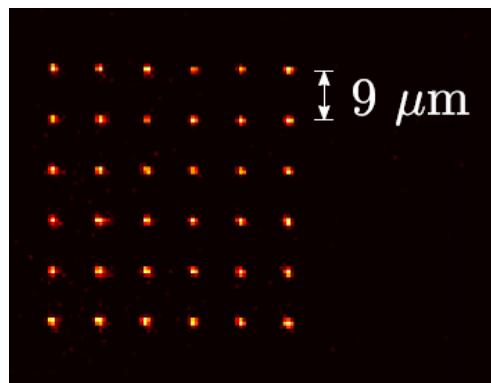
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Vary slowly Rabi freq. and detuning



$$C_6 > 0$$

$$N = 36, R_b \sim a$$



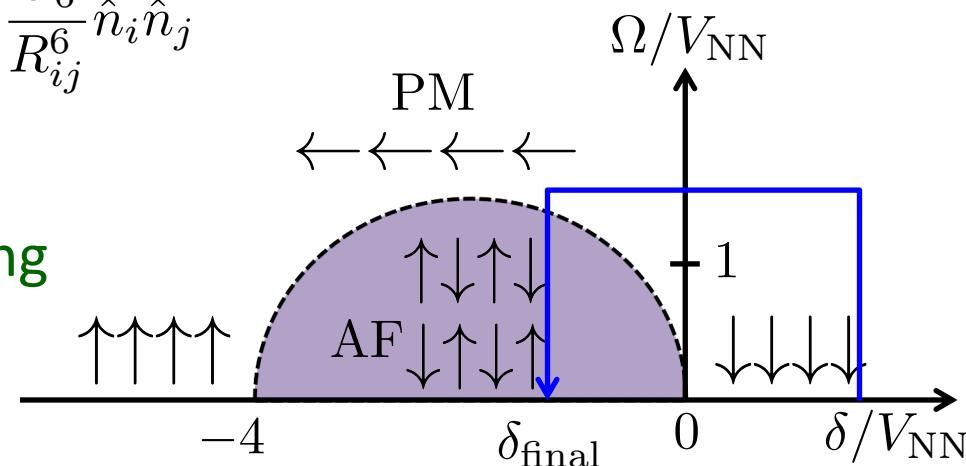


Adiabatic preparation of an Ising antiferromagnet

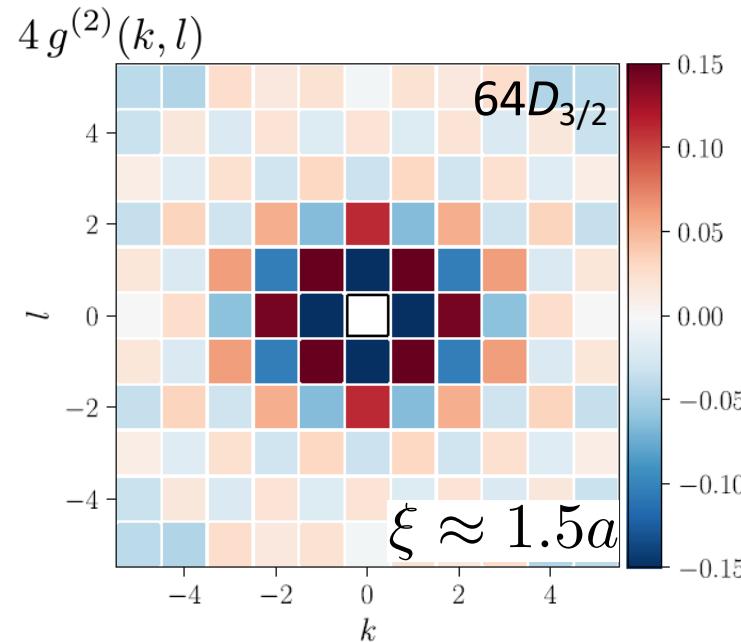
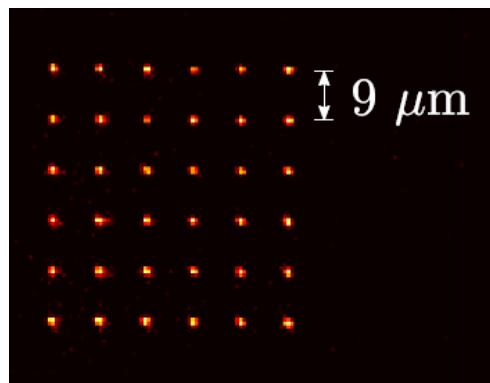
V. Lienhard, PRX 8, 021070 (2018)

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Vary slowly Rabi freq. and detuning



$$N = 36, R_b \sim a$$



Onset of AF correlations

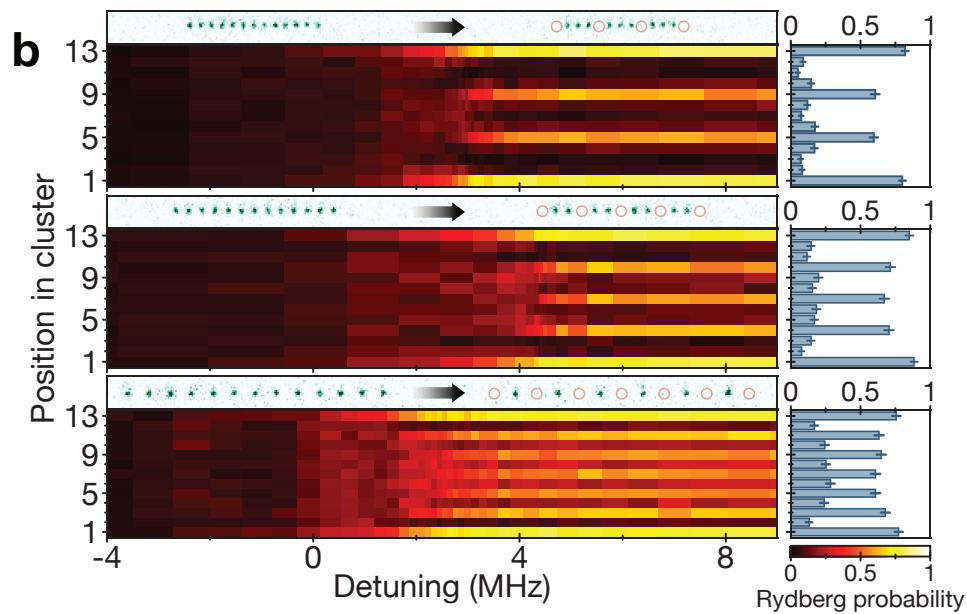
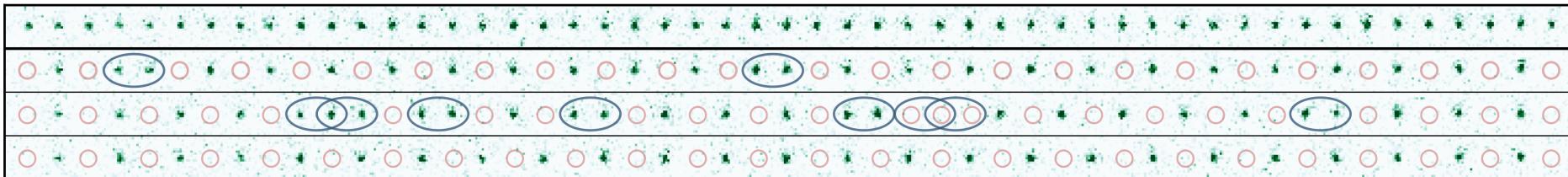
Study growth with x, t

Also: Bakr, PRX 8, 021069 (2018)

A 51-atom “quantum simulator” (Harvard-MIT)

$$R_b \sim a$$

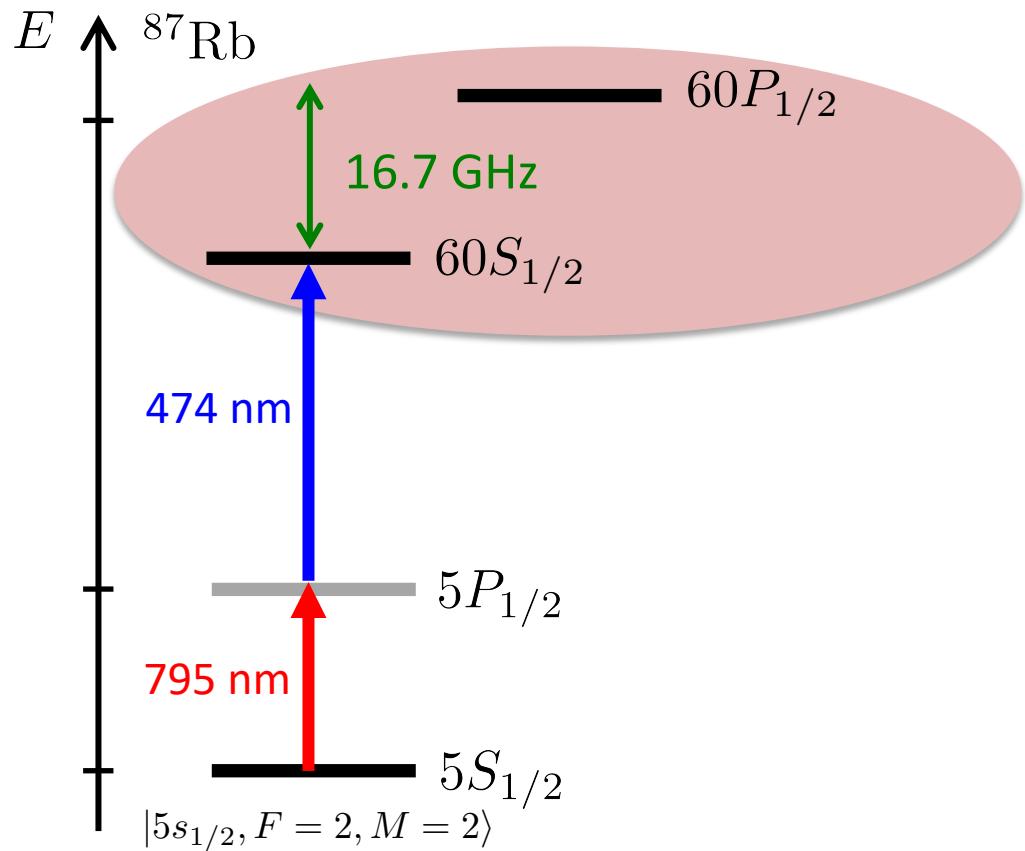
H. Bernien, Nature (2017)



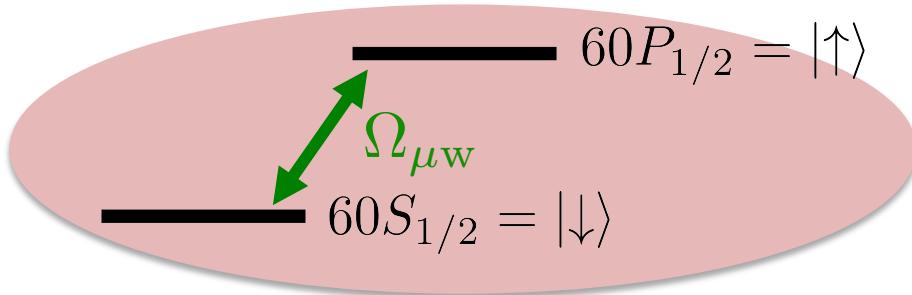
Outline

1. Quantum simulation and spin models
2. Simulation of Ising model using van der Waals interactions
3. Topological systems using resonant dipole interactions

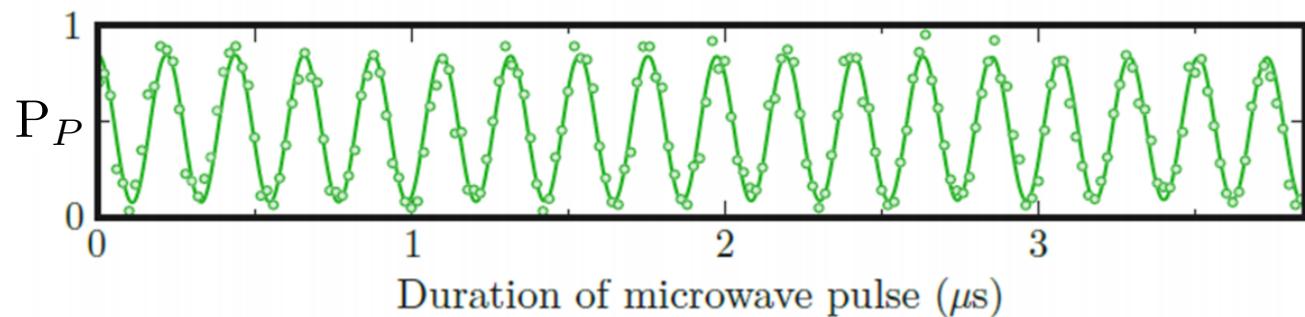
Resonant dipole-dipole interaction between Rydberg atoms



Resonant dipole-dipole interaction between Rydberg atoms

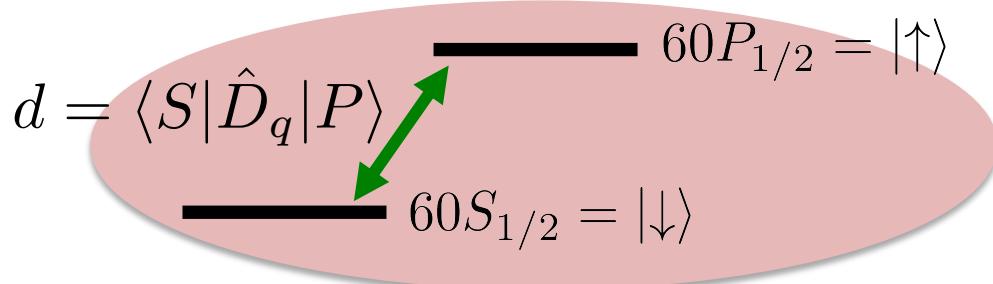


Mapping on
spin $\frac{1}{2}$ system

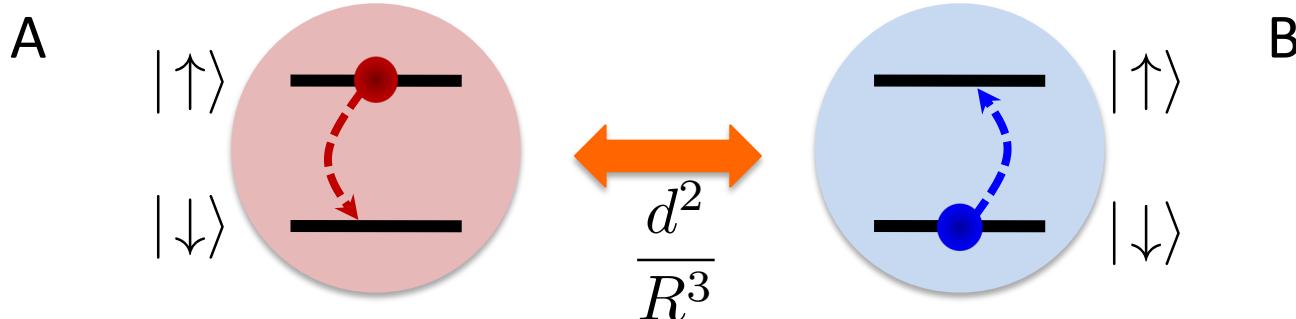


Barredo, PRL **114**, 113002 (2015)

Resonant dipole-dipole interaction between Rydberg atoms



Mapping on
spin $\frac{1}{2}$ system



$$\hat{H} = \frac{d^2}{4\pi\epsilon_0 R^3} (\hat{\sigma}_A^+ \hat{\sigma}_B^- + \hat{\sigma}_A^- \hat{\sigma}_B^+)$$

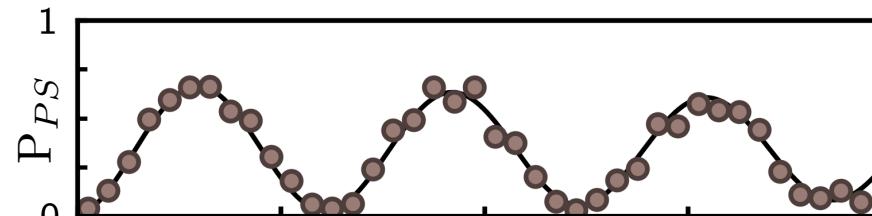
“exchange” of excitation

Observation of resonant dip.-dip. interaction with 2 atoms

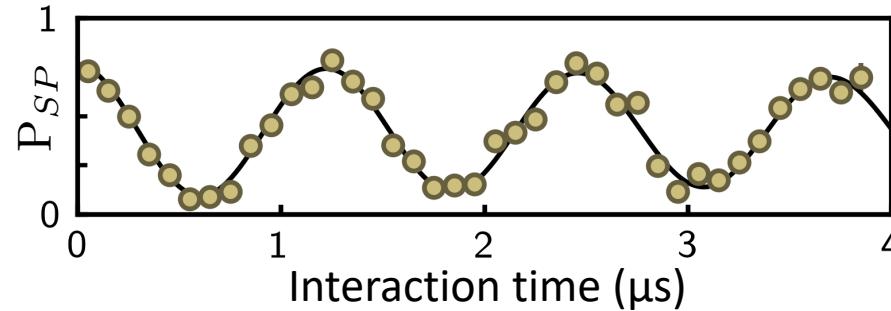
Prepare $|\uparrow\downarrow\rangle$ using microwaves + addressing beam

$$R = 30 \text{ } \mu\text{m}$$

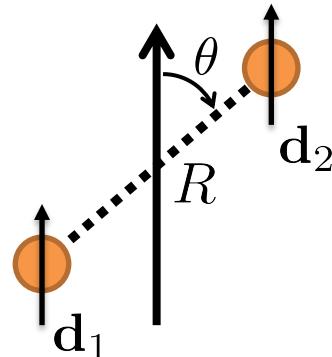
$$\text{Frequency: } \frac{2C_3}{R^3}$$



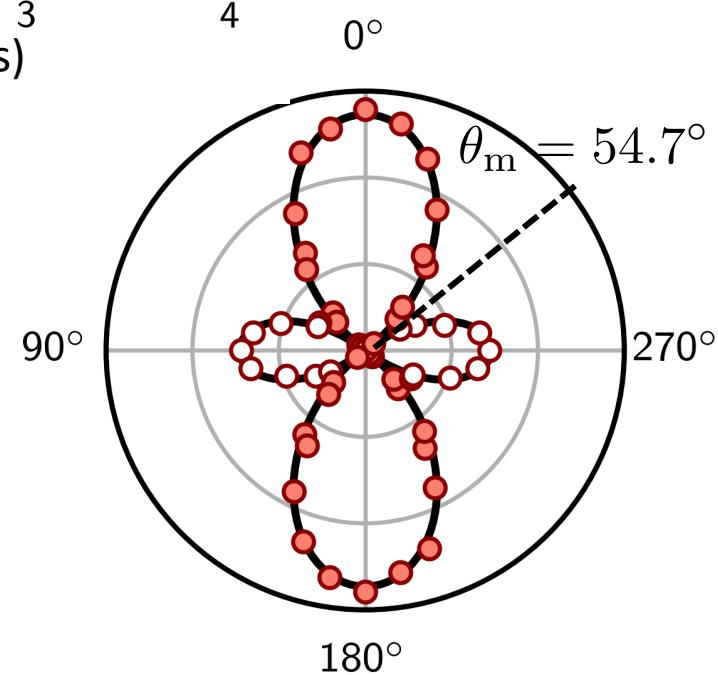
Barredo PRL (2015)
de Léséleuc, PRL (2017)



Quantization
axis (B)



$$C_3(\theta) \propto 1 - 3 \cos^2 \theta$$

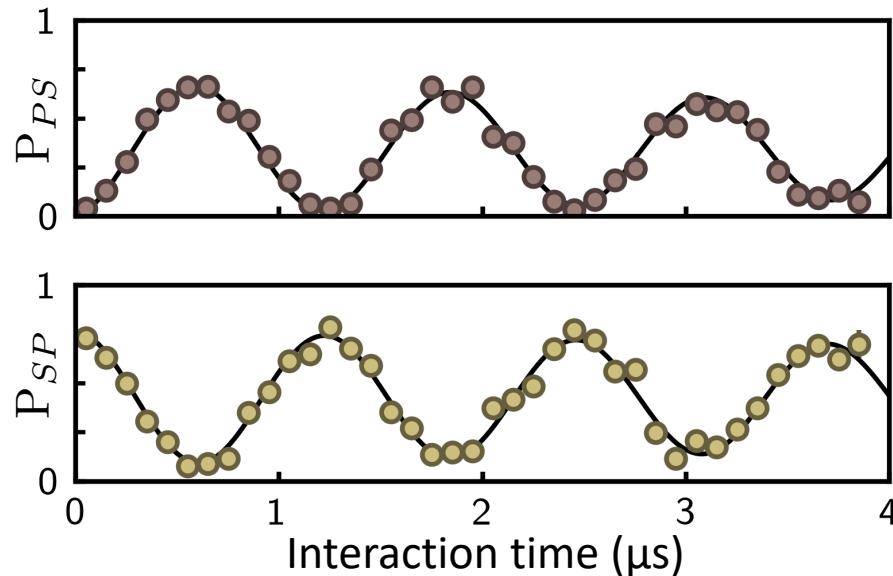


Observation of resonant dip.-dip. interaction with 2 atoms

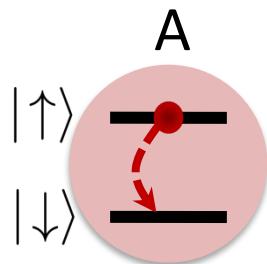
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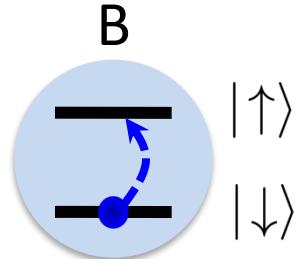
$$\text{Frequency: } \frac{2C_3}{R^3}$$



Barredo PRL (2015)
de Léséleuc, PRL (2017)



$$C_3/R^3$$



Spin excitation exchange



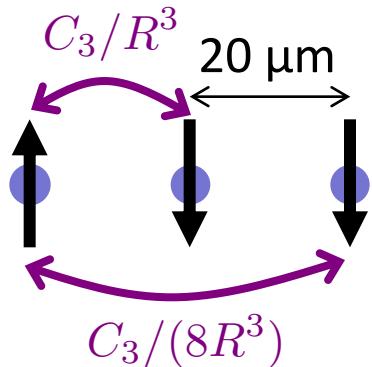
Particle hopping

$$J \hat{\sigma}_A^+ \hat{\sigma}_B^-$$

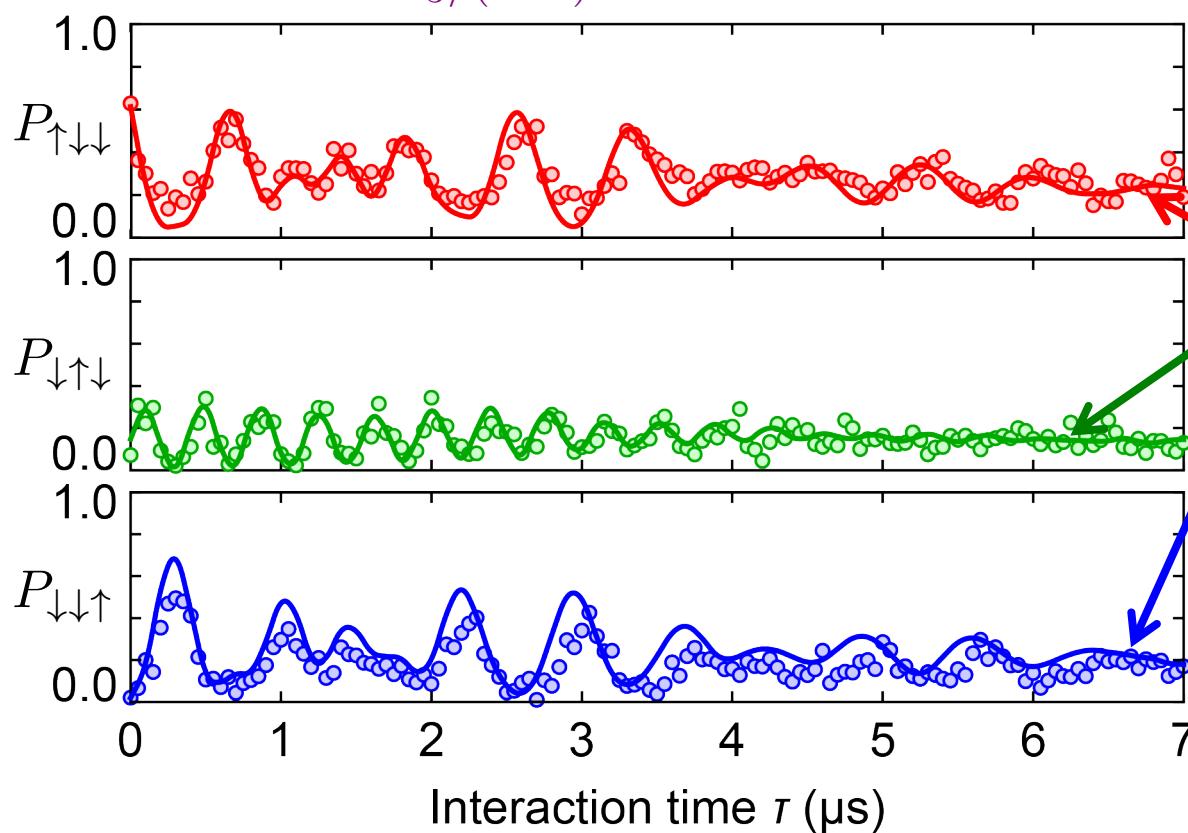
$$J|A\rangle\langle B|$$

Observation of spin exchange in a 3-atom chain

D. Barredo *et al.*, PRL **114**, 113002 (2015)



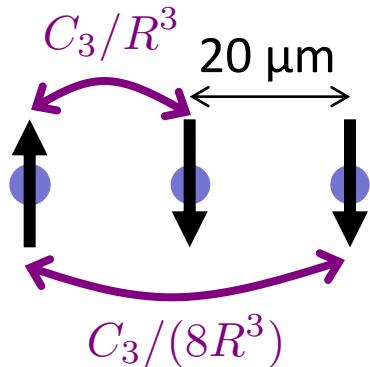
Prepare $|\uparrow\downarrow\downarrow\rangle$ at $t = 0$,
and let the system evolve



XY model
(Schrödinger)
+ imperfections

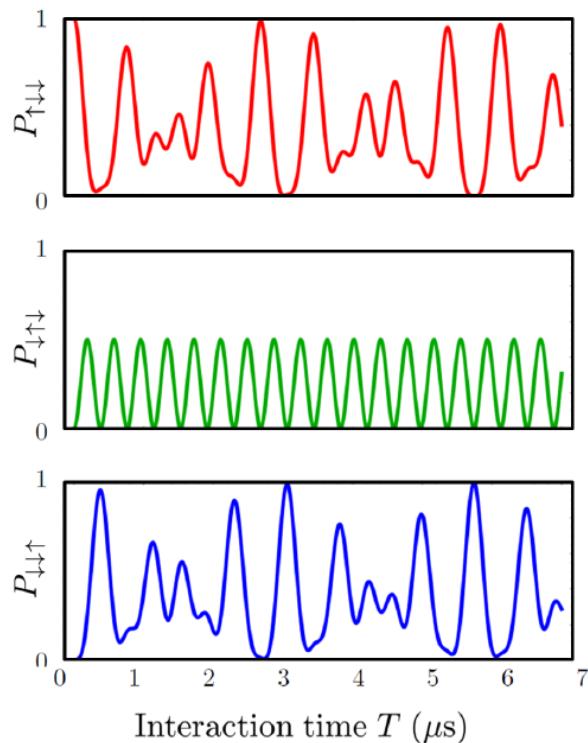
No adjustable
parameter

Three-atom “spin-chain”: what to expect (theory) ?



Prepare $|\uparrow\downarrow\downarrow\rangle$ at $t = 0$,
and let the system evolve

$1/R^3$ interaction

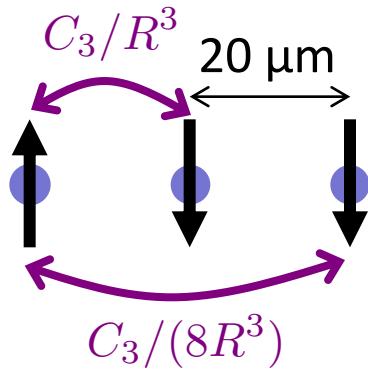


2 off-diagonal couplings: V & $V/8$

⇒ eigenvalues (incommensurate):

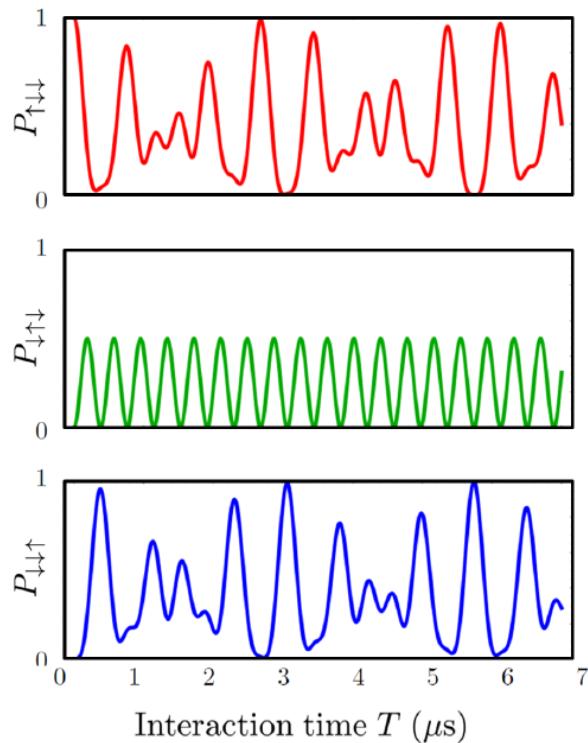
$$\frac{V}{16} \left(1 + 3\sqrt{57} \right), \quad \frac{V}{16} \left(1 - 3\sqrt{57} \right), \quad -\frac{V}{8}$$

Three-atom “spin-chain”: what to expect (theory) ?

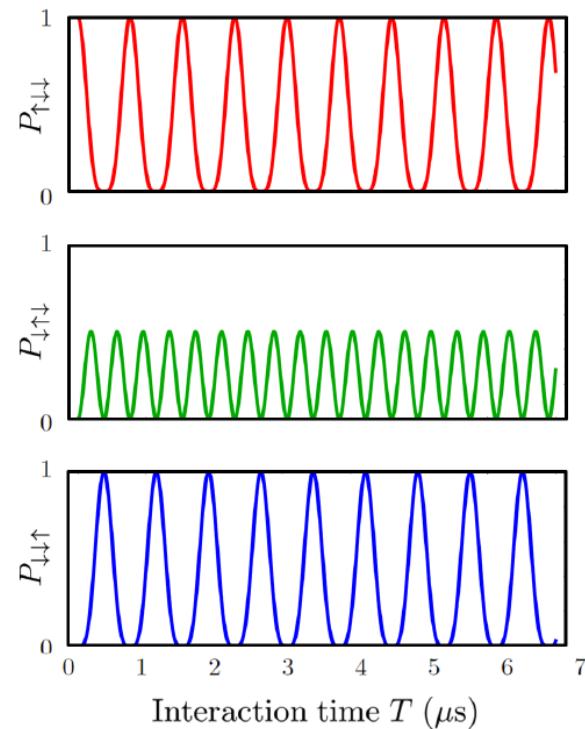


Prepare $|\uparrow\downarrow\downarrow\rangle$ at $t = 0$,
and let the system evolve

1/R³ interaction



Nearest-neighbor only



The Su-Schrieffer-Heeger model

- Introduced to explain conductivity in polymers

VOLUME 42, NUMBER 25

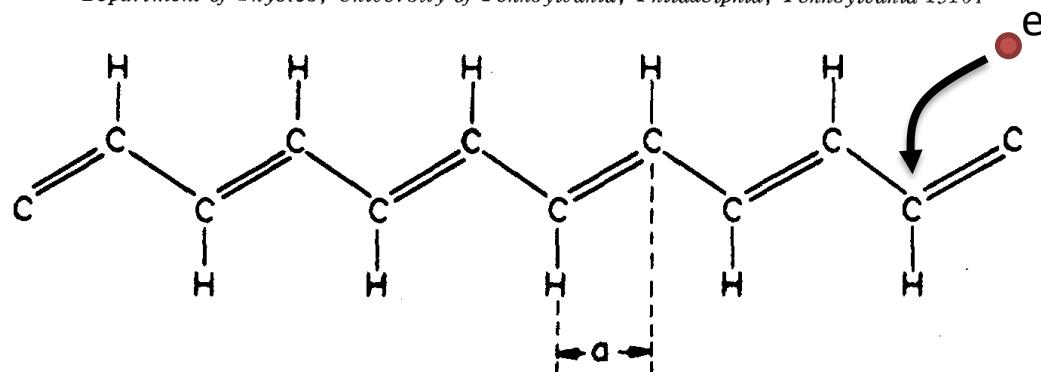
PHYSICAL REVIEW LETTERS

18 JUNE 1979

Solitons in Polyacetylene

W. P. Su, J. R. Schrieffer, and A. J. Heeger

Department of Physics, University of Pennsylvania, Philadelphia, Pennsylvania 19104



- Now, considered as simplest example of topological model

Asboth, [arXiv:1509.02295](https://arxiv.org/abs/1509.02295), Cooper, [arXiv:1803.00249](https://arxiv.org/abs/1803.00249)

- Goal:** build an **artificial** SSH system to explore role

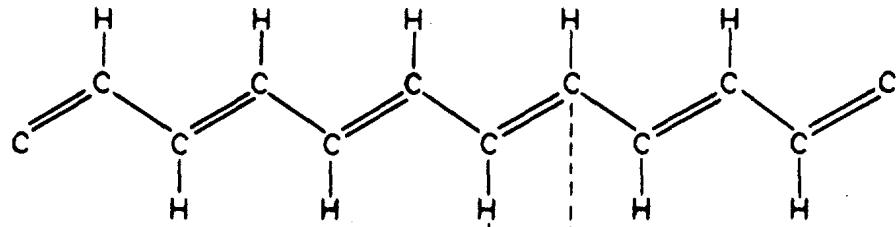
- Symmetries
- Interactions
- ...

H.-P. Büchler
S. Weber, N. Lang

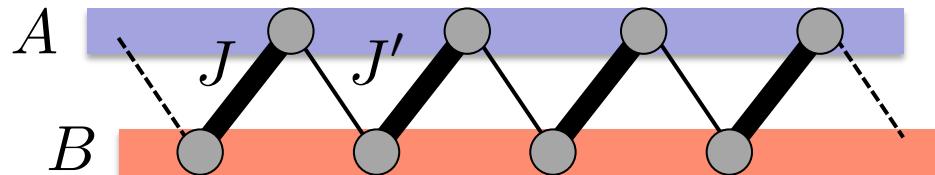
Science 365, 775 (2019)



The Su-Schrieffer-Heeger model: single-particle



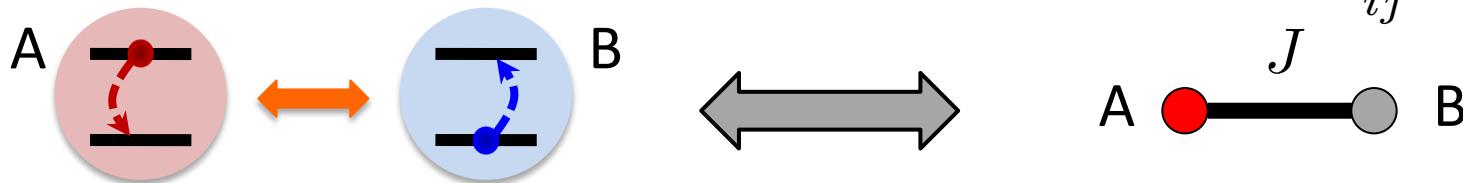
The Su-Schrieffer-Heeger model: single-particle



Model: tight-binding
dimerization: $J > J'$

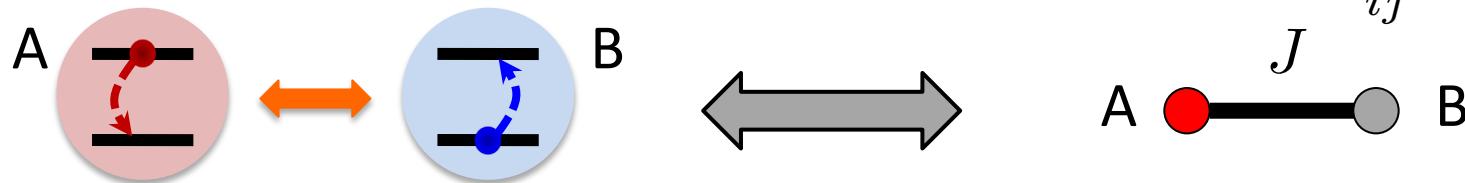
Implementation of SSH spin chain with Rydberg atoms

Couplings J_{ij} : resonant dipole-dipole interaction $\frac{C_3(\theta_{ij})}{R_{ij}^3}$



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Couplings J_{ij} : resonant dipole-dipole interaction $\frac{C_3(\theta_{ij})}{R_{ij}^3}$

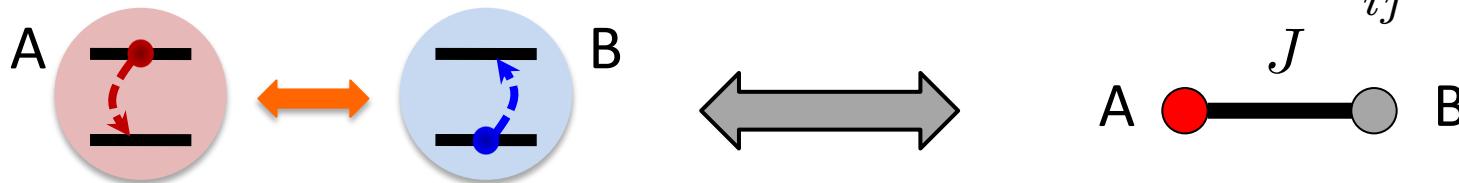


Chain at magic angle \Rightarrow chiral symmetry (no A-A or B-B hopping)

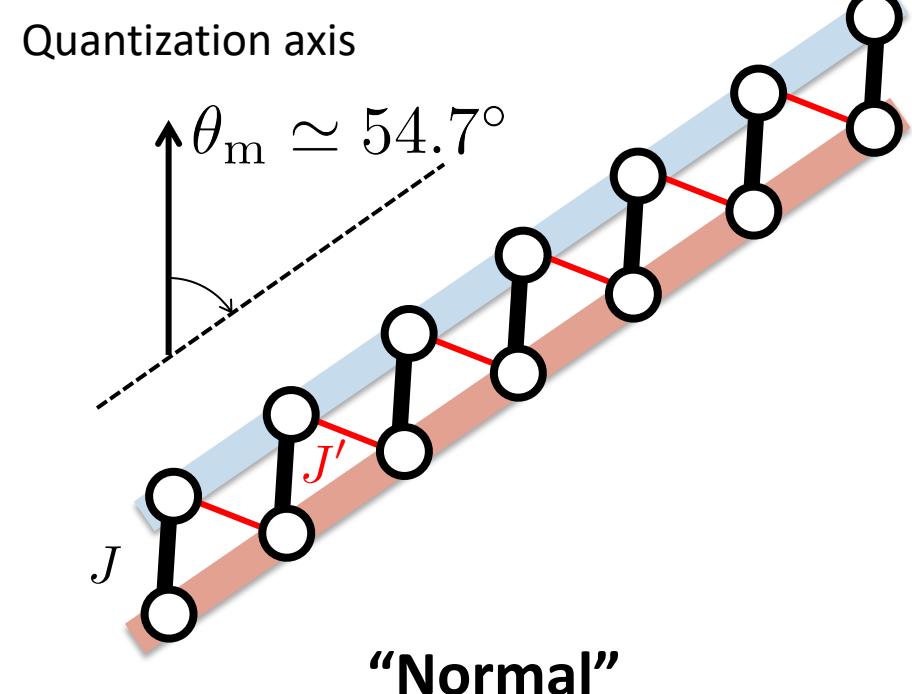
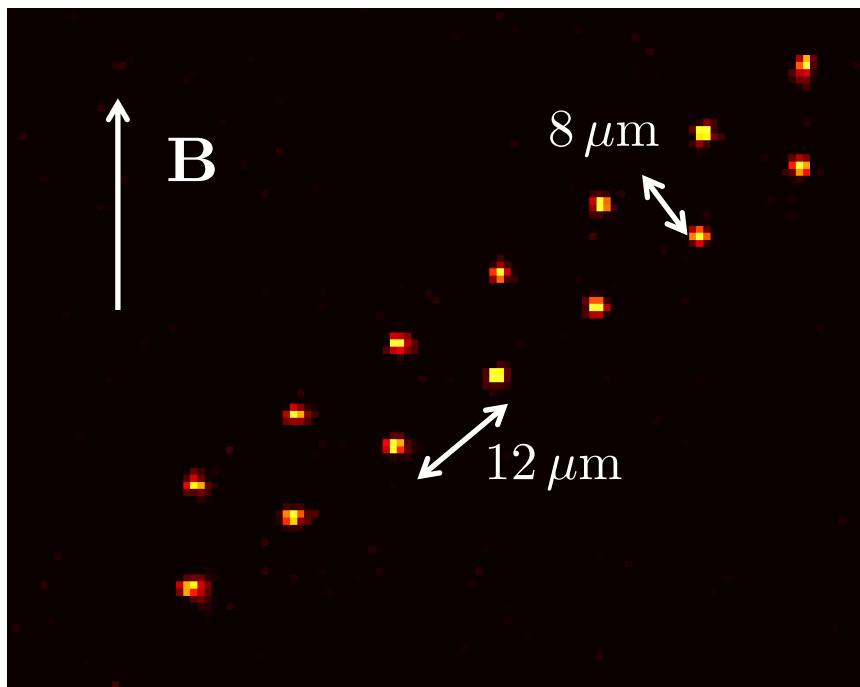
Implementation of SSH spin chain with Rydberg atoms

Couplings J_{ij} : resonant dipole-dipole interaction

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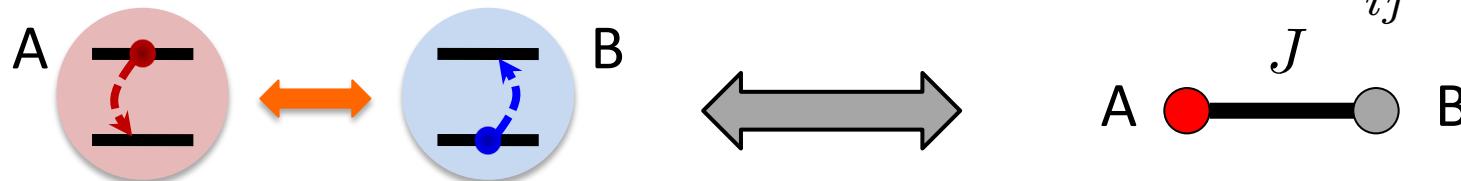


Chain at magic angle \Rightarrow **chiral symmetry** (no A-A or B-B hopping)

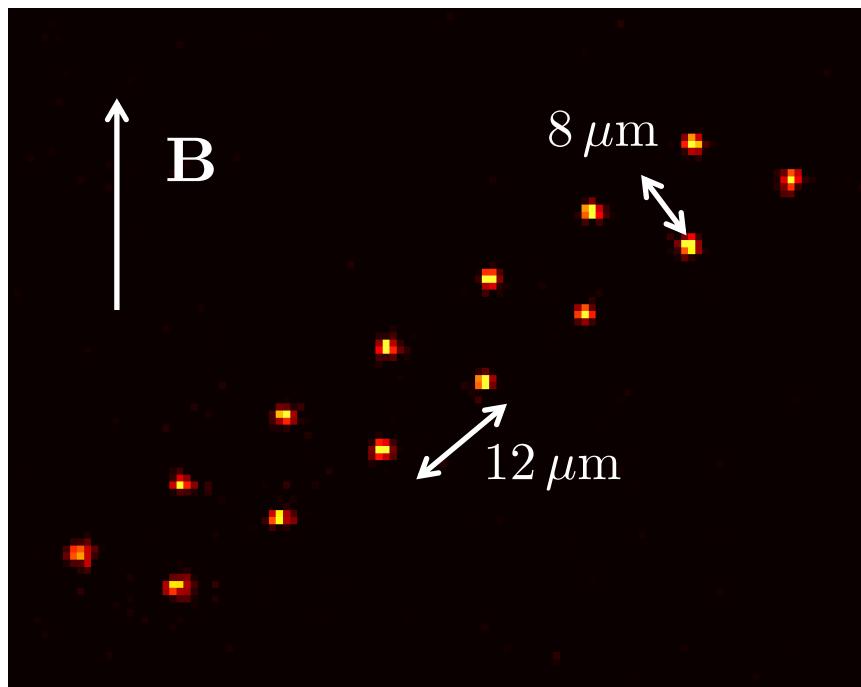


Implementation of SSH spin chain with Rydberg atoms

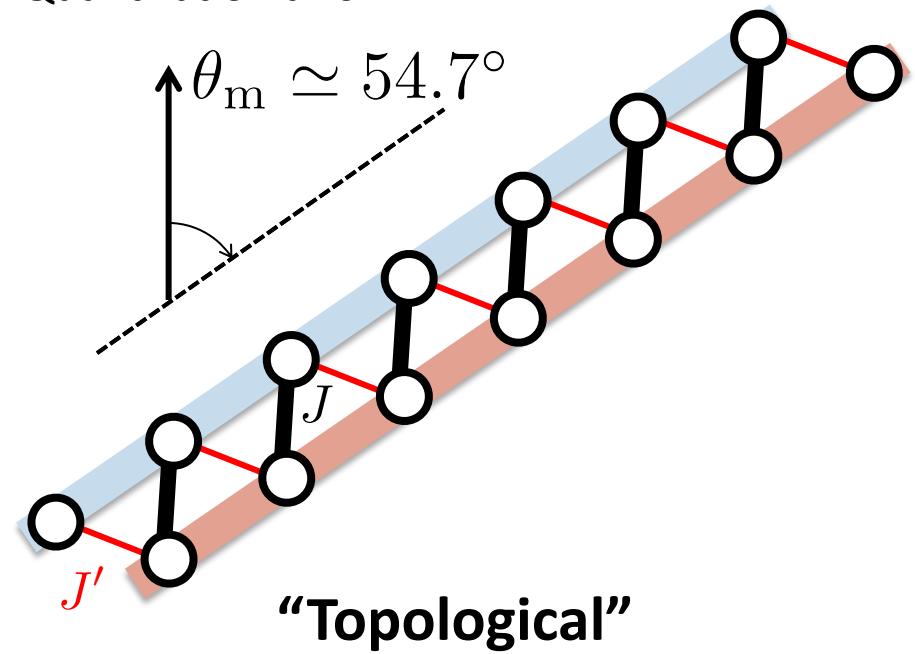
Couplings J_{ij} : resonant dipole-dipole interaction $\frac{C_3(\theta_{ij})}{R_{ij}^3}$



Chain at magic angle \Rightarrow **chiral symmetry** (no A-A or B-B hopping)



Quantization axis



Conclusion

Rydberg interactions are strong & controllable

1. Resonant interaction
2. Van der Waals

$$R = 10 \text{ } \mu\text{m} \Rightarrow V_{\text{int}}/h \sim 1 - 10 \text{ MHz}$$

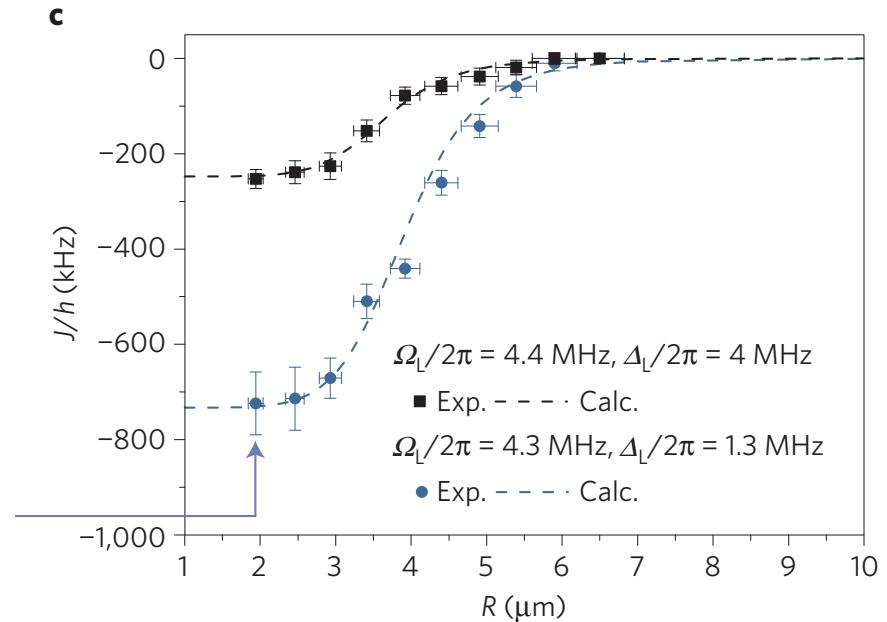
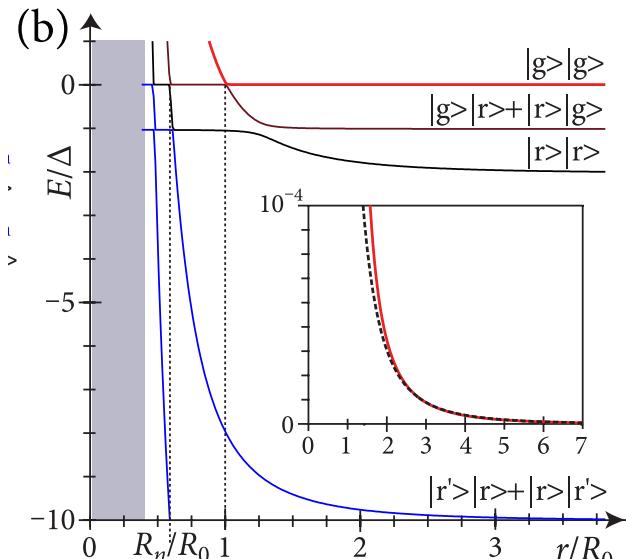
\Rightarrow timescales < μsec

Rydberg blockade useful for quantum information &
quantum optics

Control over interaction useful for many-body physics &
quantum simulation

New / other directions: tailored interactions

Bouchoule Moelmer (PRL 2002)
Pupillo, Zoller (PRL 2010)

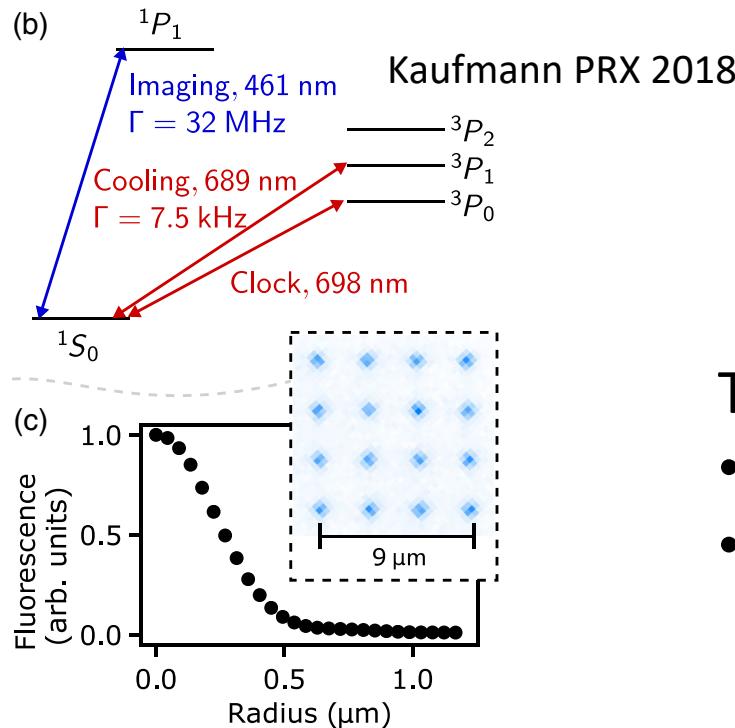


Experiments: Biedermann (Nat. Phys. 2015) 2 atoms
Zeiher (Nat. Phys. 2016) lattices
Porto (PRL 2016) ensemble + lattices

Challenging...: unwanted collective decay channels

New / other directions: 2-e atoms (Sr, Yb, Ba...)

Sr

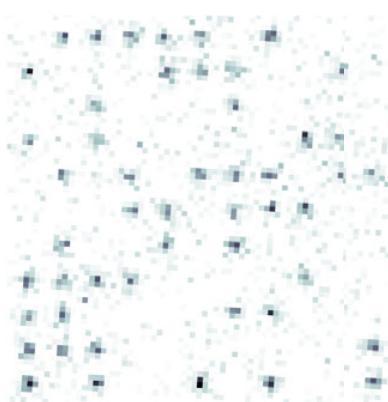


“Recent advances in Rydberg physics using alkaline-earth atoms”, Dunning, Killian, Yoshida, Burgdörfer, J. Phys. B **49**, 112003 (2016)

Two electrons = two “handles”:

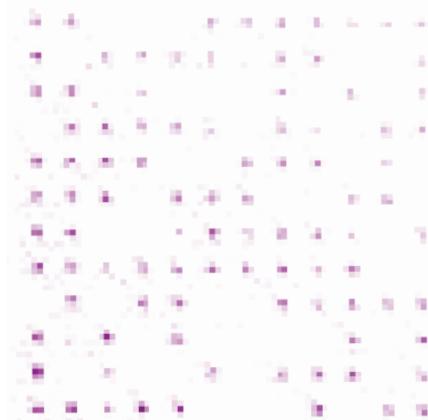
- One e- to Rydberg states
- One e- for laser trapping, or imaging

Sr



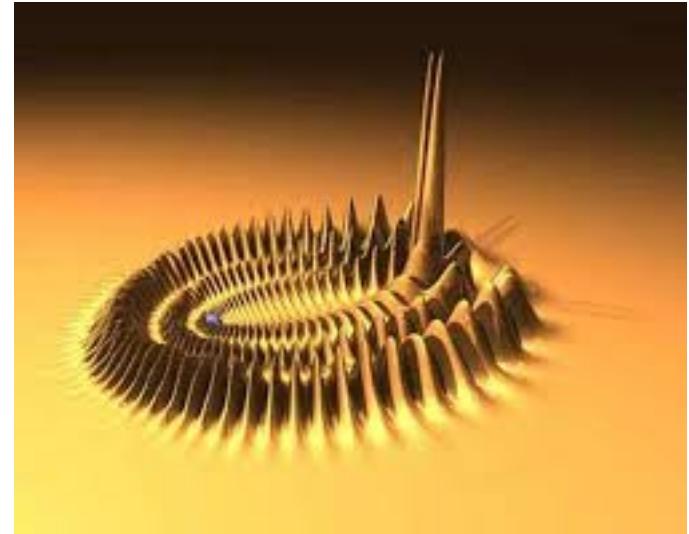
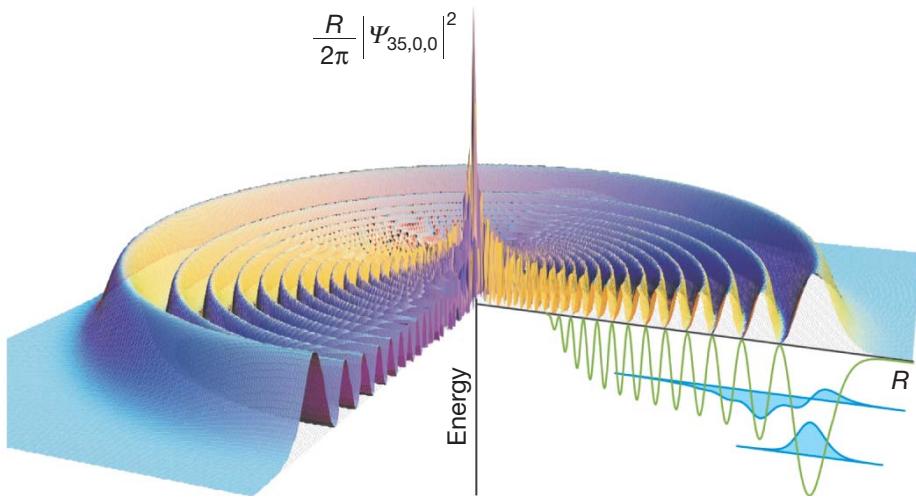
Endres PRX 2018

Yb



Thompson, PRL 2019

New / other directions: Rydberg molecules

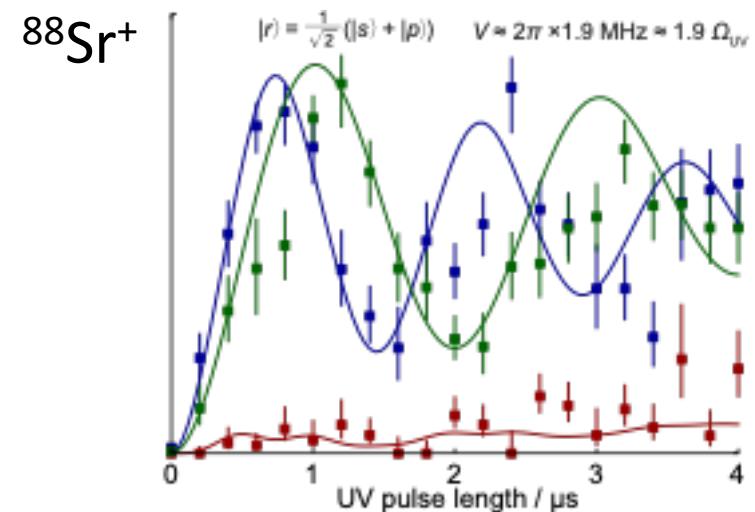
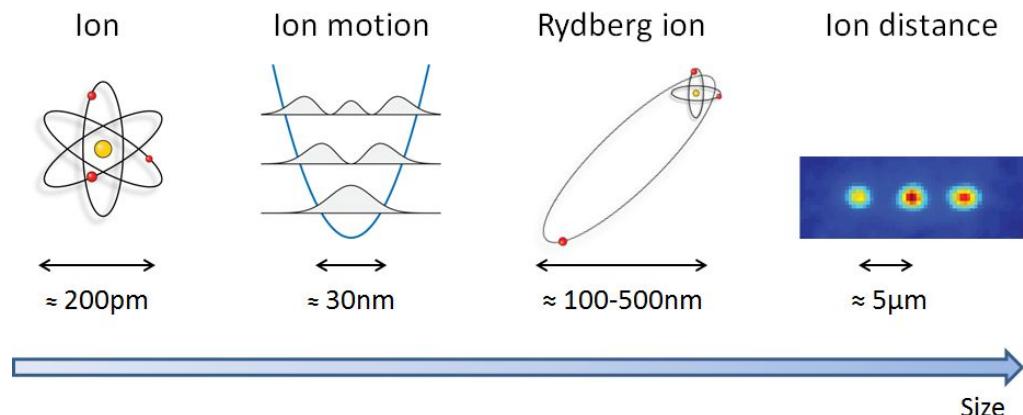
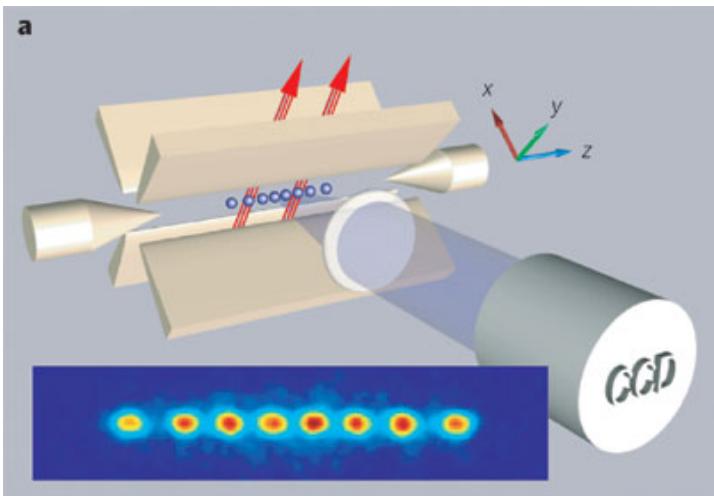


Pfau, Nature (2009)

A “new” binding mechanism beyond
Molecular (vdW) bound
Covalent bound
Ionic bound

Scattering of Rydberg e^- on atomic core [Fermi 1934...]

New / other directions: Rydberg ions



Faster gates (ns instead ms)
Scalable

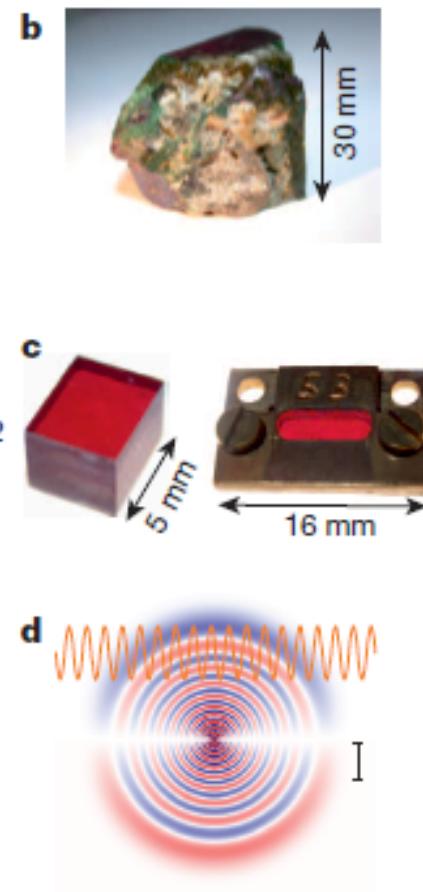
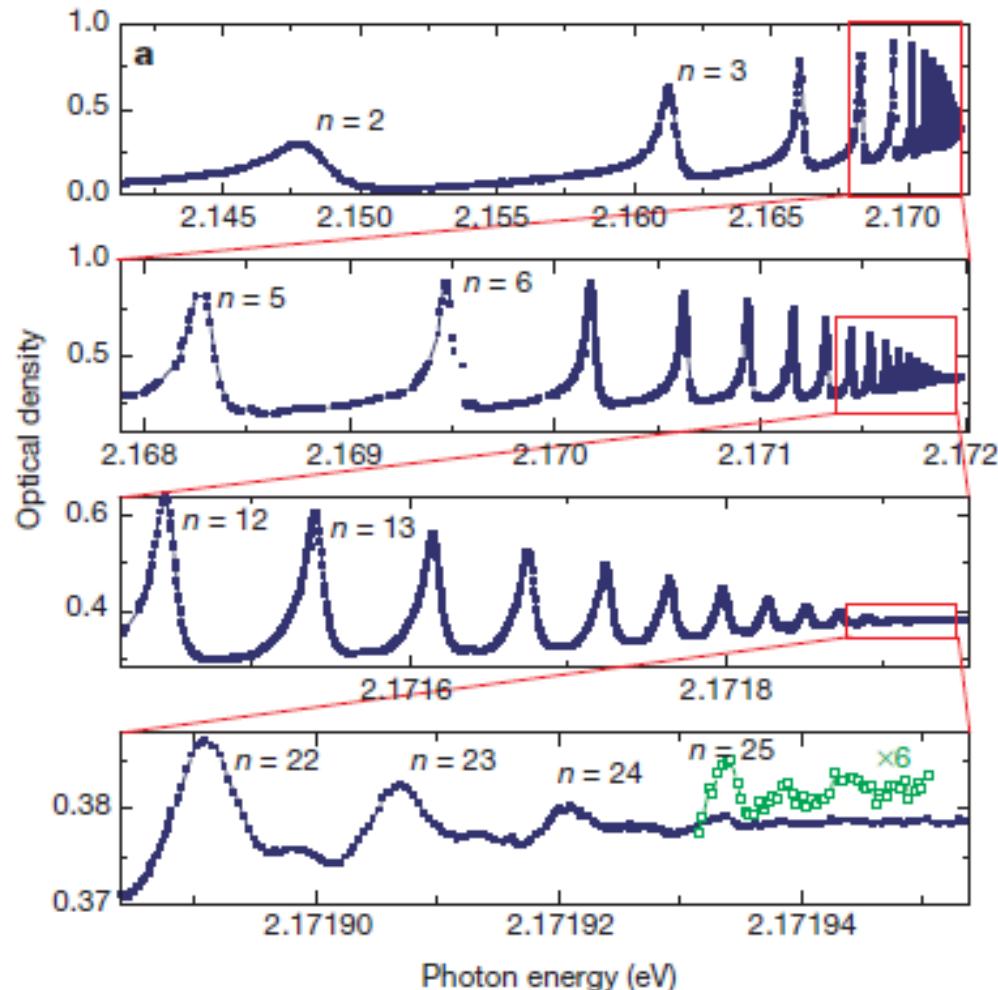
F. Schmidt-Kaller et al., NJP (2011)

F = 78%

Hennrich, arXiv 1908.11284

New / other directions: Rydberg states excitons in Cu₂O

T. Kazimierczuk *et al.*, Nature 2014



$T = 1 \text{ K}$

Evidence of blockade!

